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THE ELEMENTS

OF

MACHINE DESIGN

Part I

GENERAL PRINCIPLES, FASTENINGS, AND TRANSMISSIVE MACHINERY

BY

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SEVENTEENTH IMPRESSION

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THE THIRD SCIENCE is that of MACHINE DESIGN. This has been removed by Redtenbacher from its incorrect position as a part of Applied Mechanics, and established on a footing of its own. Its province is to show how the parts of the machine are to be proportioned so as to resist deformation.. In order to accomplish this fully, they must be considered both with reference to the external forces acting on the machine, and the corresponding molecular forces within its substance.

The former are assumed as determined by theoretical mechanics (for example, the steam pressure upon a piston, or the water pressure on the vanes of a turbine); these define the requirements of the parts as to strength. The latter, the molecular forces, transmit the force action from part to part (for example, from the piston rod to the connecting rod, or from toothed wheel to toothed wheel), and cause also friction and wear. The science of Machine Design applies the results of research in these two directions to the special problems with which it deals. When it solves these problems in accordance with technological requirements, it forms a really technical science.

REULEAUX, *Theoretische Kinematik.*

PREFACE

TO

THE PRESENT EDITION

PART I

IN re-editing MACHINE DESIGN with considerable additions, it has been found convenient to divide it into two volumes.

The present Part contains the first twelve chapters of previous editions of 'Machine Design,' except Chapter VI., which is more conveniently placed in Part II. In revising the work some discussions which appeared of comparatively little importance have been omitted, and others abbreviated. Thus room has been found for considerable additions in almost every chapter. The author has not found it necessary to alter the plan of the book or to enlarge the field it covers; but he hopes the changes now made will be found to add considerably to its completeness, accuracy, and clearness of statement.

The Second Part will contain the remainder of the original treatise dealing with machine details and especially with engine details. This part requires a proportionately greater amount of revision than the earlier part, but it is hoped it may appear in the summer.

If possible the Author would like at some time to complete the work by the addition of a Third Part, giving examples of the application of the methods and rules to complete machines.

March, 1890.

PREFACE

TO

THE FOURTH EDITION.

It may perhaps be inferred from the need of a Fourth Edition of this Treatise within less than five years from its first publication, that it has proved useful to those for whom it was intended. That an English Text-book of Machine Design was likely to be serviceable was very apparent to the author, but to write such a Text-book satisfactorily involves not only much practical knowledge, but considerable labour in comparing the dimensions which workshop experience has shown to be necessary, with those deduced on mechanical principles from the forces acting in the machine, so far as they are known. Probably no one is more aware than the author how far this Treatise falls short of what such a Treatise might be. It is pleasant, therefore, to have reason to think that, in spite of defects and omissions, it has not altogether failed in its purpose.

The author was, indeed, warned, in a not unkindly review, that from the redundancy of symbolical expressions his work would be regarded as a mere algebraical puzzle. But it appears that elementary knowledge of applied mechanics is not quite at so low a level amongst practical engineers as the reviewer supposed. When the reviewer

adds that the work, though 'unique of its kind,' 'will remain a closed book to very many practical men of little erudition but of inquiring minds, to whom it would otherwise have been of the greatest value and as good a companion as their Molesworth or other formulæ books,' it can only be replied that there may be practical engineers to whom the mathematical parts of the work will be difficult, but that is not a sufficient reason for omitting them, and that it would have been much easier to write a 'formula book,' but that it was precisely to avoid doing this that a large part of this work was written.

Long experience has enabled engineers to proportion the special machines which are daily manufactured in a very perfect way, and no great improvement is to be expected from a theoretical study of their proportions. The empirical rules, current in the workshop and drawing office, are sufficient for their design. Although some rules of this kind are given in the present Treatise, it is not merely or mainly a collection of such rules. All intelligent mechanical design must be based on a knowledge of scientific principles. If an engineer often rightly guides himself by induction from instance to instance, it is none the less desirable that a more general knowledge should control his inferences. The author's primary object was not to furnish ready-made rules of thumb, useful as these often are, but to explain the principles available as guides in machine design, by as many illustrative examples of their application as possible. The first question should always be, what are the considerations to be attended to in different cases. It is of quite secondary importance to evolve formulas which save the designer the trouble of thinking. But then the explanation of principles

does involve much that may appear obscure or difficult, to those who care only about the result and nothing about the reasoning which leads to it.

That this Treatise does contain a good many mathematical expressions is due, in part, to the fact that much has been condensed into a small space, and that the symbolical expression of the reasoning is the simplest and briefest. But the mathematics employed are, with few exceptions, of a very simple kind, and ought to present no difficulty to any one fairly acquainted with ordinary algebra and trigonometry. The use of the rules given has been made easy by copious tables.

To some it may appear that the book is too theoretical. But every engineer acts upon a theory of some kind, in proportioning machines. If a piston rod for a 48-inch cylinder is made of twice the diameter of that for a 24-inch cylinder, a theory of the piston rod's resistance is acted upon, and in fact a theory which is only approximate and safe within certain limits, for the resistance of the rod is not independent of its length, and the forces due to the inertia of the piston which act on the rod are not proportional to the piston's diameter. Hence, theory is essential to any systematic treatment of the subject, and all that ought to be required is that the theory should be accurate and free from useless refinements. It is not legitimate, or safe, to render the solutions of practical problems easier, by ignoring some essential conditions. Even when all the elements of the problem cannot in practice be taken into the reckoning, it is still important that the designer should bear them in mind.

In order to avoid constant repetition, a uniform plan is adopted, as to the units employed, which is only departed

from in a few cases for special reasons. Wherever there is no express statement to the contrary, the units adopted are as follows :—

Dimensions are in inches.

Loads or forces are in lbs.

Stresses are in lbs. per sq. in.

Fluid pressure is in lbs. per sq. in.

Velocities and accelerations are in feet per second.

Work is in foot lbs.

Speeds of rotation are in revolutions per minute, or in angular velocity per second.

Statical moments (as bending and twisting moments) are in inch lbs.

A more consistent and scientific system of units could easily be adopted, but it would involve a departure from the modes of reckoning current in the workshop.

In the second edition a somewhat stricter use of scientific terms was attempted, in Chapters II. and III. For the suggestion of this and of several other corrections then made, the author is indebted to Mr. A. G. Greenhill, M.A.

The fault of many of the terms commonly used in the workshop and in books dealing with the subject of the strength of materials is, that they are applied to express both the forces acting on a structure and the deformations which are produced. Thus, compression means in ordinary usage either the stress acting on a bar or the strain due to its action. There is a further ambiguity arising from the use of the same words for a quantity and an intensity. Thus elongation and compression are used either for the whole deformation or for the deformation per unit of length.

An attempt has been made to avoid some of the ambi-

guities arising from this double use of the same terms. The following short scheme may be useful for reference :

$$\frac{\text{Stress}}{\text{Strain}} =$$

$$\frac{\text{Tension}}{\text{Extension}} \text{ or } \frac{\text{Pressure}}{\text{Compression}} \text{ or } \frac{\text{Shearing stress}}{\text{Shearing strain}} =$$

Corresponding Elasticity.

$$\text{Also Extension} = \frac{\text{Elongation}}{\text{original length}}$$

$$\text{and Compression} = \frac{\text{Contraction}}{\text{original length.}}$$

Thanks are due to Mr. Heys, of Manchester, who read the chapter on toothed gearing, and made some suggestions noted in the text ; to Messrs. Pearce, of Dundee, who supplied data of their rope gearing ; to Messrs. Tullis, of Glasgow, who afforded information about leather belting; and to Messrs. Jackson, of Manchester, who gave information about some special forms of toothed gear.

In the present edition the chapter on riveting has been almost entirely rewritten in accordance with the results of an examination of experiments on riveted joints in the author's Report to the Institution of Mechanical Engineers. Besides additions scattered through the book, part of the theory of the strength of journals and shafts ; the method of drawing worm and wormwheel teeth ; the theory of piston rings ; the account of Zeuner's valve diagram ; and the chapters on chains, gearing chains, and lubricators, are new.

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ELEMENTS
OF
MACHINE DESIGN

CHAPTER I

MATERIALS USED IN MACHINE CONSTRUCTION

Cast Iron

1. Cast iron is a material obtained direct from the ore in a blast-furnace. It is fusible, but will not temper nor weld. According to the proportions of the charge put into the blast-furnace the pig cast iron obtained differs in appearance, in strength, and in the purposes for which it can be used. The differences of quality are due to differences of composition of the cast iron, and especially to differences in the amount and condition of the carbon it contains. In the whitest and hardest cast irons the carbon is combined with the iron. In the greyer and softer cast iron part of the carbon is separated from the iron and mixed with it in the form of particles of graphite. The quality of the iron depends in part on the amount of other constituents present, and silicon appears to have an important influence on the form the carbon takes in the cast iron.

The cast irons of commerce are generally divided into six classes. The whiter and harder cast irons are used only

for conversion into wrought iron. The greyer cast irons, classed as Nos. 1, 2 and 3, are used also for foundry purposes. The greyest iron is deficient in strength. Hence most castings are composed of mixtures of Nos. 1, 2 and 3 in proportions varied according to the judgment of the founder. The larger the casting and the stronger it requires to be, the less is the proportion of No. 1 which is used. Generally cast iron is improved by remelting.

Cast iron usually contains 3 to $4\frac{1}{2}$ per cent. of carbon. In white iron this is entirely combined with the iron. In grey iron from 0·6 to 1·5 per cent. is combined, and the remainder, 1·8 to 3·2 per cent., crystallises separately as graphite. The next most important constituent is silicon, which may vary from 0·15 to 5 per cent.

The following useful table, constructed by Mr. Turner, shows the relation of quality and composition in foundry irons :—

	Combined Carbon	Graphitic Carbon	Silicon
Greatest softness . . .	0·15	3·1	2·5
,, hardness . . .	—	—	under 0·8
,, general strength .	0·5	2·8	1·42
,, tensile strength .	—	—	1·8
,, crushing strength .	over 1·0	under 2·6	about 0·8

Cast-iron machine parts are formed by melting the pig iron in a cupola and pouring the melted cast iron into moulds.

A *pattern* is first made of the exact shape of the casting required. A *mould* is then formed from this in foundry sand or loam. Then the molten iron is poured into the mould. After solidification the sand is cleared away.

The patterns are commonly made of yellow pine, or, when small, of mahogany. Metal patterns are used when a great number of similar castings are required. As the cast iron contracts about $\frac{1}{8}$th of an inch per foot in each direction, the pattern is made larger than the required casting in that pro-

portion. The amount of contraction varies with the quality of the iron and the size of the casting, and this sometimes gives rise to much difficulty and trouble.[1] Passages and apertures in castings which are so small that the sand would not resist the scouring action of the flowing metal, are formed of loam, in wooden moulds termed core-boxes, and are baked before being used. Simple cylindrical parts can be moulded in loam, without the use of a core-box. Thus, the core of a pipe is formed of loam, plastered on to a hollow metal core-bar. By rotating the core-bar and strickling off the superfluous loam with a sharp-edged board, the exact cylindrical form is obtained. The moulds for large cylinders are formed of loam, plastered over roughly built brick cylinders, strickled to the required form and dried.

Although simple forms are more easily moulded and cast than more complex forms, the skill of the moulder enables him, when necessary, to mould castings of very complicated and difficult shapes. Hence, the cast parts of machines may be more complicated in form than those which are forged. Castings, however, do not retain an altogether sharp and accurate impression of the mould. The corners of castings are usually somewhat blunt and ragged, deep hollows partially filled up, and straight lines slightly twisted. Hence, for appearance' sake, castings should have broad and rounded surfaces with well-rounded edges and filleted hollows. Architectural mouldings are not suitable for castings.

2. Cast iron is stronger than wrought iron under pressure,

[1] The following small table gives the average contraction in casting of the materials most used in machines :—

Material	Linear Contraction	Cubic Contraction
Cast iron	$\frac{1}{97}$	$\frac{1}{32}$
Brass	$\frac{1}{64}$	$\frac{1}{22}$
Gun-metal	$\frac{1}{130}$	$\frac{1}{44}$

and much weaker under tension. Hence, it is more suitable for compressed than for stretched machine parts. Within a limited range of stress, it is tougher than wrought iron, or undergoes a greater deformation. But its range of deformation is not great. Hence, it is not so safe as wrought iron when subjected to impact.

When castings are contracted for, it is usual to stipulate that test bars shall be cast at the same time and of the same metal as the castings. These test bars are very commonly $3\frac{1}{2}$ ft. × 2 ins. × 1 inch. They are laid on supports 3 ft. apart, with the deeper side vertical, and loaded at the centre till they break. Such bars should carry from $1\frac{1}{4}$ to $1\frac{5}{8}$ tons before breaking, and will deflect before fracture from 0·2 to 0·5 inch. Generally it is desirable that the iron should be ductile, and the deflection should not be less than 0·3 inch. The tensile strength of cast iron, which is really a much better criterion of its usefulness in construction than its transverse strength, varies from 7 to $11\frac{1}{2}$ tons per square inch. Exceptionally good cast iron may have a tenacity of 15 tons or even more.

3. The special difficulty and danger in the use of cast iron is its liability to be put into a state of internal stress, in consequence of its contraction when cooling. That contraction varies with the size and thickness of the casting and with the quality of the iron. Thus it has been found that thin locomotive cylinders contract only $\frac{1}{16}$th of an inch per foot. Heavy pipe castings and girders contract $\frac{1}{8}$th inch in 12 inches or $\frac{1}{8}$th inch in 15 inches. Small narrow wheels contract as little as $\frac{1}{25}$th inch per foot, while large and heavy wheels contract $\frac{1}{10}$th inch per foot or more. If some parts of a casting contract more than others, the thick parts, for instance, more than the thin parts, the casting is twisted and strained. If some parts of a casting solidify while others are still fluid, the former attain nearly their final dimensions, while the contraction of the latter has still to be effected. That contraction therefore strains the parts already set, and

their resistance to deformation gives rise to stresses in the parts which are contracting. Thus a condition of initial stress is induced, sometimes great enough to fracture the casting without the application of any external cause, and in all cases reducing the effective strength of the casting. The danger of initial stress is less when the form of the casting is simple and the thickness uniform and not excessive. It appears that the initial stress is to some extent gradually removed by molecular yielding, the alteration going on for months after the casting is made.

Suppose a casting of the form shown at *A*, Fig. 1. The thin side would solidify, while the greater body of heat in the

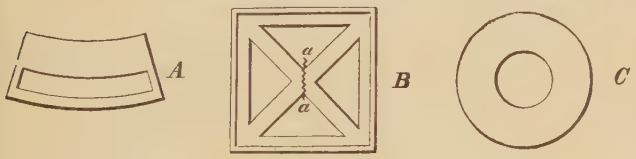


Fig. 1

thick part still retained it in a fluid condition. When the thick part contracted, it would necessarily curve the bar and induce compression in the thin part and a corresponding extension in the thick part. In a panel of the form shown at *B*, with a thin but rigid flange, the contraction of the diagonals takes place more slowly than that of the rim surrounding them, and is very liable to cause fracture at *a a*. In a thick cylinder, such as a press cylinder, Fig. 1, *C*, the outer layers solidify and begin contracting first. The contraction of the inner layers, after that of the outer layers is completed, induces pressure in the outer layers ; and the rigidity of the outer layers, causing a resistance to the contraction of the inner layers, puts them into tension. Such a cylinder will not bear so great a bursting pressure as if there were no initial strain. In fact, to obtain the greatest resistance to an internal bursting pressure, the reverse distribution of initial stress is necessary. This has sometimes

been obtained by casting the cylinder with a water core, or hollow core having a water circulation through it. The interior is then cooled most rapidly. Compression of the inner layers and extension of the outer layers is the result of this mode of cooling. Castings in the form of wheels and pulleys often give much trouble. In pulleys which have a thin but rigid rim, the rim contracts first, and the subsequent contraction of the arm breaks it by tension along the line *a a a*, Fig. 2. In some cases, however, the rim

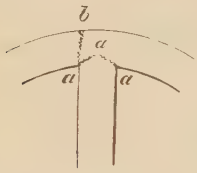


Fig. 2

Fig. 3.

breaks across near the arm, at *a b*. This appears to be due to the arms setting first. They then form a rigid abutment resisting the contraction of the rim, and bending stress is produced in the rim, causing fracture to begin outside and extend inwards.

It is because of these incalculable initial strains that cast iron is an unreliable material, where great strength is required, in structures of irregular form. The danger may be partially removed by the skill of the founder, who, by various devices, ensures as far as he can an uniform rate of cooling. But generally cast-iron structures must have excessive dimensions in order to ensure safety.

At sharp corners a plane of weakness is formed, in consequence of the way in which the crystals arrange themselves, normally to the surfaces through which heat is transmitted. This is one reason why all corners should be well rounded. Fig. 3 shows roughly the crystalline structure.

4. *Chilling.* When castings are rapidly cooled during solidification, the separation of the graphite from the iron is

prevented. The casting has then a silvery fracture and is extremely hard. To effect this chilling, as it is termed, the mould is made of a thick block of cast iron, the surface in contact with the molten iron being protected by a wash of loam. The iron mould abstracts the heat much more rapidly than a sand mould.

5. *Malleable Cast Iron.*—This is made by surrounding a casting with oxide of iron or powdered red hæmatite, and keeping it at a high temperature, for a time varying with the size of the casting, from two or three, to thirty or forty hours, or even longer. Part of the carbon is eliminated, and the casting is converted, partially or wholly, into a tough material resembling wrought iron. Malleable castings stand blows much better than ordinary castings, but they should only be hammered when cold. The decorative parts of iron-work and pinions of wheels are often thus treated.

Wrought Iron

6. *Wrought iron* is a silvery metal, fusing with difficulty, moderately hard, very strong and very tough. It is obtained from cast iron by eliminating the greater part of the carbon, and during the process passes into a pasty condition, so that it cannot be cast into an ingot. At a temperature of 1,500° or 1,600° Fahr. wrought iron softens, and can then be welded, a property of great importance and value. Wrought iron is used for parts of machines requiring strength and toughness, and such parts should generally have as simple a form as possible. Wrought-iron parts are first shaped by hammering or rolling at welding heat, and are then reduced to the exact form required by cutting tools. In some cases dies or swages are used to facilitate the forging of difficult forms. Large wrought-iron structures are built up of bars or plates riveted together. Wrought iron easily oxidises, and must be polished bright, and oiled, or painted.

The different qualities of wrought iron are commercially

distinguished as merchant bar, best iron, double best, and treble best. These terms refer to the amount of working the iron has received in manufacture, and are only rough indications of quality. 'Best iron' is iron which after first rolling into merchant bars is cut up, faggotted and re-rolled. 'Best Best' is twice faggotted and re-rolled. To ensure a given quality the iron used should be tested. Its strength is usually determined by subjecting it to tensile stress. Its ductility and toughness may be deduced from its elongation and contraction of area before rupture. Workmen test its toughness by bending it over a sharp corner with the hammer.

7. The forms in which wrought iron is most easily procured are the following :—

Bar iron. Round bars $\frac{1}{8}$ in. to 7 ins diameter. Square iron up to 5 ins. or 6 ins. each side. Flat iron from $\frac{1}{4}$ in. thick and $\frac{1}{2}$ in. to 6 ins. wide, to $1\frac{1}{2}$ in. thick and 3 to 10 ins. wide. Lengths usually 20 to 30 ft.

Plates $\frac{3}{16}$ to 1″ thick, and usually not exceeding 24 sq. ft. area. Angle iron, Tee iron, and double Tee iron, in bars, usually not exceeding 8 ins. in the sum of the widths.

Various other forms are made, as half-round iron, channel iron, grate bar iron.

The quality of wrought iron varies greatly, and for some purposes strength is most important, while for others capability of being worked under the hammer without cracking or losing strength is more important. The following is a rough classification of the qualities usually met with :—

(*a*) Iron easily worked hot, and hard and strong when cold ; used for rails.

(*b*) Common iron, used for ships, bridges, and sometimes for shafting.

(*c*) Single, double, and treble best iron, from Staffordshire and other parts, where similar qualities are made. The single or double best is used for boilers. Double and treble best are used for forging.

(*d*) Yorkshire iron, from Lowmoor, Bowling, or other forges where only fine qualities are made. The best Yorkshire iron is very reliable and uniform in quality. It is used for tyres, for difficult forgings, for furnace plates exposed to great heat, for boiler plates which require flanging, &c.

(*e*) Charcoal iron. Very ductile, and of the best quality.

Wrought iron elongates about $\frac{1}{10000}$th of its length for each ton per sq. in. of tension up to the limit of elasticity. Beyond that limit it elongates much more, the greater part of the elongation being permanent. When a bar is broken by tension the transverse area of the bar is reduced in the neighbourhood of the fracture and the contraction of area is sometimes taken as a measure of the ductility or toughness of the material.

The ultimate tensile strength of wrought iron ranges from 18 to 28 tons per sq. in. Generally bar iron is stronger than plate iron, and bars of simple round or square section stronger than L, T, I, or other more complex sections. The following short table gives the ultimate strength and the ductility, as indicated by the contraction of area at the fractured section :—

	Tenacity. Tons per sq. in.	Contraction of area per cent.
Bar iron	from 24 to 22	40 to 20
Plate iron, best, in direction of rolling .	23	25 to 20
„ „ across „ „ .	20	15 „ 12
„ ordinary, in direction of rolling	21	15 „ 12
„ „ across „ „ .	19½	9 „ 7

8. *Case Hardening.*—The surface of wrought iron may be hardened by partially converting it into steel. This can be effected to a slight extent by making the surface bright, heating it to a red heat, then rubbing it with prussiate of potash, and quenching in water. It is far more completely

effected by heating the iron in a close box, filled with bone dust and cuttings of horn and leather.

9. *Cold Rolled Iron.*—Wrought iron rolled cold under great pressure gets a smooth polished surface, and is found to have a greatly increased tenacity. Its ductility and toughness are, however, much diminished. Hammering iron when cold produces a similar effect. Annealing, or heating the iron to red heat and allowing it to cool slowly, restores it to its original condition.

All mechanical compression of iron when cold appears to increase its strength at the expense of its toughness and ductility. On the other hand, annealing reduces the strength, but increases the ductility and toughness. In rolling or hammering when hot, mechanical compression and annealing are going on simultaneously.

Steel

10. *Steel* is pure iron combined with a proportion of carbon, silicon, manganese, phosphorus and other constituents. If the pure iron were combined with carbon alone, the properties would depend directly on the percentage of carbon. But in actual steels the case is much more complex. Silicon, manganese, phosphorus, and some other constituents influence its physical properties and to some extent produce the same effect as carbon. The mildest steels contain 0·15 to 0·4 per cent. of carbon. The hardest 1·2 to 1·6 per cent.

The commercial term 'steel' includes widely different materials. Steel containing little carbon (less than $\frac{1}{2}$ per cent.) resembles wrought iron and has now superseded it for many purposes. Like wrought iron it welds easily and is not hardened when heated and suddenly cooled. This steel would be most conveniently termed 'ingot iron,' so as to mark the fact that it has been cast into a malleable ingot, while, like wrought iron, it does not temper or harden. Steel with

somewhat more carbon is harder and stronger and is used for tyres and rails. Steel containing one per cent. of carbon or more has the striking property of hardening when heated and suddenly cooled and of softening again if heated and slowly cooled.

With good steel of this character, any desired hardness can be obtained either by heating to some definite temperature and suddenly cooling it, or more commonly by first heating it strongly and cooling so as to make it excessively hard, and then tempering it or letting down the temper. The steel is reheated to a temperature indicated by the colour of the oxide which forms on its surface (straw-yellow to blue). It is then cooled in water or oil. This property of hardening makes steel suitable for tools. In its hardened state it has great strength, a high elastic limit, and little ductility.

Steel is more fusible than wrought iron and can be cast in moulds like wrought iron. At first great difficulty was found in obtaining sound castings, and the great contraction of steel in solidifying gives rise to excessive internal stresses. The difficulties in obtaining good steel castings have been for the most part overcome. By using silicon, aluminium and other constituents, the casting is rendered sound ; and by prolonged annealing, the internal stresses are destroyed. Sir J. Whitworth introduced a process of casting under pressure which secures a very admirable quality of steel.

In welding steel it is important that the pieces to be united should contain the same amount of carbon. If they do not, their welding temperatures are different. Steel requires more care in smithing than wrought iron, and it is more liable to injury if worked at an improper temperature. All straining of the material when cold by hammering or punching injures steel more than wrought iron, but the damage is entirely removed by annealing.

Bauschinger found the tenacity of steel to increase with the amount of carbon nearly in the following proportion :—

$$\text{Breaking strength} = 27{\cdot}6\ (1 + C^2) \text{ tons per sq. in.}$$

where C is the percentage of carbon. But the constituent of steel vary so much that no single rule will cover all cases.

In steel plates the strength in the direction of rolling and across it is almost exactly the same. One great advantage of steel over wrought iron in construction is that plates of much greater area and weight and bars of much greater length can be obtained without extra cost. Steel plates of 70 sq. ft. area or 15 cwt. in weight are obtained with little difficulty, and rail bars are rolled up to 150 feet in length.

The following are the qualities of steel most used in construction :—

	Tenacity. Tons per sq. in.	Contraction of area per cent.
Best quality, hard	38	25
„ average	35	35
„ ductile	29	45
Ordinary quality, hard	35	20
„ „ ductile	28	30

In compression the harder qualities are probably preferable.

Good hard cast steel, capable of tempering, may have a tenacity of 50 tons per sq. in. before hardening.

Copper.

11. *Copper* is a reddish metal, of inferior tenacity to wrought iron, but of greater ductility. It can be cast, but is seldom used in that way, as it fills the mould badly, and the castings are porous and unsound. It is usually rolled into plates and hammered to shape. It can be welded if proper precautions are taken, or joints in this material can be made by a process of brazing so as to be little inferior in strength to the solid sheet.

Copper castings have a tenacity of 10 tons per sq. in. ; sheet copper about 13½ tons per sq. in. ; annealed copper

wire about 18 tons per sq. in. It is an expensive material, and is chiefly used for pipes which require to be bent cold, for bolts in positions where corrosion must be prevented, and for firebox plates where its ductility and power of resisting great heat are of value. After hammering cold it loses ductility and requires to be annealed. Its tenacity diminishes with increase of temperature to a greater extent than is the case with wrought iron.

Bronze or Gun-metal

12. *Bronze* or *gun-metal* is harder and less malleable than copper. It is fusible, and makes excellent castings. It varies in quality according to the proportion of tin. Thus :—

Soft gun-metal contains .	8 tin to	92 copper
Hard gun-metal . . .	18 „	82 „
Bell metal, from . . .	$23\frac{1}{2}$ „	$76\frac{1}{2}$ „
to . . .	23 „	77 „

Some zinc is often added to facilitate casting. Ordinary bronze is not uniform in texture. Whitish spots of alloy, rich in tin, are distributed through the mass. It has been found that when it is rapidly cooled after casting the composition is more uniform, the density greater, and the strength and toughness are increased. This rapid cooling is best effected by using thick cast-iron moulds or chills, the process being analogous to the chilling of cast iron. The best alloy for guns contains 8 to 10 parts of tin and 100 of copper. Such an alloy, when cast in sand moulds, breaks with about 11 tons per sq. in. of tension, and its limit of elasticity is reached at 5·6 tons. Cast in chills, its tenacity is 17·6 tons, and its limit of elasticity is raised to 6·7 tons per sq. in.

The friction between bronze and wrought iron is moderate and regular, and the bronze being softer wears most rapidly. Hence it is very suitable for the steps upon which

rotating pieces are supported. Gun-metal for bearings often contains 82 per cent. of copper and 18 per cent. of tin. The softest bronze is used for cocks and small fittings. Bronze is tougher than cast iron, and is sometimes used for gearing subjected to shocks.

Brass

13. *Brass* contains from 66 per cent. copper and 34 per cent. zinc to 70 per cent. copper and 30 per cent. zinc. A little lead is often added. Common brass for cheap brass-work contains a larger proportion of zinc. Muntz metal, which can be rolled hot, contains 60 per cent. of copper and 40 per cent. of zinc, or sometimes 66 per cent. of copper, 33 of zinc, and one of lead. It is used for sheathing-plates for ships and for the tubes of locomotives. Brass is extensively used on account of its easy working and good colour. It is cheaper but less strong and tough than gun-metal. The tenacity of brass is from 8 to 13 tons per sq. in., and that of Muntz metal somewhat greater.

Other Alloys

14. *White Brass.*—Various alloys have been used for bearings containing large proportions of tin or lead. The alloys in which tin is the chief ingredient contain 40 to 90 parts of tin, 5 to 17 parts of antimony, and 1·5 to 17 parts of copper. Alloys in which lead is the chief ingredient contain 66 to 88 parts of lead with 4 to 20 parts of antimony and 12 to 20 parts of tin. The object of trying these very various alloys is to obtain a metal for bearings which is cheaper and softer than ordinary gun-metal, and which works with less friction. The friction depends on the way in which the step wears. If it is soft and of uniform texture, and wears with a smooth and polished surface, the friction may be expected to be small.

The following table gives the composition of some alloys which have been used for railway and other bearings :—

	Parts by Weight								
Lead . . .	70	—	42·5	37·5	—	—	—	84
Zinc . . .	—	82	42·5	—	—	—	—	—
Tin . . .	—	—	—	37·5	66·7	90	85	—
Antimony . .	20	11	15	25 0	11·1	7	10	16
Copper . .	10	7	—	—	22·2	3	5	—

Some of these alloys are fusible at a low temperature, and are cast in position round a smooth mandril. Then they do not require turning. Most of them are too soft to be used for large bearings ; in such cases, a thin sheet of the alloy is cast in recesses in an ordinary gun-metal step. One objection to very soft alloys is that they crush and clog the oil channels.

Phosphor Bronze. – Under this name an alloy has been introduced composed of copper and tin with a small proportion of phosphorus, which is of great service in machine construction. It appears to owe its properties in part to the great care exercised in its manufacture and the accurate proportions of the constituents. Its qualities can be varied at will, so that it may be either very strong and hard, or, with less strength, ductile and very tough. Unlike ordinary bronze, it can be re-melted without deterioration of quality.

As to its strength and ductility, various tests show a tenacity of from 22 tons per sq. in. in the softer qualities to 33 tons in the hardest. The elastic limit of the former is about 5 tons, and the latter 25. The former elongate 30 per cent. or more before fracture, and the latter 3 to 4 per cent. The contraction of area at fracture ranges from 4 to 30 per cent. Unannealed wire (16 B. W. G.) broke with from 102 tons per sq. in. to 151 tons per sq. in., and the same wire after annealing carried from 48 to 74 tons per sq. in.

In some Belgian experiments, railway axle bearings of

phosphor bronze were found to wear much longer than gun-metal bearings, and this bronze has also been used for the large crank bearings of marine engines, where ordinary gun-metal has failed. Its great strength and toughness render it especially suitable for gearing subjected to shocks. It has been used in place of steel for tools in gunpowder factories, and it can be drawn into wire and used for rigging and perhaps for wire-rope belting.

Silicium Bronze has a much higher electrical conductivity than phosphor bronze and at the same time equal strength. It is therefore very suitable for telegraph wire in towns, where its resistance to corrosion is of great value.

Manganese Bronze is now made by introducing a proportion of ferro-manganese in bronze or brass. The manganese, like phosphorus, clears the alloy of oxide. Several qualities are made. No. 1, containing a good deal of zinc, can be forged or rolled hot. Simply cast it has a tenacity of 24 tons per sq. in. When rolled it has a tenacity of 28 to 32 tons and an elastic limit of 15-23 tons per sq. in. No. 2 is stronger and is used for castings. No. 3, which is without zinc, replaces ordinary bronze and has great strength and toughness. It has been used for large bearings and for screw propellers.

Delta Metal. This is a brass containing a proportion of iron. It can be worked hot or cold, or brazed. Cast in sand its tenacity is 21 tons per sq. in. ; after forging 33 to 35 tons per sq. in. It is specially useful for forming special shapes by hot stamping.

Aluminium Bronzes.—Bronzes with a small proportion of aluminium have been made of great strength and toughness.

Effect of High Temperatures on the Tenacity of Materials. —As parts of machines are exposed to high temperatures—in the case of boiler shells and valves exposed to high-pressure steam, for instance—the influence of temperature on the strength cannot always be overlooked. Fortunately, in the case of iron and steel at temperatures ordinarily reached,

the strength is not seriously impaired. Most experiments show for iron or steel a small gain of tenacity between 60° and a temperature of 400° or 500° F. This gain amounts in extreme cases to 12 to 20 per cent. At temperatures above 600° the strength diminishes rapidly with increase of temperature, about 50 per cent. being lost at temperatures of 1,000° or 1,100°.

In the case of copper and the copper alloys a much more marked influence of temperature is observed. If f is the tenacity in tons per sq. in. at a temperature $t°$, F., then the author has found approximately

$$f = a - b\,(t - 60)^2$$

where a and b are constants.

	$a =$	$b =$
Copper	14·8	·000014
Rolled yellow brass . .	24·1	·000028
„ delta metal . .	31·3	·000041
„ Muntz metal . .	24·7	·000029
Cast gun-metal . .	12·5	·000050
„ brass . . .	12·5	·000024
„ phosphor bronze .	16·1	·000026

Tenacity of wire. When a metal is drawn down to a comparatively small section, its strength per unit of section increases. This is more apparent in unannealed than in annealed wire. Karmarsch has found that the strength in lbs. is given nearly by the equation

$$P = a\,d + b\,d^2$$

where d is the diameter in inches.

	Unannealed Wire		Annealed Wire	
	a	b	a	b
Steel.	1160	69000	165	62000
Iron, from . . .	690	69000	165	36000
to	1000	50000	275	31000
Brass	440	60000	300	31000
Copper	410	38000	0	25000

15. *Protection of Iron from Corrosion.*—One of the difficulties in the use of iron or steel is the corrosion to which these materials are liable. In many cases, as, for instance, in steam boilers, the corrosion, if allowed to proceed, may greatly weaken and endanger the structure. The corrosion is most rapid on surfaces which are alternately wet and dry, and less rapid on surfaces entirely covered by water. Cast iron obtains in the sand mould a covering of silicates which, if unbroken, is less liable to corrosion than clean surfaces of the metal. Cast iron and steel are more rapidly attacked in sea-water than wrought iron. The acids present in some woods (as, for instance, oak) cause rapid corrosion of iron in contact with them. Hence, in oak copper bolts are generally used. The modes of protecting iron from corrosion are as follows: (1) Heating the iron to 310° F. and immersing it in a bath of pitch maintained at a temperature of at least 210°. A little oil is generally added to the pitch. This process, known as Dr. Angus Smith's, is commonly employed for protecting water pipes. The pitch used is coal tar, from which the naphtha has been removed by distillation. (2) A tar varnish for application to surfaces which cannot be heated, consists of tar with a little tallow and resin. (3) Painting with oil paint, especially with paints which have oxide of iron as a basis. (4) Certain transparent varnishes are manufactured which protect clean iron surfaces without altering their appearance. (5) Mr. Barff protects iron by forming on its surface a coating of magnetic or black oxide of iron. This is effected by subjecting the iron for some time to the action of superheated steam at a high temperature. (6) Temporary protection is obtained by a coating of tallow. (7) The most complete protection is obtained by immersing the iron in a bath of melted zinc, a process which is termed 'galvanising.'

Where iron is in contact with a metal electro-negative to it, and both are immersed in water, there is a voltaic action which causes rapid corrosion. If the water contains acids,

as is the case sometimes with the feed water of boilers, the action is still more rapid. The irregular corrosion known as pitting and furrowing, is probably due to portions of the surface exposed being electro-negative to others, either from want of homogeneity in the material or from other causes. On the other hand, if a metal electro-positive to iron is placed in contact with it, the iron is protected from corrosion. Thus boilers are now sometimes protected by suspending inside them blocks of zinc. The zinc gradually disappears, but the iron is protected.

Machine Drawing.—All machines are designed as drawings from which the actual machines are constructed, the business of the workman being to carry out accurately the instructions shown by the drawing. Mechanical drawing is for the machine designer a kind of written language, and it is by means of the drawing that he makes his ideas intelligible. On the drawing the engineer studies the relative motion of the parts of his machine, determines their size to ensure strength, arranges them so that they do not interfere with each other, and considers all the modifications which tend to greater efficiency of working or less cost of manufacture. The drawing also serves to indicate how the work is to be distributed to different shops and to indicate the order in which the several parts will be required. Class letters and numbers on the drawings, which can also be placed on the work if necessary, serve to prevent confusion, and facilitate the keeping of time and cost records.

Shop Processes.—Pattern-making is the making of wood models of those machine parts, of brass or cast iron, which have to be cast. The pattern-maker has to consider the allowance to be made for shrinkage in casting and for turning, boring, or other subsequent operations of manufacture. He has also to consider very carefully how the moulding of the machine part from the pattern can be most cheaply carried out, and to arrange the pattern accordingly. Parts of the pattern must be left loose or

moulding is impossible. Parts can only be moulded by the use of cores. Parts which have to be finished by cutting tools must be so placed that they are not likely to be unsound from blow holes or dirt. *Moulding* and *Founding* or *Casting* are the operations of forming the sand or loam mould of a machine part, and running into it the melted iron or brass. In green-sand moulding the mould is made from a complete wood pattern. In loam moulding the pattern is strickled to shape by boards which have the form of a generator of the surface required. Whether of sand or loam, the mould must be excessively porous to allow the escape of the steam and gases generated by the heat of the melted metal. The founder requires to know how to produce different qualities of metal, softer or stronger, by mixing different proportions of pig iron of different brands.

Forging is the operation of shaping wrought iron or steel, materials which become plastic without fusing. To a large extent forging consists of welding on piece by piece and shaping gradually the large pieces required. In smith work, the pieces being small, the forging is done by hand ; in larger forgings machine driven or steam hammers are required. In heavy forging the highest skill is required to arrange the operation so that the material is not injured, and the mass after the operation is homogeneous and sound. *Fitting and finishing* are the operations of cutting to accurate dimensions the rough products of the foundry and forge and fitting them together. The cutting is done by steel tools which cut the metal when cold. Cutting operations include chipping and filing, boring, drilling, turning, planing, shaping and milling, and also in some cases abrading operations such as grinding. The surfaces most easily obtained by cutting operations are cylindrical surfaces and plane surfaces, and most ordinary cutting operations are intended to form truly cylindrical or truly plane surfaces. Conical and screw surfaces are, however, also produced, and occasionally other forms. In the best work the dimensions

of fitted work are determined by standard gauges, which ensure far greater accuracy than any system of measurement.

Sectional Shading

Fig. 4 shows the sectional shading adopted in this treatise to indicate the materials most commonly used.

Fig. 4

CHAPTER II

ON THE STRAINING ACTIONS TO WHICH MACHINES ARE SUBJECTED

16. The forces acting on the parts of a machine comprise :—

(1) The useful load due to the effort transmitted from the driving to the working point to accomplish useful work. (2) Prejudicial resistances due to friction in the machine itself or work expended otherwise than at the working point. (3) The weight of parts of the machine. (4) Reactions of inertia due to changes of velocity of parts of the machine. (5) Centrifugal forces due to changes of direction of motion of parts of the machine. (6) Occasionally there are stresses due to constraint preventing expansion with changes of temperature. The total action on any member of a machine of these forces may be called the straining action on that member.

In each part of a machine the straining action varies with the fluctuations of the useful load and with the variations of position and velocity of the different parts of the machine. Each member must be capable of resisting the maximum straining action on that part of the machine. For each part of the machine, therefore, it is necessary to consider under what conditions the straining action is greatest. If in consequence of changes of position or velocity the straining action produces stresses of different kinds at different times, the member must be capable of sustaining the maximum stress of each kind. Lastly, as will be more fully explained presently, the amount of variation of the straining action

affects the endurance of the material, and therefore requires also to be considered.

17. *Ultimate or Breaking Strength.*—If the straining action on a bar is gradually increased till it breaks, the load which produces fracture is termed the ultimate (statical) or breaking strength of the bar. Many experiments on the breaking strength of bars of different materials, subjected to different kinds of straining action, have been made. From these it is possible to infer, by the rules of applied mechanics, the statical breaking strength of any member of a machine.

Now obviously the ordinary working load on a machine part must be less than the breaking load. Practical experience has shown that it must be very considerably less. But in general it is not possible to determine by direct experiments what should be the working load, while it is easy to determine directly or from recorded experiments what is the breaking strength. Hence has arisen the custom of ascertaining the statical breaking strength of machine members and dividing it by a factor, termed a *factor of safety*, to find the proper working load. The factor of safety has been ascertained by comparing the working and breaking strength in actual cases. Some machine members have been subjected to a given straining action for a long time and have shown no sign of failing. Others have broken down. In the former cases the factor of safety must have been sufficient, in the latter it must have been too low. By comparison of such cases values of factors of safety suitable for different materials and in different circumstances have been ascertained.

The following is a table of such factors of safety. They are ratios of the statical breaking stress under a load steadily and gradually increased till fracture occurs to the ordinary working stress. They include some allowance for contingencies which may cause the actual stress to be greater than the intended working stress. But where the straining action can only be very imperfectly known larger factors of safety must be adopted.

TABLE OF FACTORS OF SAFETY

Material	Factors of Safety for			
	A dead load	A live or varying load producing		In structures subject to varying loads and shocks
		Stress of one kind only	Equal alternate stresses of different kinds	
Cast iron	4	6	10	15
Wrought iron and steel .	3	5	8	12
Timber	7	10	15	20
Brickwork and masonry .	20	30	—	—

In cast iron the factors are higher to allow for unknown internal stresses due to contraction in casting. In timber they are higher because a gradual increase of deformation goes on in course of time. In brickwork and masonry the weakness of the joints has to be allowed for, and the distribution of stress is probably very irregular.

It will be seen that the determination of the working strength by dividing the breaking strength by a so-called factor of safety is a purely empirical method. It remains to be seen whether a study of the properties of materials will furnish a more rational and satisfactory method.

18. *Steady or Dead Load, and Variable or Live Load.*— A steady load is one which is invariable during the life of the structure, and which produces a permanent and unvarying amount of straining action. The weight of a fixed part of a machine is such a dead load. A variable or live load is a load which is alternately imposed and removed, and which produces a constantly varying amount of straining action. A steady load can generally be very exactly estimated, and when the load is entirely of this kind, a comparatively low factor of safety affords a sufficient guarantee

of security. A live load is often less easily estimated, and a load of this kind produces much more injurious effects on a structure than a dead load of the same amount. Hence, for a double reason, a higher factor of safety must be used for a live than for a dead load.

A suddenly applied load is a load imposed on an unstrained structure, without velocity, but at one instant. Practical cases rarely approximate to these conditions. Such a load accumulates, in deflecting or elongating the structure, a certain amount of energy of motion, which is ultimately expended in increasing the deformation beyond the amount due to a steady load. If the stress does not exceed the limit of elasticity of the material, a suddenly applied load produces twice the stress which would be produced by the same load gradually applied or resting on the structure.

If the load impinges on the structure with an amount of energy of motion previously accumulated, the stress produced will exceed that due to the same load applied steadily, to an extent which depends on the original energy of motion. Such a load may be termed an impulsive load.

Strain and Stress.—Every load which acts on a structure produces a change of form which is termed the *strain* due to the load. The strain may be either a vanishing or elastic deformation, that is, one which disappears when the load is removed ; or a permanent deformation or set, which remains after the load is removed. In general, machine parts must be so designed that, under the maximum straining action, there is no sensible permanent deformation.

The molecular actions within the material, which are called into existence by external forces or loads, and which resist deformation, are called *stresses.*

19. *Elastic and Plastic Condition of Materials.*—An elastic material is one which, though it is deformed by any straining action, recovers its original condition if the straining action is removed. A plastic material is one which, when deformed by a straining action, does not recover when the straining action

is removed. Speaking broadly, any strain or deformation consists of two parts—one, an elastic deformation which is proportional to the straining action, and which vanishes if the stress is removed; the other, a plastic deformation or permanent set. In all materials ordinarily used in construction, the elastic deformation is small, and though not to be neglected, yet produces no alteration of figure or dimension which the engineer cannot provide against. The plastic deformation, however, may be very large and prejudicial. Further, for most ordinary materials there is a certain range of straining action for which the deformation is wholly, or almost wholly, elastic. But for greater straining action the deformation is largely plastic or permanent. Hence it is common to say that the working stresses must not exceed the limits within which the deformations are almost wholly elastic, or must be such as to produce no sensible permanent set.

Relation of Stress and Strain. Stress Strain Diagram.—Much of the material an engineer uses is almost perfectly elastic with moderate stresses and very plastic with greater stresses. Suppose a bar of wrought iron or mild steel subjected to a gradually increasing stress, and suppose the strains corresponding to each stress are accurately observed. The relation between stress and strain during such a test may be shown by a diagram, which may be called a 'stress strain diagram,' such as fig. 5.

Let a pair of rectangular axes be taken and let stresses be plotted as ordinates, strains as abscissæ. Then the relation between stress and strain will be indicated by points along a curve. For convenience we may plot tensions upwards, pressures downwards, extensions to the right, compressions to the left. The curve for a material such as wrought iron or mild steel is similar to *g f O a b c d e*. Attending at present to tension only, for a certain range of stress, the curve is almost exactly a straight line, *O a*; that is, for stresses which do not exceed the value *a a*, the strain

is proportional to the stress and the strain is wholly temporary or elastic strain. At the stress indicated by the point *a*, the proportionality sensibly ceases. The strains increase faster than the stresses, and part of the strain is permanent strain or set. The point *a* is termed the *limit of proportionality* or true elastic limit of the material in the given conditions. For rolled or hammered materials there is often, but not always, a second well-marked point, *b*, where the material takes a large increase of strain without any increase

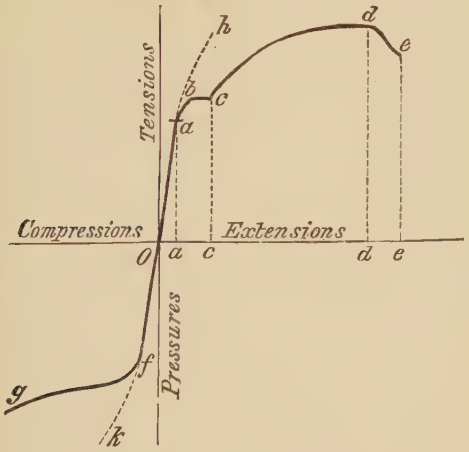


Fig. 5

of stress. This point is the *yield-point*, and is often commercially called the 'elastic limit.' It is a point where the defect of elasticity can be first very easily observed, even by very rough instruments. At the point *d* the maximum stress is reached. With greater stresses, a local drawing down of the material occurs which reduces its section so that it will no longer sustain so great a stress. Finally, the bar breaks with the stress *e e*, the total extension being then *O e*. It is well to note that the so-called breaking stress of

a material is the stress $d\,d$ reckoned per unit of area of the primitive section of the bar. The real stress as the section reduces, reckoned per unit of reduced section, increases to the end of the test, and is much greater than the so-called breaking stress. For compression the curve is like $O\,a\,f\,g$, with a less well-marked yield-point than in tension. In compression the area increases as the block is compressed, and hence the straining action increases up to the end of the test.

For cast materials like cast iron, the stress-strain curve is like $k\,f\,O\,a\,h$, with no well-marked elastic limit and no great amount of permanent or plastic elongation. Such materials are said to be brittle or wanting in toughness.

20. *Working Limits of Stress.*—There is no clear agreement at present as to the principle on which the greatest safe-working stress should be fixed. The condition that any large deformation is practically inconvenient fixes the yield-point, for such materials as have a strongly marked deformation at that point, as a limit of stress which should not be exceeded. But many materials have no such yield-point. Next the limit of proportionality has been fixed on as marking the safe limit of working stress. It is clear that, if the working stress produces only a temporary or elastic deformation, then the material is unchanged by the stress, and may be expected to bear any number of repetitions of the stress.

Hence most of the older writers fixed on the elastic limits a and f on the diagram as marking out limits to the stresses which would be safe. The stresses corresponding to the points a and f are called the elastic strengths of the material. The small defects of elasticity observable with very refined measurements with stresses less than the elastic strength, and which generally disappear after a few successive loadings, may, with a good deal of probability, be ascribed to initial defects of homogeneousness in the material.

Unfortunately, however, so simple a view of the limits of

safe stress can no longer be held without modification. The 'limits of elasticity determined in a new bar are found to be changed if the bar is subjected to continued variations of stress. Application of stress of one kind raises the elastic limit for stress of that kind and lowers the elastic limit for stress of the opposite kind. When a material is subjected to opposite stresses alternately, it is not safe, if the range of stress is as great as that from *a* to *f*. On the contrary, with such a range of stress, the bar actually breaks after a not very considerable number of repetitions of loading.

21. *On the Peculiar Action of Live Loads.*—The researches of Wöhler, since repeated by others, show that the safety of a structure, subjected to a varying amount of straining action, depends on the *range of variation* of stress to which the structure is subjected, and on the number of repetitions of the change of load. It has been hitherto assumed that it depends only on the maximum intensity of the stress ; but this must now be considered to be erroneous. Every machine, subjected to a constant variation of load, must be designed to resist a practically infinite number of changes of load. In order that it may do so, the greatest intensity of stress must be less than for a steady load, and less in some ratio which depends on the amount of variation the stress undergoes in its successive changes.

A steady load has already been defined as one which remains invariable during the life of the structure. Let the intensity of stress required to fracture a given material under a steady load be denoted by K, so that K is what is commonly termed the breaking strength of the material. In designing a machine part to sustain a steady load, the greatest safe stress is generally taken at about $\frac{1}{2}$ to $\frac{1}{3}$ K. Here the factor 2 or 3 is a real factor of safety which allows for unknown contingencies. With a live or variable load it has been usual to take a higher factor of safety, and to restrict the working stress to $\frac{1}{4}$ or $\frac{1}{6}$ K, or to some other limit, ascertained by practical experience in special cases.

Wöhler's researches show that this is not a scientific way of dealing with the question. Suppose that under the action of the live load the stress varies from $k_{\text{max.}}$ to $k_{\text{min.}}$, and that the range of variation $= \Delta = k_{\text{max.}} - k_{\text{min.}}$ In using this expression, if tensions are reckoned positive, pressures must be reckoned negative, so that, if the two stresses are of different sign, the range of stress is equal to their sum $[k_{\text{max.}} - (-k_{\text{min.}}) = k_{\text{max.}} + k_{\text{min.}}]$. Let the number of changes of load be indefinitely great. Then Wöhler's researches show that fracture will occur, for some value of $k_{\text{max.}}$ less than κ, and so much smaller, the greater the range of stress Δ. That is, the breaking strength for the live load is less than κ. Hence, in designing a structure for such a varying load, the ultimate strength is to be taken at some value k less than κ, which is determined with reference to Δ.

For example, Wöhler found that a bar was equally safe to resist varying bending and tensile straining actions, or sustained an equal number of repetitions of loading, before breaking, when the maximum and minimum stresses had the following values :—

For Wrought Iron

	Pounds per sq. in.		
	k max.	k min.	Δ
In tension only	18713 to	31	18682
In tension and compression alternately .	8317 to	−8317	16634

For Cast Steel

In tension only	34307 to	11436	22871
In tension and compression alternately .	12475 to	−12475	24950

These experiments are sufficient to show that the breaking and therefore also the working stress depends very much on the range of variation of stress, being much greater when a bar is subjected to stress of one kind than when subjected alternately to stresses of opposite kinds, and in both cases being less than for a steady load with no variation.

Unfortunately, Wöhler's experiments, although extensive, were not made on a sufficient variety of materials to furnish decisive rules for practical guidance. They afford an explanation of the apparently high factors of safety, which, in certain cases, experience has shown to be necessary, but they are not complete enough to indicate precisely the factor of safety to be chosen in different cases. Nor indeed could rules be obtained, without the most careful comparison of the results of researches, of the kind begun by Wöhler, with the actual stresses found to be safe in practice, in a great variety of cases.[1]

Let, as before, κ be the breaking strength per unit of section, for the given material, and for a load once gradually applied. Let $k_{\max.}$ be the breaking strength for the same material subjected to a variable load ranging between the limits $k_{\max.}$ and $\pm k_{\min.}$, and repeated an indefinitely great number of times. $k_{\min.}$ is $+$ if the stress is of the same kind as $k_{\max.}$ and κ, and $-$ if the stress is of the opposite kind, and it is supposed that $k_{\min.}$ is not greater than $k_{\max.}$ Then the range of stress is $\Delta = k_{\max.} \mp k_{\min.}$, the upper sign being taken if the stresses are of the same kind, and the lower if they are different. Hence Δ is always positive.

[1] Wöhler's experiments agree with and confirm the earlier experiment of Sir W. Fairbairn, communicated to the Royal Society, on the effect of continuous changes of load on a riveted girder. In Germany the breaking strength receives different names according to the conditions in which the piece is placed. The stress at which fracture occurs by a single application of a steady or very gradually applied load is called the *statical breaking strength* (Tragfestigkeit). If after each application of a load the bar reverts to its original condition, and if all stresses are in the same sense (that is, all tensions, compressions, or shears in one direction), then the greatest stress which can be sustained for a specified number of repetitions is termed the *primitive strength* (Ursprungfestigkeit). If the stresses are alternately of opposite senses, that is, alternately tensions and compressions, or shears in opposite directions, the stress which can be sustained is termed the *vibration strength* (Schwingungsfestigkeit). See Weyrauch, *Proc. Inst. Civil Eng.* vol. lxiii.

Then Wöhler's experiments appear to suggest a rule of the following kind, for the relation between K and $k_{max.}$:—

$$k_{max.} = \frac{\Delta}{2} + \sqrt{K^2 - n\Delta K}. \quad . \quad . \quad (1)$$

If $\Delta = 0$, we get $k_{max.} = K$, the load being then a steady one. Further, by choosing a suitable value for n, we can make the decrease of $k_{max.}$ for increasing values of Δ correspond with the observed values in Wöhler's experiments. For ductile iron and steel the average value of n appears to be about 1·5. For hard qualities $n = 2$.

The special cases most useful to consider are the following :—

(1) The load is invariable ; then $\Delta = 0$.

(2) The load is entirely removed and replaced. Then $k_{min.} = 0$ and $\Delta = k_{max.}$

(3) The stress is alternately a compressive and tensile stress of the same magnitude. Then $k_{max.}$ and $k_{min.}$ are equal in magnitude and of opposite sign, and $\Delta = 2\,k_{max.}$

For these cases the formula gives the following values : —

	Greatest Stress.	Least Stress.	Range of Stress. Δ	
(1)	$k_{max.}$	$k_{max.}$	0.	Then $k_{max.} = K$.
(2)	$k_{max.}$	0	$k_{max.}$	Then $k_{max.} = 2(\sqrt{n^2+1} - n)K$.
(3)	$k_{max.}$	$-k_{max.}$	$2k_{max.}$	Then $k_{max.} = \frac{1}{2n}K$

Putting $n = 1{\cdot}5$, we get, in Case 1, $k_{max.} = K$; in Case 2, $k_{max.} = 0{\cdot}6054\,K$; in Case 3, $k_{max.} = \frac{1}{3}K$. In round numbers the stresses at which the bar breaks are in the proportion of 3 : 2 : 1 in these three cases.

From an examination of all the trustworthy series of experiments on repeated stress in which bars stood at least two or three million changes of load, the values of n and K in formula (1) above were deduced. Then from these values the breaking strengths given in the following table for each of the materials, and for different ranges of stress, were cal-

TABLE *giving the* MAXIMUM *and* MINIMUM STRESS, *and range of stress for bars which stood at least two or three million changes of load before breaking. Stresses in tons per sq. in.*

Material	Minimum Stress	Maximum Stress	Range of Stress	Authority
Phœnix iron .	−8·6	+8·6	17·2	Wöhler
	+0	+15·25	+15·25	
	12	20·5	10	$n = 1·33$, K = 22·8
	22·8	22·8	0	
Krupp's axle steel	−14·05	+14·05	+28·10	Wöhler
	0	20·5	26·5	
	17·5	37·75	20·25	$n = 1·83$, K = 52
	52	52	0	
Untempered spring steel	0	25·5	25·5	Wöhler
	12·5	35·0	22·5	$n = 2·14$, K = 57·5
	20	40	20	
	30	45	15	
	57·5	57·5	0	
Wrought iron plate	−7·15	+7·15	14·30	Bauschinger
	0	13·10	13·10	
	11·4	19·2	7·8	$n = 1·60$, K = 22·8
	22·8	22·8	0	
Bessemer steel	−8·55	+8·55	17·10	Bauschinger
	0	15·7	15·7	
	14·3	23·8	9·5	$n = 1·68$, K = 28·6
	28·6	28·6	0	
Bar iron .	−7·85	+7·85	15·7	Bauschinger
	0	14·4	14·4	
	13·3	22·02	8·72	$n = 1·67$, K = 26·0
	26·0	26·0	0	
,, .	−8·65	+8·65	17·3	Bauschinger
	0	15·75	15·75	
	13·2	21·92	8·72	$n = 1·53$, K = 26·4
	26·4	26·4	0	
Steel axle .	−10·5	+10·5	21·0	Bauschinger
	0	19·7	19·7	
	20	32·1	12·1	$n = 1·91$, K = 40
	40	40·0	0	
Steel rail .	−9·7	+9·7	19·4	Bauschinger
	0	18·4	18·4	
	19·5	30·85	11·35	$n = 2·00$, K = 39
	39	39	0	
Steel boiler plate	−8·65	+8·65	17·3	Bauschinger
	0	15·8	15·8	
	13·3	22·55	9·25	$n = 1·53$, K = 26·6
	26·6	26·6	0	

Where the range of stress is zero the stress given is the ordinary statical breaking weight.

culated. A fuller account of the experiments will be found in the author's treatise on 'The Testing of Materials of Construction.'

Determination of Safe Working Stresses from the Results of Wöhler's Experiments.—From the tabulated results given above or from the formula (1) it is now possible to determine the real breaking strength of the most ordinary materials of construction, when subjected to an indefinitely great number of repetitions of loading with a known variation of stress. The safe working stress in corresponding conditions of loading for a machine part must be determined by dividing the breaking stresses by a real factor of safety, that is, by a factor which allows for probable contingencies and provides an intended margin of safety between the working and breaking stress.

For structures such as railway bridges, the load on which is known with reasonable accuracy, the contingencies to be allowed for are that, from imperfect workmanship, the stresses in different bars may be somewhat different from the calculated stresses, and that from imperfect conditions of the roadway there will be impacts and vibrations not allowed for in the calculations. In such cases it appears that a real factor of safety of 3 is amply sufficient, or the same factor of safety as is ordinarily adopted for a steady load. The ordinary working stresses in actual bridges are about one-third of those given by equation (1). Hence, the same factor of safety would appear to be sufficient for machinery, where the actual straining action is known with fair accuracy. Thus, for wrought iron in ordinary cases a working stress of 19,000 lbs. per sq. in. would be safe with a steady load. With a load alternately replaced and removed, the stress should not exceed 12,000 lbs. per sq. in., and with a stress constantly reversed, as in the case of axles, the stress should not exceed 6,000 lbs. per sq. in. Where exceptional contingencies must be allowed for, a larger factor of safety must be taken.

21*a*. *Fatigue of Materials.*—In many cases materials are subjected to impulsive loads, and a gradual deterioration of strength is observed. Thus a crane chain sometimes breaks when carrying a load which it has often before carried safely. In part, this deterioration of strength may be due to the ordinary action of a live or repeated load; but it appears to the author to be more often due directly to the gradual loss of the power of elongation in consequence of the slow accumulation of the permanent set. Suppose a crane chain carrying a load, W, surges, so that the load falls a distance, h, and let the elongation of the chain under the action of this impulsive load be l. Then the work done by the load in falling is $W(h+l)$. The work absorbed in elongating the chain is $k p l A$, where A is the area of the section of the chain, p is the maximum intensity of the stress induced, and k is a constant, which would be $\frac{1}{2}$ if the stress p were within the elastic limit, but which lies between $\frac{1}{2}$ and 1, if p exceeds the elastic resistance. Equating these, we get—

$$W(h+l)=k p l A$$

$$p=\frac{W(h+l)}{k l A}.$$

Hence, if h is not small compared with l, the stress p will be greater the smaller the elongation of the chain l. In a new chain, l will include the permanent set as well as the elastic elongation. In an old chain, which has already elongated permanently, and thus become less tough, the power of elongation before fracture is diminished. Hence the stress p induced by a load W (capable of producing stresses somewhat beyond the elastic limit) increases for any given chain as the chain gets older, and may ultimately reach the breaking stress. It is believed that annealing a crane chain restores its power of elongation and its original power of resisting impulsive loads. The term *fatigue*, which has been somewhat loosely applied to various kinds of deterioration

of resistance, may be conveniently restricted to this, which is a removable deterioration.

22. *Straining Action due to Power transmitted.*—When HP horses' power are transmitted through a link or connecting rod moving with velocity v, in ft. per second, the straining force, parallel to the axis of the rod, due to the work transmitted, is

$$P = \frac{550\ HP}{v} \text{ lbs.}$$

There will be in this case other straining actions, due to the reactions of the supports of the link, if the link is not moving parallel to its axis.

When HP horses' power are transmitted through a rotating piece, making n revolutions per second, the twisting moment, about the axis of the piece, is given by the equation

$$M = \frac{550\ HP}{2\pi n} = 1050{\cdot}4\frac{HP}{n} \text{ inch lbs.}$$

Or if N=revolutions per minute,

$$M = 63024\frac{HP}{N} \text{ inch lbs.} \quad . \quad . \quad (2)$$

Straining Actions due to Variations of Velocity.—When a heavy body is accelerated or retarded, straining actions are produced, due to its inertia. If w lbs. acquire an increase of velocity $d\,v$, in the time $d\,t$, the stress due to acceleration estimated in lbs. weight is

$$-\frac{w}{g} \cdot \frac{d\,v}{d\,t} \quad . \quad . \quad . \quad . \quad (3)$$

Graphic Method of determining the Stress due to Inertia.—In most cases the algebraic expression of the velocity of machine parts is rather complicated, but it is easy to draw a curve representing very approximately the velocity at any instant. Suppose such a velocity curve, having for abscissa the distance moved, and for ordinate the velocity, to have been drawn. Then the acceleration per unit of mass at any

point of the path is the subnormal of the velocity curve at that point.

Thus, let Ox be the path of a heavy piston, and let Obx be the velocity curve; that is, when the piston has moved through a distance Oa, let ab be the velocity v. If now the normal bd is drawn to the curve, ad represents the acceleration p per unit of weight, at the moment the piston is at a, and if ad is twisted to the position ac, c is a point in a curve representing the accelerations. The scale of the acceleration curve is the same as the scale of the velocity curve. The normal bd is easily drawn with sufficient accuracy by using a straight edge and set square, or by trial the centre may be found for a circular arc agreeing with the given curve in the neighbourhood of b. If W is the weight of the piston, and p is the ordinate ac, then $\frac{W}{g}p$ is the total acceleration. This is the straining action due to inertia mentioned in the last paragraph, and is + or − according as the motion is accelerated or retarded. This construction is due to Dr. Proll.[1]

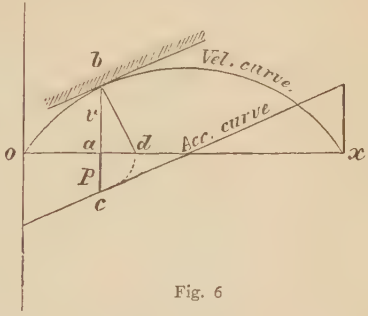


Fig. 6

Straining Effects due to Change of Direction of Motion.— When a mass is forced to move in a curved path, it exerts, in consequence of its inertia, a force equal and opposite to

[1] It is easy to see that $\frac{dv}{dt}=\frac{dv}{ds}\cdot\frac{ds}{dt}=v\frac{dv}{ds}$. But $\frac{dv}{ds}$ is the trigonometrical tangent of the angle between the tangent to the curve and ox, or, what is the same thing, the angle abd. Putting $v=ab$, $\frac{dv}{ds}=\frac{ad}{ab}$. Consequently, $\frac{dv}{dt}=v\frac{dv}{ds}=ad=p$.

the constraint which deflects it. If a weight w moves in a circular path of radius r, with the angular velocity ω, its centrifugal force, which is equal and opposite to the force deviating it, is

$$\frac{w}{g}\omega^2 r \quad . \quad . \quad . \quad . \quad (4)$$

23. *Resilience. Resistance to Impulsive Loads.*—The quantity of work expended in deforming a bar (provided the stress does not exceed the elastic limit) is equal to the product of the deformation and the mean load producing it. Thus, if a bar is elongated or deflected a feet, by a force gradually increased from nothing to P, the work done in deformation is $\frac{1}{2}$ P a ft. lbs. W pounds moving with velocity v have $\frac{W}{g} \cdot \frac{v^2}{2}$ ft. lbs. of work stored in them. Hence the relation between the impulsive load and the resistance of the bar, when the direction of the impulse coincides with the direction of the deformation, is

$$\frac{W}{2g} v^2 = \tfrac{1}{2} P a \quad . \quad . \quad . \quad (5)$$

If a bar is twisted, the work done is equal to half the twisting moment, multiplied by the angle of torsion.

The work done in deforming a bar up to the elastic limit is termed the resilience of the bar.

Stress due to Change of Temperature.—Let $t°$ be the change of temperature in degrees centigrade, α the expansion of the material per unit length per degree, and E the coefficient of elasticity of the material. A bar of the length l at the colder temperature would become $(1+\alpha t)\ l$ at the hotter, if unhindered. If an obstacle is opposed to the expansion a stress is induced which may reach the value given by the relation

$$at = f/E$$

$$f = at\,E \quad . \quad . \quad . \quad (6)$$

If $\alpha = 0{\cdot}0000108$ for iron ; $E = 29000000$; then $f = 313t$ in lbs. per sq. in.

CHAPTER III

RESISTANCE OF STRUCTURES TO DIFFERENT KINDS OF STRAINING ACTION

Physical Constants for Ordinary Materials

The change of form of a body under the action of external forces consists of two parts, one proportional to the loading force, which vanishes if the load is removed; the other, a permanent strain or set which does not vanish when the load is removed. For any given condition of variation of loading there is a limit of stress up to which the permanent set is zero or very small, beyond which it very rapidly increases. This is called the 'limit of elasticity.' Working stresses should not exceed this limit. The limit of elasticity is different when a bar is subjected to tension or pressure only, from what it is when the bar is subjected to pressure and tension alternately.

24. The table given on pp. 40, 41 shows the elastic and ultimate strength of different materials, when the stress is simple tension, pressure, or shearing stress. The values given are either average or maximum, mean and minimum values, selected from the most trustworthy experiments. In different specimens of the same material, and even in different pieces of the same bar or plate, there are often considerable differences of elasticity and strength, and the judgment of the engineer must be relied on in deciding how far average values of this kind are applicable in any given case. The first two columns of the table relate to direct stresses of pressure or tension produced by straining actions normal to

TABLE I.—*Ultimate and Elastic Strengths of*

Material	Breaking Strength		
	Tension	Pressure	Shearing
Cast iron	30,500	130,000	12,000
	17,500	95,000	10,500
	10,800	50,000	8,700
Wrought-iron bars	67,000	—	49,000[2]
	57,600	50,000	40,000
	33,500	—	22,400
Iron ship plates ‖	49,000	—	—
Iron boiler plates ‖	47,000	—	36,000
,, ,, ⊥	41,500	—	—
Steel plates, ¼% carbon	65,000	—	50,000
,, ,, ½% ,,	78,000	—	56,000
,, ,, 1% ,,	110,000	—	83,000
Steel boiler plates	66,000	—	56,000
Rivet steel	65,000	—	55,600
Cast steel untempered	150,000	—	—
	120,000	—	—
	84,000	—	—
Cast steel tempered	—	—	—
Steel castings from	63,000	—	—
to	34,000	—	—
Copper cast from	19,000	—	—
to	23,000	58,000	—
,, rolled plates	31,000	—	—
,, annealed wire	45,000	—	—
,, hard drawn wire	58,000	—	—
Brass from	17,500	—	—
to	29,000	—	—
Gun-metal or bronze	52,000	—	—
	27,000	—	—
	23,000	—	—
Delta metal, cast	36,000	—	—
,, ,, rolled	74,000	—	—
Phosphor bronze	58,000	—	43,000
Muntz metal	49,000	—	—
Cast zinc	7,500	—	—
Lead	2,500	7,000	—
Tin	4,700	—	—
Wood, pine from	6,700	3,400	—
to	13,000	7,000	630[3]
,, oak	15,000	10,000	2,240[3]
Leather	4,200	—	—

[1] Cast iron has properly no elastic limit.
[3] These are along the fibres of the wood.

Materials and Coefficients of Elasticity, in lbs. per sq. in.

Elastic Strength			Coefficient of Elasticity	
Tension	Pressure	Shearing	Direct E	Transverse C
—	—	—	23,000,000	7,600,000
10,500[1]	21,000[1]	8,000[1]	17,000,000	6,300,000
—	—	—	14,000,000	5,000,000
—	—	—	31,000,000	—
30,000	30,000	22,000	29,000,000	10,500,000
—	—	—	27,000,000	
—	—	—	—	
24,000	24,000	15,000	26,000,000	14,000,000
—	—	—	27,000,000	—
42,000	38,000	—		—
47,000	49,000	—	31,000,000	13,000,000
67,000	71,000	—		—
36,000	—	—	30,000,000	13,500,000
46,000	—	—	30,000,000	13,000,000
—	—	—	—	—
80,000	80,000	64,000	30,000,000	12,000,000
—	—			—
190,000	190,000	145,000	36,000,000	14,000,000
34,000	—		30,000,000	—
20,000	—	—	20,000,000	—
—	—	—	12,000,000	—
—	—	—	—	—
5,600	4,000	3,000	15,000,000	5,600,000
—	—	—	16,000,000	—
—	—	—	17,000,000	—
—	—	—	13,500,000	—
—	—		—	—
—	—		—	—
6,200	—	4,150	13,500,000	—
—	—		—	—
17,000	—	—	12,000,000	—
51,000	—		13,000,000	—
19,700	—	14,500	14,000,000	5,250,000
—	—	—	—	—
3,200	—	—	—	—
1,500	—	—	2,500,000	—
—	—	—	—	—
—	—	—	1,000,000	—
—	—	—	1,600,000	—
—	—	—	1,450,000	
—	—	—	25,000	—

[2] The shearing resistance of wrought iron is much less along the fibre than across it.

the sections considered. The third relates to tangential or shearing stress produced by straining action parallel to the section. The elastic strength is that stress per unit of area at which the strains cease to be sensibly proportional to the stresses, and the values given are those obtained when a bar is first loaded with a gradually increasing stress of one kind only. The ultimate strength is the intensity of stress at the moment preceding rupture, and this depends in some degree on the manner of carrying out the experiment and the proportions of the bar tested. The more rapid the loading of the bar and the less vibration induced, the greater is the load carried before rupture ensues. Nevertheless, if the experiment is carried out with proper care, the breaking strength is a definite measure of the properties of the material The elastic and breaking strengths are expressed in lbs. per sq. in.

A coefficient of elasticity is the ratio of the intensities of stress and strain of some given kind, when the elastic limit is not passed. Thus the coefficient of direct elasticity of a material is the ratio of the stress p, per unit of section of a bar, to the extension or compression, l, per unit of length, produced by the stress, when the bar is not subjected to lateral constraint; that is, the coefficient of direct elasticity $= E = p/l$ where p is expressed in lbs. per sq. in. and l in inches per inch of length. The bar is supposed to be free laterally. For ordinary metals the coefficient of direct elasticity may be taken to be the same for tension and pressure. The coefficient of transverse elasticity is the ratio of the shearing stress q per unit of area to the distortion n; the distortion being measured by the tangent of the difference of the angles of an originally square particle before and after the stress is applied. Hence the coefficient of transverse elasticity $= G = q/n$. The ratio G/E for ordinary materials of construction is about $\frac{3}{8}$ to $\frac{2}{5}$.

25. *Working Stress.*—It has been pointed out that the working stress of a material must be less than the elastic

TABLE II.—*Ordinary Working Stress*

a. For dead load. *b*. For a varying load producing stress of tension or pressure only. *c*. For a varying load producing equal maximum stresses in tension and pressure.

Material	Tension or Pressure			Shearing			Torsion		
	a	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*
Cast iron [1] . . .	4,200	2,800	1,400	—	2,300	—	2,100	1,400	700
Bar or forged iron . .	15,000	10,000	5,000	12,000	8,000	4,000	6,000	4,000	2,000
Iron plate ∥ . . .	15,000	10,000	5,000	—	—	—	—	—	—
,, ,, ⊥ . . .	12,000	8,000	4,000	10,000	6,600	3,300	—	—	—
Mild steel . . .	20,000	13,200	6,600	16,000	10,600	5,300	8,000	5,400	2,700
Cast steel . . .	24,000	16,000	8,000	20,000	13,200	6,600	10,000	6,600	3,300
Phosphor bronze . .	10,000	6,600	3,300	7,000	4,600	2,300	4,200	2,800	1,400
Gun-metal . . .	4,200	2,800	1,400	—	—	—	—	—	—
Rolled copper . .	4,000	2,600	1,300	2,400	1,600	800	—	—	—
Brass	3,000	2,000	1,000	—	—	—	—	—	—

[1] Cast iron in compression carries about three times as much stress as in tension.

strength, to allow for straining actions which cannot be taken into account, for imperfections of workmanship and for other sources of danger. The TABLE II. on p. 43 gives values of the ordinary working stress allowed in designing machinery. The safe working stress has been fixed with attention partly to practical experience, partly to the results of Wöhler's experiments. The cases for which the working stress is given are those discussed on p. 32.

The following table gives the working stress very commonly adopted in building construction, in cases where the load is largely a dead load :—

	Tension lbs. per sq. in.	Compression lbs. per sq. in.
Wrought iron	14,000	14,000
Cast iron	3,360	10,080
Oak	1,400	940
Pine	1,120	800
Brickwork, lime mortar	—	36 to 72
,, cement mortar	—	72 to 108
Rubble masonry, lime mortar	—	58
,, ,, cement mortar	—	72
Portland cement concrete	—	100
Pressed bricks in cement mortar	—	114 to 128

RESISTANCE TO SIMPLE TENSION AND COMPRESSION

26. A bar is in tension or compression when the load acts parallel to its axis, and the stress on any section of the bar is uniformly distributed or not, according as the line of action of the load does or does not pass through the centre of figure of that section. Cases in which the stress is a varying stress will be treated as cases of compound stress. At present only cases of uniformly distributed stress are considered.

Let AB, fig. 7, be a section, of area a (in sq. ins.), on which a load P (in lbs.) acts, normally to the section. Then

TABLE III.— *Weights of Materials*

	lbs. per c. foot.	lbs. per c. inch.
Gaseous Bodies.		
Air at 32° and one atm.	·0807	—
Steam at 212° and one atm.	·0378	—
Liquids.		
Pure water at 39°·1 F.	62·425	0·036
,, ,, 60° F.	62·373	—
River water, mean	63·0	—
Sea water, mean	64·05	—
Timber.		
Oak or teak	50	—
Pine or fir	44	—
Greenheart	72	—
Brick and Stone.		
Ordinary brick	112	0·065
Brickwork (mean)	106	0·062
Rubble masonry (mean)	140	0·081
Concrete (mean)	150	0·087
Ashlar, sandstone	150	0·087
,, limestone	165	0·096
Granite	175	0·102
Earths.		
Chalk	150	0·087
Moist clay	163	0·094
Mud or moist sand	102	0·059
Compact moist soil	140	0·081
Metals.		
Wrought iron	490	0·283
Cast iron	468	0·271
Sheet lead	711	0·411
Copper	555	0·321
Zinc	449	0·260
Cast steel	496	0·287
Soft steel	480	0·278
Gun-metal	546	0·316
Cast brass	518	0·299
Rolled brass	526	0·305

the intensity of normal or direct stress is $f = P/a$ (in lbs. per sq. in.). If the section C D is not perpendicular to the direction of the load, let θ be the angle between the normal to the section and the direction of P. Then the stress on C D consists of a normal or direct stress $f_n = f \cos^2 \theta$, and a tangential or shearing stress $f_t = f \sin \theta \cos \theta$.

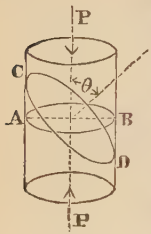


Fig. 7

To determine the section of a bar for a given load P, a value must be selected from the preceding tables for the working stress f, suitable for the material and the kind of loading to which the bar is subjected. Then the section of the bar normal to the direction of P is

$$a = \frac{P}{f} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Very long bars bend under the action of a longitudinal compressive force, and must be treated by special rules (§ 37).

From the definition of the coefficient of elasticity already given, it is obvious that the extension or compression l, of a uniform bar of length L in inches, is given by the equation

$$l = \frac{f_L}{E} \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

where f is the intensity of stress on sections perpendicular to the axis of the bar, and E is the coefficient of direct elasticity for the material of the bar. This equation ceases to be accurate if f exceeds the limit of elasticity.

In simple tension there is a lateral contraction, and in simple pressure a lateral dilatation which bears a definite ratio to the longitudinal extension or compression. The ratio of the transverse to the longitudinal strain estimated per unit length is termed Poisson's ratio. Its value for most engineering materials such as metals lies between $\frac{1}{3}$ and $\frac{1}{4}$.

Work done in Extending or Compressing a Bar.—During the extension of the bar from L to $L + l$ the stress increases from zero to f proportionately to the elongation. Hence the mean stress during the operation is $\frac{1}{2}f$, and consequently the work done in extending the bar is

$$W = \tfrac{1}{2} f a l = \tfrac{1}{2} f^2 \frac{a L}{E} \text{ inch lbs.} \quad . \quad (1b)$$

That is, for a given intensity of stress the work varies as the volume of the bar. The same formulæ are applicable for compression. The bar is assumed to be uniform in section, and not strained beyond the elastic limit.

26*a*. *Resistance of thin Cylinders to an internal bursting Pressure.*—Consider a thin cylindrical shell of diameter d, length l, and thickness t, in inches, subjected to a uniform internal pressure of p lbs. per sq. in. Let the cylinder be cut by a diametral plane $a\,b\,c\,d$, fig. 8. The resultant force P acting on either side of that plane = $p \times$ area $a\,b\,c\,d$. Hence, $P = p\,d\,l$. The molecular tensions which resist the bursting force act on the sections of the shell at $a\,b$ and $c\,d$, and are equal to the intensity of stress induced × sectional area of $a\,b$ and $c\,d$, that is, $2\,t\,l$. Putting f for the intensity of tensile stress, the total force resisting the bursting pressure is $2\,f\,t\,l$. Equating the load and resistance

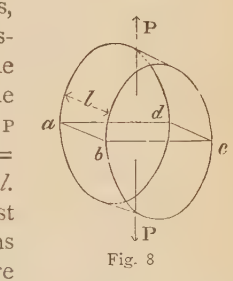


Fig. 8

$$2\,f\,t\,l = p\,d\,l = P$$

$$f = \frac{p\,d}{2\,t} \quad . \quad . \quad . \quad . \quad (2)$$

If the cylinder consists of riveted plates, the section $a\,b\,c\,d$ should be taken so as to pass through the rivet holes. Then the area of the rivet holes must be deducted from

$2\,t\,l$, before equating the internal and external forces. If the cylinder is thick relatively to its diameter, the mean stress is unaltered, but the inner layers are more severely strained than the outer layers. In that case the thickness necessary to resist a bursting pressure p, with a maximum intensity of stress f, is found by Grashof to be

$$t=\frac{d}{2}\left\{-1+\sqrt{\frac{3f+2p}{3f-4p}}\right\} \quad . \quad . \quad . \quad . \quad (3)$$

it being assumed that p is less than $\frac{3}{4}f$. If $\frac{p}{f}$ is a small ratio—

$$t=\frac{d}{2}\frac{p}{f}\left(1+\tfrac{5}{6}\frac{p}{f}\right) \text{ very nearly} \quad . \quad . \quad . \quad . \quad (3a)$$

In a thin spherical shell, the tension is half as great as in a thin cylindrical shell of the same diameter and thickness, exposed to the same pressure.

In a cylindrical shell the intensity of longitudinal stress is only half as great as the intensity of circumferential stress.

Resistance to Bending.

27. A bar is subjected to simple bending when the following conditions are fulfilled :—(1) The axis of the bar is straight ; the axis of the bar being a line connecting the centres of figure of parallel transverse sections ; (2) The bar is symmetrical about a plane passing through the axis ; (3) All the external forces act in such a plane of symmetry normally to the axis. If these conditions are not fulfilled, the action of the straining forces is more complex, and some cases in which this happens will be considered under the head of Compound Stress.

Consider the case represented in fig. 9, where, in the lower figure, the flexure is exaggerated for the sake of clearness. In this case, a bar originally straight, and having transverse sections symmetrical about the plane of the paper, in which the bending forces act, is subjected to flexure, under the action of two equal couples of forces applied to

its ends. Then the curvature from c to d is circular, and the effect of the bending is to lengthen the upper parts of the bar, and to shorten the lower parts. If the flexure is very small, so that the straining forces are sensibly parallel, a plane normal to the paper, through the axis of the bar, will divide the parts in tension from those in compression. The length of the bar measured along that surface will be unaltered by the flexure, and hence it is termed the neutral surface.

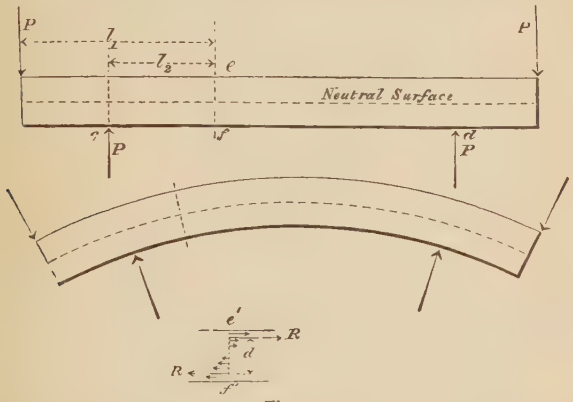


Fig. 9.

The amount of the bending action, at any section $e f$ of the bar, is measured by the resultant moment of the straining forces on either side of that section, which is termed the bending moment. Taking the forces to the left of $e f$, the bending moment is $P l_1 - P l_2$. The molecular stresses in the bar, developed by the external actions, form at any section a couple, whose moment is equal and opposite to the bending moment, and which is termed the moment of resistance of the section. The action of the molecular stresses is represented at $e' f'$. The tensions above and the compressions below the axis have resultants R, R, whose moment is $R d$. Equating this to the bending moment

$$P(l_1 - l_2) = R d \quad . \quad . \quad . \quad . \quad (4)$$

In other cases the action is a little more complex. Suppose the force P acts at the end of a bar (fig 10) solidly fixed at the other end, and let it be required to find the straining action at *e f*. Equilibrium is not disturbed, if we introduce two equal and opposite forces P′ P″, in the direction *e f*. Then the action of P on the section *e f* is equivalent to that of a couple, P, P″, and an unbalanced force P′. The couple has a moment P *l*, which produces simple bending, and is in equilibrium with a couple formed by molecular stresses at *e f*, parallel to the axis of the bar, precisely similar to those described in the previous case. The remaining force P′ produces a shearing stress on the section *e f*. The two actions are independent, and the bar must be strong enough to resist both the bending moment and the shearing action. In a large number of cases the amount of material necessary to resist the bending moment is much more than sufficient to resist the shearing action, so that the latter may be left out of consideration.

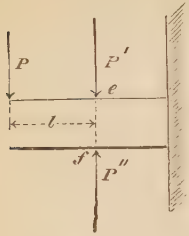


Fig 10.

If several forces act to the left of *e f*, we may take their resultant, and then proceed as if only a single force required to be dealt with.

It will alter nothing in the conditions of the stresses of the bar in fig. 9, if we suppose it to form part of a longer bar bent to a complete circle of the same curvature, by the action of the external forces. It can then be seen that fibres originally straight in the unstrained bar become coaxial circles in parallel planes in the strained bar, and plane transverse sections become plane radial sections, across which there is no shearing stress, but a bending moment only, the resultant of the tensions and pressures of the fibres.

Let ρ be the radius of the layer of fibres which are

neither extended nor compressed by the bending of the bar. Then the length of all the fibres before bending was $2\pi\rho$. After bending, a fibre at radius $\rho+y$ has the length $2\pi(\rho+y)$, and its extension (or compression if y is negative) is $2\pi y$. Assuming the formulæ in § 26, the stress f due to this extension is given by the equation

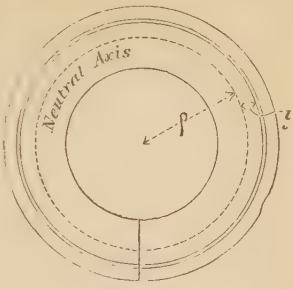


Fig. 11.

$$2\pi y = \frac{f}{E} 2\pi\rho.$$

Hence $$f = \frac{Ey}{\rho}$$

The total stress on an element of area a, at radius $\rho+y$, is therefore $fa=\frac{Eya}{\rho}$, and the total stress on the whole section is $\Sigma\left(\frac{Eya}{\rho}\right)$ But since the pressures and tensions across the section form a couple,

$$\Sigma\left(\frac{Eya}{\rho}\right)=0;$$

or since $\frac{E}{\rho}$ is a constant,

$$\Sigma ya = 0.$$

This equation is only true if the distances y are measured from a line passing through the centre of figure of the section. Hence the neutral axis of the bar passes through the centres of figure of the cross sections.

The moment of the stress fa about the neutral axis of the section is $fay=\frac{Ey^2a}{\rho}$. The total moment of the couple formed by the tensions and pressures at the section is $\Sigma\left(\frac{Ey^2a}{\rho}\right)$ or $\frac{E}{\rho}\Sigma ay^2$. Now the quantity Σay^2 is known

as the moment of inertia of the section, and is usually denoted by the symbol I. Hence, putting M for the moment of the actions producing bending on one side of the section, which are in equilibrium with the stresses at the section,

$$M = \frac{E\,I}{\rho},$$

which expresses the relation between the bending moment and the curvature of the bar. Let f_t f_c be the tension and pressure at points distant y_t and y_c from the neutral axis. Then

$$f_t = \frac{E}{\rho} y_t \text{ and } f_c = \frac{E}{\rho} y_c.$$

Therefore

$$M = f_t \frac{I}{y_t} = f_c \frac{I}{y_c}.$$

$$f_t = M\frac{y_t}{I} \text{ and } f_c = M\frac{y_c}{I}.$$

It is generally necessary to find the greatest tension and pressure in the bar, and we must then take for y_t and y_c the distances of the parts of the bar farthest from the neutral axis on the extended and compressed sides. Then the quantities $\frac{I}{y_t}$ and $\frac{I}{y_c}$ may be termed the moduli of the section with respect to tension and pressure, and putting z_t and z_c for these moduli, the equations may be more simply written

$$M = f_t z_t = f_c z_c.$$

28. Let M be the moment of the external forces on one side of any transverse section, or bending moment, estimated relatively to the section.

Z=the modulus of the section, that is a function of the dimensions of the section, which is proportional to the moment of resistance of the section. The value of z for various sections is given in the following tables.

f_t and f_c=the greatest safe working stress in tension and compression for the material of the bar. Then the bar will be safe, if

$$\left.\begin{array}{r} \text{M is not greater than } f_t z \\ \text{and also is not greater than } f_c z \end{array}\right\}$$

If we put fz for the lesser of the two values of the moment of resistance, the bar will be of adequate strength when

$$M = fz \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and the greatest stress due to bending is

$$f = \frac{M}{z} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

29. When the section is not symmetrical about the neutral surface, z has two values, z' corresponding to the part above, and z'' to the part below the neutral surface. Then two cases arise :—

(1) The part of the bar above the neutral axis in tension, the part below in compression,

$$\text{Moment of resistance} = f_t z' \text{ or } f_c z''.$$

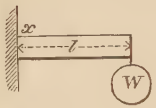
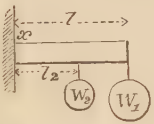
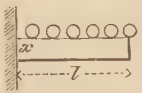
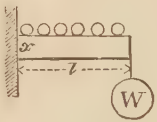
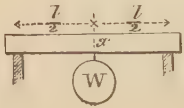
(2) The part of the bar above the neutral axis in compression, the part below in tension,

$$\text{Moment of resistance} = f_c z' \text{ or } f_t z''.$$

The smaller of the two values is to be taken in either case. If the straining forces act successively in opposite directions, the least of the four values is the effective moment of resistance.

In many cases, bars subjected to bending are necessarily uniform in section. Then it is only necessary to consider the greatest bending moment, and to design the section of the bar for that moment. In other cases, the bar varies in section, and the moment of resistance at each section must be, at least, equal to the bending moment at that section. The section at which the bending moment is greatest is sometimes termed the dangerous section. The table on p. 54 gives, for various loads and modes of support, the greatest bending moment; the position of the dangerous section; the greatest shearing force; and the working load corresponding to a given moment of resistance fz, at the dangerous section.

TABLE IV.—*Bending Moment and Shearing Force corresponding to different Loads and for different modes of Support.*

		Greatest bending moment (at x)	Working load for given moment of resistance	Greatest shearing force	Remarks
	BEAMS FIXED AT ONE END.				
I.		$W l$	$W = \frac{f z}{l}$	W	Loaded at free end.
II.		$W_1 l_1 + W_2 l_2$		$W_1 + W_2$	More than one load.
III.		$w \frac{l^2}{2}$	$w = \frac{2 f z}{l^2}$	$w l$	Uniform load w lbs. per in. run.
IV.		$\frac{w l^2}{2} + W l$		$l + W$	Load partly uniform, partly concentrated.
	BEAMS SUPPORTED AT BOTH ENDS.				
V.		$W \frac{l}{4}$	$W = 4 \frac{f z}{l}$	$\frac{W}{2}$	Loaded at centre.

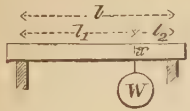
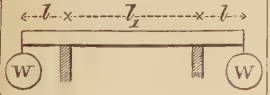
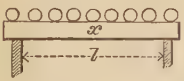
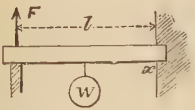
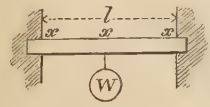
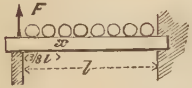
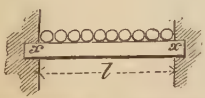
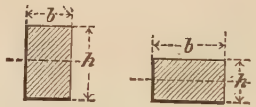
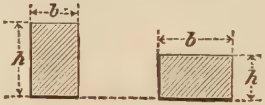
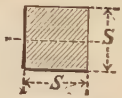
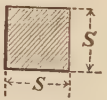
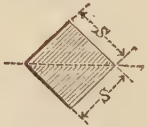
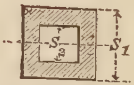
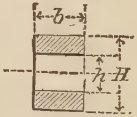
		Greatest bending moment (at x)	Working load for given moment of resistance	Greatest shearing force	Remarks
VI.		$W\frac{l_1 l_2}{l}$	$W=\frac{f z l}{l_1 l_2}$	$\frac{W l_1}{l}$ & $\frac{W l_2}{l}$	Load not at centre.
VII.		$W l$	$W=\frac{f z}{l}$	W	Two equal couples. Uniform bending moment, and no shearing force between the supports.
VIII.		$\frac{w l^2}{8}$	$w=\frac{8 f z}{l^2}$	$\frac{w l}{2}$	Uniformly distributed load. w lbs. per in. run.
	BEAMS FIXED AT ONE END AND SUPPORTED OR FIXED AT THE OTHER.				
IX.		$\frac{3}{16} W l$	$W=\frac{16 f z}{3 l}$		Load at centre. Pressure on support = $F=\frac{5}{16}$ W.
X.		$\frac{W l}{8}$	$W=\frac{8 f z}{l}$		Load at centre. Equal bending moments at ends and centre.
XI.		$\frac{w l^2}{8}$	$w=\frac{8 f z}{l^2}$		Greatest bending moment at fixed end.
XII.		$\frac{w l^2}{12}$	$w=\frac{12 f z}{l^2}$		Greatest bending moment at ends.

TABLE V.—*Area, Moment of Inertia*

The plane of bending is supposed parallel to the side of the page. figure of the section, except in the cases where the axis is stated to be given for the modulus, z′ applies to the fibres at the top edge and z″ to

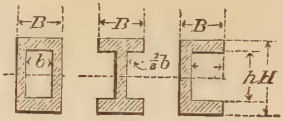
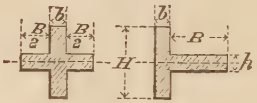
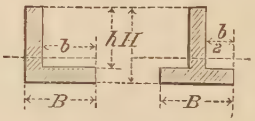
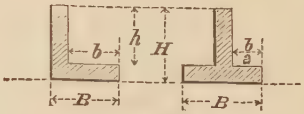
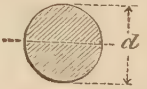
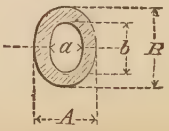
Form of Section	Diagram of Section Axis of moment shown by dotted line zz	Area of Section A
Rectangle		bh
" (axis at base)		bh
Square		s^2
" (axis at base)		s^2
" (axis diagonal)		s^2
Hollow square		$s_1^2 - s_2^2$
Pierced rectangle		$b(H-h)$
Triangle		$\frac{1}{2}bh$

and Modulus of different sections

The axis of moments is the horizontal dotted line through the centre of the horizontal through the base of the section. Where two values are the fibres at the bottom edge of the section.

Moment of Inertia of Section about axis zz I	Square of radius of gyration of Section $\rho^2=\frac{I}{A}$	Modulus of Section $Z=I/y$
$\frac{1}{12}bh^3$	$\frac{1}{12}h^2$	$\frac{1}{6}bh^2$
$\frac{1}{3}bh^3$	$\frac{1}{3}h^2$	
$\frac{1}{12}s^4$	$\frac{1}{12}s^2$	$\frac{1}{6}s^3$
$\frac{1}{3}s^4$	$\frac{1}{3}s^2$	
$\frac{1}{12}s^4$	$\frac{1}{12}s^2$	$0{\cdot}118s^3$
$\frac{1}{12}(s_1{}^4-s_2{}^4)$	$\frac{1}{12}(s_1{}^2+s_2{}^2)$	$\frac{1}{6}\frac{s_1{}^4-s_2{}^4}{s_1}$
$\frac{b}{12}(H^3-h^3)$	$\frac{1}{12}(H^2+hH+h^2)$	$\frac{1}{6}\frac{b}{H}(H^3-h^3)$
$\frac{1}{36}bh^3$	$\frac{1}{18}h^2$	$\begin{cases}Z'=\frac{1}{24}bh^2\\ Z''=\frac{1}{12}bh^2\end{cases}$

TABLE V. – *Area, Moment of Inertia and*

Form of Section	Diagram of Section Axis of moment shown by dotted line zz	Area of Sectio A
Hollow rectangle, tee, or channel		$BH - bh$
Triangle (axis at base)		$\frac{1}{2}bh$
Cross		$bH + Bh$
Angle iron		$BH - bh$
" (axis at base		$BH - bh$
Circle		$\frac{\pi}{4}a^2 = .785d^2$
Hollow circle		$\frac{\pi}{4}(D^2 - d^2)$
Ellipse		$\frac{\pi}{4}ba$
Hollow ellipse		$\frac{\pi}{4}(BA - ba)$

Modulus of different sections — Continued

Moment of Inertia of Section about axis zz I	Square of radius of gyration of Section $\rho^2 = \frac{I}{A}$	Modulus of Section $Z = I/y$
$\frac{1}{12}(BH^3 - bh^3)$	$\frac{1}{12}\frac{BH^3 - bh^3}{BH - bh}$	$\frac{I}{6H}(BH^3 - bh^3)$
$\frac{1}{12}bh^3$	$\frac{1}{6}h^2$	
$\frac{1}{12}(bH^3 + Bh^3)$		$\frac{I}{6H}(bH^3 + Bh^3)$
$\frac{(BH^2 - bh^2)^2 - 4BHbh(H-h)^2}{12(BH - bh)}$		$\frac{(BH^2 - bh^2)^2 - 4BHbh(H-h)^2}{6(BH^2 + bh^2 - 2bHh)}$
$\frac{1}{3}\{(B-b)H^3 + b(H-h)^3\}$		$\frac{(BH^2 - bh^2)^2 - 4BHbh(H-h)^2}{6(BH^2 - bh^2)}$
$\frac{\pi}{64}d^4 = .0491\,d^4$	$\frac{1}{16}d^2$	$\frac{\pi}{32}d^3 = .0982d^3$
$\frac{\pi}{64}(D^4 - d^4)$	$\frac{1}{16}(D^2 + d^2)$ If $D - d$ is small $= \frac{1}{8}D^2$ nearly	$\frac{\pi}{32}\frac{D^4 - d^4}{D}$
$\frac{\pi}{64}ba^3$	$\frac{1}{16}a^2$	$\frac{\pi}{32}ba^2$
$\frac{\pi}{64}(BA^3 - ba^3)$		$\frac{\pi}{32}\frac{BA^3 - ba^3}{A}$

It is convenient for some purposes to state the moment of inertia for an unsymmetrical angle-iron and T-iron section more fully.

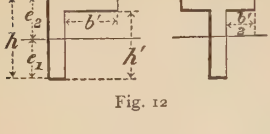


Fig. 12

Let z z be the axis through the centre of figure of the section (fig. 12), then

$$e_1 = \frac{b\,h^2 - b'h'^2}{2\,b\,h - b'h'};$$

$$e_2 = h - e_1$$

$$\text{I} = \frac{(b\,h^2 - b'h'^2)^2 - 4\,b\,h\,b'h'\,(h - h')^2}{12\,(b\,h - b'h')};$$

and dividing this by the values of e_1 and e_2 we get the two moduli of the section.

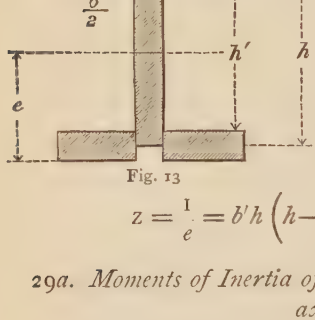


Fig. 13

The symmetrical I section is most conveniently dealt with approximately. Suppose it reduced to the form shown in fig. 13, the small omitted parts being neglected and the heights measured to the centres of the flanges. Then—

$$Z = \frac{\text{I}}{e} = b'h\left(h - h'\right) + \tfrac{1}{6}\,h^2\left(b - b'\right).$$

29*a*. *Moments of Inertia of plane figures about parallel axes*

Let I be the moment of inertia of the figure about an axis $x\,x$ passing through its centre of figure and I_1 the moment of inertia about a parallel axis $z\,z$. Let A be the area of the figure and y the distance between the axes.

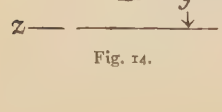
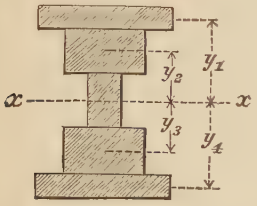


Fig. 14.

Then $\text{I}_1 = \text{I} + \text{A}y^2$.

This result is convenient in finding the moment of inertia of complex sections which can be cut up into a series of rectangles. For instance, the I-section, fig. 15, in which xx is an axis through the centre of figure of the whole section. Neglecting at present the small piece of the web in the middle, the areas a_1 a_2 a_3 a_4 of the rectangles can be written



Areas	Moments of inertia
a_1	I_1
a_2	I_2
a_3	I_3
a_4	I_4

Fig. 15

down and the moments of inertia of these rectangles about axes through their centres of figure. Then the moment of inertia of the whole section about $x\,x$ is $I_1+I_2+I_3+I_4+a_1y_1^2+a_2y_2^2+a_3y_3^2+a_4y_4^2$. The neglected part of the web can be treated in the same way or as two rectangles for which the moments of inertia can be written down at once from the value in Table V.

30. *Continuous Beams.*—When a beam rests on more than two supports, the ordinary statical conditions of equilibrium do not suffice to determine the reactions of the supports. Recourse may then be had to a relation due to

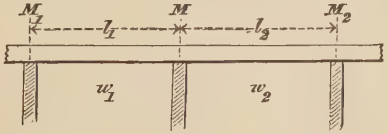


Fig. 16

Clapeyron, which is termed the Theorem of three moments. Let fig. 16 represent two consecutive spans of a beam resting on several supports, at the same level; let l_1, l_2, be the

lengths of the spans; w_1, w_2, the loads per unit of span; M_1, M, M_2, the bending moments over the supports. Then

$$8(l_1+l_2)M+4l_1M_1+4l_2M_2=w_1l_1^3+w_2l_2^3 \quad . \quad . \quad (7)$$

The equation assumes that the beam is of uniform section. But it may be applied in cases where the beam is approximately a beam of uniform strength. This theorem furnishes, for a beam of n spans, $n-1$ equations. In addition to these, the condition that a beam simply supported at the ends has no bending moment at the ends, furnishes two additional equations, $M_0=0$, $M_n=0$. There are then $n+1$ equations, to determine the $n+1$ bending moments at the points of support. By then reversing the ordinary process, the reactions can be found from the bending moments and loads. The following are some of the simplest results of applying this theorem to beams uniformly loaded with w lbs. per inch run.

	Reactions at supports
Beam of 2 equal spans	$\frac{3}{8}wl$; $\frac{5}{4}wl$; $\frac{3}{8}wl$.
„ 3 „ . .	$\frac{4}{10}wl$; $\frac{11}{10}wl$; $\frac{11}{10}wl$; $\frac{4}{10}wl$.

31. *Relative Economy of different forms of Section.*—The weight of a bar is proportional to its sectional area, its resistance to bending to its section modulus. Of two bars of different forms, subjected to the same loading, that will be the more economical of material which, with a given value of the modulus of resistance z, has the lesser sectional area A. Hence the more economical the form of the bar, the greater will be the ratio $\frac{Z}{A}$.

In a prismatic bar, of circular or rectangular section, only the material at the extreme top and bottom of the section is fully strained. Nearer the neutral surface the material is less strained, and at the neutral surface it is not strained at all by the direct stresses due to bending. Such a bar would be made stronger by removing some of the material from the neighbourhood of the neutral surface

towards the top and bottom of the section. We thus arrive at the excellent form of section known as the ⌶ or double T section. The material is chiefly collected in the top and bottom flanges, which bear nearly the whole of the direct stresses due to bending; the remainder forms a vertical web, whose chief function is to resist the shearing action.

32. *Flanged Sections when both Flanges are strained to the Working Limit.*—In order that the stress at the stretched edge of the bar may be at the working limit of tension, and the stress at the compressed edge may be at the working limit of pressure, we must have

$$f_t z' = f_c z'' \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where z' is the modulus, corresponding to the part in tension, and z'' that corresponding to the part in compression, and f_t, f_c, are the working intensities of tension and pressure resistance. If $f_t = f_c$, then $z' = z''$, or the modulus must be the same, both for the upper and lower parts of the section, and this will be the case when the section is symmetrical about the neutral surface.

If f_t and f_c are not equal, both edges cannot be fully strained when $z' = z''$, and the material is not used in the most economical way. In that case, it is better to adopt a section unsymmetrical with respect to the neutral surface.

Fig. 17

Let a_t be the area of the tension flange, a_c the area of the compression flange, and a the area of the web of a beam of ⌶-shaped section. Let h be the depth, measured from centre to centre of the flanges. Then if the area a of the web is small compared with that of the flanges the required condition is nearly fulfilled, when

$$f_t a_t = f_c a_c$$

$$a_t = \frac{M}{f_t h} \text{ and } a_c = \frac{M}{f_c h} \quad . \quad . \quad . \quad . \quad (9)$$

Further, if F is the total shearing action, and f_s the safe shearing stress, the strength of the web is sufficient when

$$a = \frac{F}{f_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

For practical reasons, especially in cast beams, a has often to be made of larger area than is given by this equation.

In the foregoing equations the resistance of the web to bending is neglected, and in cast-iron beams especially this introduces considerable error. If the web is taken into account we proceed as follows :

To design a beam of unsymmetrical ⌶ *section for a given bending moment.*—Cast iron beams are very commonly of ⌶-shaped section with unequal flanges, the ultimate resistance of cast iron to pressure being five or six times its resistance to tension. It is not usual, however, to proportion the beam with so great an inequality of flange section as it would have if the working stresses were proportional to the ultimate stresses in tension and compression. With great inequality of flange section the beam is difficult to cast. More commonly the limits of working stress are taken at $f_t = 2$ tons for tension and $f_c = 4$ tons or 5 tons per sq. in. for pressure. It is also desirable to facilitate casting that the web should be of not greatly less thickness than the flanges.

Let fig. 18 represent a diagram of the section required. The depth h of the beam between the centres of the flanges is assumed. It is often 1–10th to 1–15th of the span. Assume also from practical considerations a thickness for the web. Then the area a of the web is known and the areas a_c and a_t of the pressure and tension flanges remain to be determined. The small omitted parts of the section may be treated as negligible for simplicity.

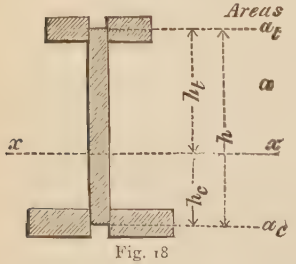


Fig. 18

If $x\,x$ is a line through the centre of figure of the section, then the flanges will have the required stresses if

$$h_c : h_t : h :: f_c : f_t : f_c + f_t$$

$$\left.\begin{aligned} h_c &= \frac{f_c}{f_c + f_t}\, h \\ h_t &= \frac{f_t}{f_c + f_t}\, h \end{aligned}\right\} \quad . \quad . \quad . \quad (10a)$$

Taking moments about $x\,x$

$$a_c\, h_c + a \cdot \frac{h_c}{h} \cdot \frac{h_c}{2} = a_t\, h + a \frac{h_t}{h} \cdot \frac{h_t}{2}$$

$$a_c\, h_c + a \frac{h_c^2}{2h} = a_t\, h_t + a \cdot \frac{h_t^2}{2h}$$

$$a_c = a_t \frac{h_t}{h_c} + a \left(\frac{h_t^2 - h_c^2}{2\, h\, h_c} \right)$$

Replacing by the relations above

$$\left.\begin{aligned} a_c &= a_t \frac{f_t}{f_c} + a \frac{f_t - f_c}{2 f_c} \\ \text{and} \quad a_t &= a_c \frac{f_c}{f_t} + a \frac{f_c - f_t}{2 f_t} \end{aligned}\right\} \quad . \quad . \quad (11)$$

Now taking the second moments of the areas about $x\,x$ the moduli of the section are

$$\left.\begin{aligned} z_c &= h \left\{ a_c + \left(2 - \frac{f_t}{f_c} \right) \frac{a}{6} \right\} \text{ for pressure} \\ z_t &= h \left\{ a_t + \left(2 - \frac{f_c}{f_t} \right) \frac{a}{6} \right\} \text{ for tension} \end{aligned}\right\} \quad (12)$$

From the known bending moment M and the relation

$$M = f_c\, z_c = f_t\, z_t$$

we can determine one modulus of the section, either z_c or z_t. Then one of the equations (12) furnishes one of the flange areas, and one of the equations (11) the other flange area. Knowing the areas, the width and thickness of the flange can be decided by practical considerations. In completing the design of the section, the web may be tapered so as to be a little stronger at its junction with the

larger flange, and the flanges may be tapered (without altering their area) so as to draw easily from the sand.

By applying the same process to sections of the beam at some distance from the centre, the amount of tapering of the flanges towards the ends can be determined. Some surplus strength is, however, provided at the ends to allow for inequality of loading and tendency to twist.

In consequence of the tendency in wrought-iron beams to a vertical or lateral buckling of the compressed flange, the working stress, in compression, is sometimes taken $\frac{2}{5}$ths less than the working stress in tension.

33. *Distribution of Bending Moment and Shearing Action.*—When the magnitude and position of the greatest bending moment and shearing action are known, the bending moment and shearing action, at any other point, can often be found very easily, by a simple graphic construction. Let O A be equal, on any scale, to the span of a beam. If at any point, *a*, a perpendicular is erected, and *a b* is made equal, on any scale, to the bending moment at the section of the beam which corresponds to *a*, *b* is a point in a curve, termed the curve of bending moments. Similarly, if *a b* were made equal to the shearing action at *a*, then *b* would be a point in a curve of shearing action. The curves might be constructed by finding the moments and the shearing actions at a sufficient number of points, setting them off on a diagram in the way just described, and then connecting the points of the curve, so found, by a line. In the simpler cases of loading, however, these curves can be more simply constructed. When the curve (which in some cases becomes a straight line) is drawn, the moment, or shearing action, at any point is obtained by scaling off the ordinate corresponding to that point. The following table (p. 67) gives the form of the curves of bending moment and shearing action, in the simpler cases

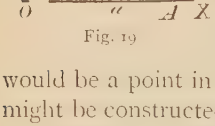
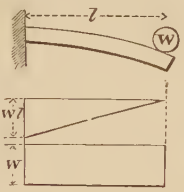
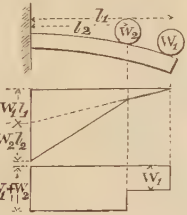
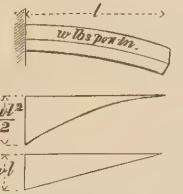
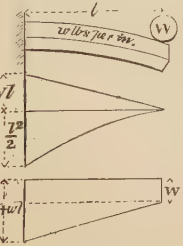
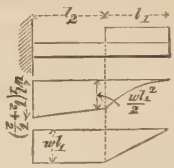
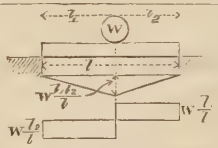
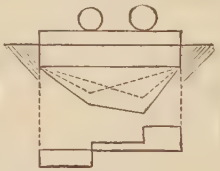
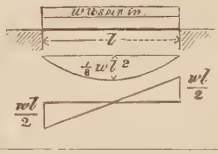


Fig. 19

TABLE VI.—*Distribution of Bending Moment and Shearing Force.*

	Loading	Diagram of load, and of bending moment and shearing force curves	Bending moment curve	Shearing force curve
		BEAMS ENCASTRE AT ONE END.		
I.	Load at free end		Straight line	Straight line
II.	Two loads		Broken line	Broken line
III.	Uniform load		Parabola, vertex at free end, axis vertical	Straight line
IV.	Uniform load and load at free end		Obtained by combining the curves in I. and III.	

	Loading	Diagram of load, and of bending moment and shearing force curves	Bending moment curve	Shearing force curve
V.	Partial uniform load		Parabola, with vertex at free end and straight line	Broken line
	BEAMS SUPPORTED AT BOTH ENDS.			
VI.	Single load		Broken line	Broken line
VII.	Two loads		Obtained by adding ordinates due to each separate load	
VIII.	Uniform load		Parabola, vertex at centre, axis vertical	Straight line

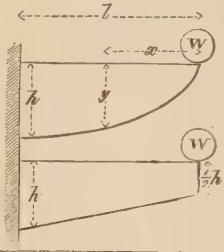
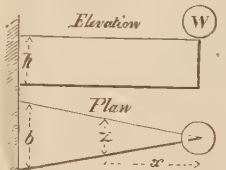
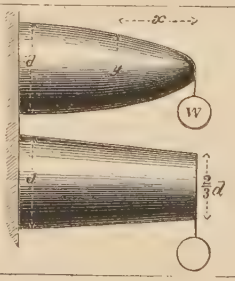
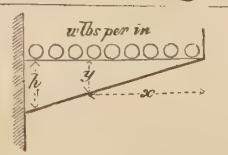
The cases given in this Table should be compared with the corresponding cases in Table IV.

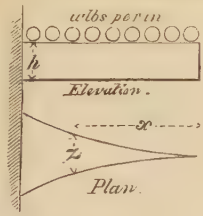
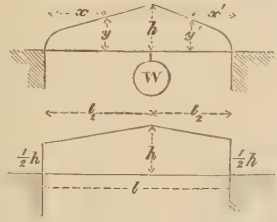
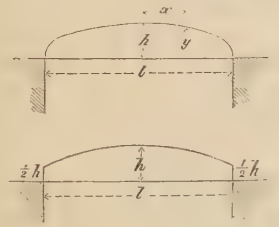
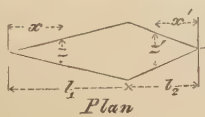
34. *Beams of Uniform Resistance to the direct Stresses due to Bending.*—Except in one special case, the bending moment varies at different points in the length of the beam. At the point where the bending moment is greatest, the section must be designed for that maximum moment. For practical reasons, it is frequently necessary to make the beam, or bar, uniform, and then the section where the bending moment is greatest determines the section of the rest of the bar. In other cases, the section of the bar may be diminished in parts where the bending moment is less, and material is then economised. The best distribution of material, so far as the direct stresses are concerned, is that which fulfils the condition

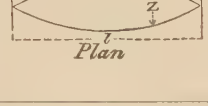
$$M = f z \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

for every transverse section, M being the bending moment at any section, and z the modulus of that section. Beams so designed are often termed beams of uniform strength. The theoretical form thus obtained requires, in some cases, to be modified for practical reasons. Approximate forms fulfilling the necessary conditions are given with the theoretical forms in the following Table. Table VII. gives some examples, partly selected from Reuleaux.

TABLE VII.—*Forms of Beams of Uniform Strength.*

		Longitudinal elevation of beam	Form of transverse section	Bounding lines of elevation or plan	Equation for determining the dimensions
BEAMS FIXED AT ONE END.					
I.	Load at free end		Rectangle of uniform breadth, b, and variable depth, y	Straight line and parabola. Approximate form, a truncated pyramid	$y^2 = \frac{6\,W}{b\,f}\,x$
II.			Rectangle of uniform depth, h, and variable breadth, z	Straight lines forming a wedge	$z = \frac{6\,W}{h^2\,f}\,x$
III.			Circle of variable diameter, y	Cubic parabola. Approximate form, a truncated cone	$y^3 = \frac{32\,W}{\pi\,f}\,x$
IV.	Uniform load		Rectangle of uniform breadth, b, and variable depth, y	Straight lines forming a wedge	$y^2 = \frac{3\,w}{f\,b}\,x^2$

		Longitudinal elevation of beam	Form of transverse section	Bounding lines of elevation or plan	Equation for determining the dimensions
V.	Uniform load		Rectangle of uniform depth and variable breadth, z	Parabolas with vertex at free end	$z = \frac{3w}{fh^2}x^2$
		BEAMS SUPPORTED AT EACH END.			
VI.	Single load		Rectangle of uniform breadth, b, and variable depth, y	Two parabolas and straight line. Approximate form, truncated pyramids	$y^2 = \frac{6\,W\,l_2}{bfl}x$ $y'^2 = \frac{6\,W\,l_1}{bfl}x'$
VII.	Uniform load		Rectangle as above (w lbs. per inch uniformly distributed)	Ellipse and straight line. Approximate form, circular arc and straight line	$y^2 = \frac{3}{4}\frac{w}{fb}(l^2 - 4x^2)$
VIII.	Single load		Rectangle of uniform depth, h, and variable breadth, z'	Straight lines	$z = \frac{6\,W\,l_2}{h^2fl}x$ $z' = \frac{6\,W\,l_1}{h^2fl}x'$

		Longitudinal elevation of beam	Form of transverse section	Bounding lines of elevation or plan	Equation for determining the dimensions
IX.	Uniform load	 Plan	Rectangle as above. (Load, w lbs. per inch, uniformly distributed)	Two parabolas	$z = \frac{3(l^2 - 4x^2)w}{4fh^2}$

A beam, supported at each end, is equivalent to two beams encastre at the point where the bending moment is greatest. The forms given for beams encastre at one end, may be used for each segment of a beam supported at both ends.

Resistance and Deflection of Springs

34*a*. *Straight Springs.*—The best form for a straight spring, or that which gives the greatest deflection for a given strength, is the form shown in Case VI. Table VII., the greatest depth, h, being at the centre, where the load is applied. The breadth is uniform. Let l be the length of the spring, h its depth, and b its breadth, in inches. Let P be the force acting on the spring, and δ the deflection due to P

$$\delta = \frac{1}{2} \frac{P l^3}{E b h^3}$$

where E is the coefficient of elasticity of the material. The deflection should not exceed $\frac{1}{10}$th of the length.

Helical and Spiral Springs.—Fig. 20 shows at *a*, *c*, forms of helical, and at *b* a form of spiral spring. Let r be the radius of the cylindrical surface passing through the centre of the coils of the helix; n the number of coils; d the diameter of the wire of which the spring is made; G the coefficient of transverse elasticity of the material; f the greatest safe shearing stress; W the load acting axially, and

not exceeding the greatest safe load; δ the extension or compression due to w; w_1 the greatest safe steady load,

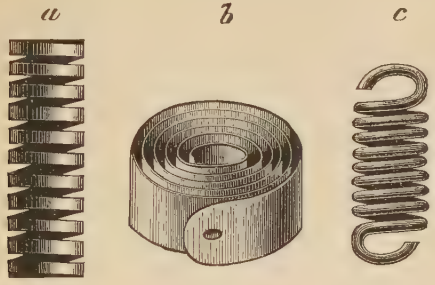


Fig. 20

and δ_1 the corresponding extension or compression. Then, according to Rankine :—

$$\delta = w \frac{64\, n r^3}{G d^4}$$

$$w_1 = \frac{0{\cdot}196 f d^3}{r}$$

$$\delta_1 = \frac{12{\cdot}566\, n f r^2}{G d}$$

Values of G are given in Table I, page 40.

Mr. Hartnell has experimented on the helical springs used for governors and valves. He finds such springs safe with a stress of 60,000 to 70,000 lbs. per square inch for $\frac{3}{8}$-inch wire and with 50,000 lbs. for $\frac{1}{2}$-inch wire. The coefficient of transverse elasticity, G, he found to vary from 13,000,000 for $\frac{1}{4}$-inch wire to 11,000,000 for $\frac{3}{8}$-inch wire. For springs with wire under $\frac{3}{8}$-inch diameter he takes $w_1 = 12{,}000 \frac{d^3}{r}$ lbs. The weight which deflects the spring one inch is $180{,}000 \frac{d^4}{n r^3}$. Hence the deflection with any load $w = \delta = \frac{w\, n r^3}{180{,}000\, d^4}$.

Resistance to Shearing

35. An action which causes sliding parallel to the section considered is termed a shearing action. Thus, the pressure of the cutting edges of an ordinary shearing machine, fig. 21, induces a shearing stress in the plane *a b*. The mean in-

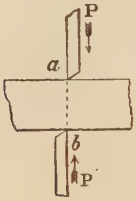


Fig. 21

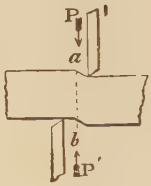


Fig. 22

tensity of the shearing stress is the shearing force P, divided by the area *a* of the section *a b* of the bar. In the case shown, the forces P P act exactly in the plane of the section, and the shearing stress is uniformly distributed, and at all parts of the section $= P/a$. But if the forces P P do not act exactly in the plane of the section, the bar tends to bend as well as to shear (fig. 22). The effect of this is to alter the distribution of the shearing action at all sections between *a* and *b*. Near the middle of the section the shearing stress is greater than the mean shearing stress, and at the upper and lower boundary of the section it becomes zero. A rivet connecting two plates (fig. 23) is almost always in shear, and a bolt is very often so. A cotter or key is similarly intended to resist shear. In these cases, the shearing forces do not act in the plane of the section *b c*, but along the centres of the plates connected. In consequence, how-

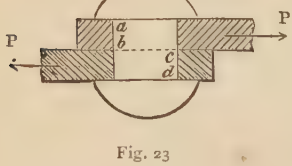


Fig. 23

ever, of the rigidity and friction of the edges $a\ b$ and $c\ d$ of the plates, the points of application of the forces P P on the surface of the rivet may very nearly approach the plane $b\ c$, and then the shearing stress is uniformly distributed on $b\ c$. If, however, the rivet fits very loosely in the rivet holes, fig. 24, the rivet bends and the distribution of stress becomes more or less unequal. For rivets, it is usual to assume that they fit their holes tightly, and that the shearing stress is simply $f_s = P/a$. But for bolts and cotters, it is safer to assume that the stress is unequally distributed, and the maximum stress may then reach the values

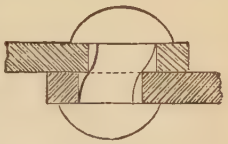


Fig. 24

$f_s = \frac{3}{2}\frac{P}{a}$ if the section is rectangular and P perpendicular to one side;

$= \frac{4}{3}\frac{P}{a}$ if the section is circular or elliptical;

$= \frac{9}{8}\frac{P}{a}$ if the section is square and P acts parallel to a diagonal.

Resistance to Torsion

36. A bar is subjected to simple torsion when two equal and opposite couples act upon it in two planes perpendicular to its axis, instead of being, as in the case of bending, in the plane of the axis. When the bar is subjected to straining action of this kind, any two transverse sections rotate slightly relatively to each other, and on any one transverse section the stress is a simple tangential or shearing stress, varying in intensity as the distance from the centre of the bar, where it is zero, to the circumference, where it is greatest. Of the two couples one, P P′, is usually due to motive forces applied to the bar. The other, P″ P‴, is due to the passive reaction of the parts to which the bar is

attached, or to the resistances which are being overcome. Further, of the two forces P, P′, constituting the former couple, one of the two forces, for instance, P′, may be due

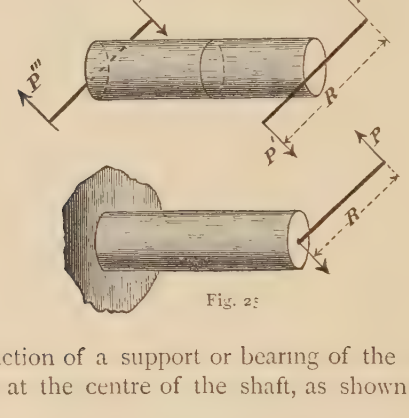


Fig. 25

to the reaction of a support or bearing of the shaft, and it then acts at the centre of the shaft, as shown in the lower figure.

The amount of straining action at any section $a\,b$ is measured by the moment of the couple on either side of the section. In this case that moment, termed the twisting moment, is $T = P\,R$. If several couples act on one side of the section, the algebraic sum of the moments of all those couples is to be taken, right-handed couples being considered positive, and left-handed couples negative.

When the bar is kept in rotation overcoming a resistance, and the amount of work transmitted is known, the twisting moment is easily found. Let HP be the number of horse power transmitted, N the number of revolutions of the bar per minute. Then the work expended in inch lbs. per minute is $12 \times 33000 \times HP$, and this is equal to the twisting moment T in statical inch lbs., multiplied by the angular motion $2\,\pi\,N$ of the bar in the same time. Hence

$$T = \frac{12 \times 33000 \times HP}{2\,\pi\,N} = 63024\,\frac{HP}{N} \text{ inch lbs.} \quad . \quad (14)$$

The moment of resistance of any section to twisting is proportional to the greatest stress at any part of the section, and to a function of the dimensions, which is termed the modulus of the section with respect to torsion. Let f be the greatest shearing stress, and z_t the modulus : —

$$T = f z_t \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

For cylindrical bars of diameter d,

$$z_t = \frac{\pi}{16} d^3 = \frac{d^3}{5{\cdot}1} = 0{\cdot}196\, d^3.$$

For hollow cylindrical bars having d_1, d_2, for outside and inside diameters,

$$z_t = \frac{\pi}{16} \cdot \frac{d_1^4 - d_2^4}{d_1} = 0{\cdot}196 \frac{d_1^4 - d_2^4}{d_1}.$$

The older theory of torsion is known to give inaccurate results for sections not circular. The following are results of the newer theory.

For bars of square section, s being the side of the square

$$z_t = 0{\cdot}208\, s^3$$

For bars of rectangular section, breadth b and height h,

$$z_t = \tfrac{2}{9}\, b^2 h$$

It is sometimes necessary to know the angle through which one end of a bar rotates, relatively to the other end, that is the rate of twist when subjected to torsion. For a cylindrical bar, let d be the diameter, l the length and G the modulus of transverse elasticity ; then the angle of torsion is, in circular measure,

$$\theta = \frac{2\,T\,l}{G\,z_t\,d} = \frac{2\,f\,l}{G\,d} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where f is the greatest actual stress due to the twisting moment. Values of G are given in Table I, page 40.

Resistance to Combined Compression and Bending

37. When a bar, of moderate length, is subjected to a thrust, acting in the direction of its axis, the stress on each

cross section is a simple pressure. The working strength of the bar is given by the equation

$$P_1 = f A \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where P_1 is the greatest safe load, A the sectional area of the bar, and f the safe working compressive resistance of the material.

38. If, however, the bar is of great length, it gives way, ultimately, under the action of a thrust, by lateral bending ; the stress, at the section, where fracture occurs, being a compound stress, due both to the longitudinal pressure, and the curvature of the bar. Rules for the ultimate resistance of long bars to compression were first obtained theoretically by Euler and experimentally by Professor Hodgkinson. Hodgkinson's formulæ have been very generally used in this country, in designing compression bars. These rules are inconvenient in form, and they can only be extended to many cases of common occurrence by theoretical assumptions, which are probably only approximately true. An expression more convenient in form than Hodgkinson's was proposed by Tredgold, revived by Gordon, and afterwards modified by Rankine, so as to be applicable to bars of all forms of section. The reasoning on which this rule is based is, however, not satisfactory. All these rules are intended to give the ultimate strength of the bars, and in applying them it is necessary to divide the resistance thus calculated by an arbitrary factor of safety. Actual compression bars are not intended to be loaded beyond their elastic limit ; and hence it may be urged, with reason, that the theoretical formulæ of Euler, which Hodgkinson discarded, as not agreeing with his experiments on ultimate strength, are more strictly applicable to the circumstances in which long compression bars are used than Hodgkinson's rules. They are simpler and include all cases. Euler's rules assume the bar to be initially straight and homogeneous, the load exactly axial, and the elasticity to be unimpaired

with the greatest load. In that case, no increase of the load would directly cause bending, but a point is reached at which the equilibrium of the bar becomes unstable. With less loads the bar, if slightly bent temporarily, will restore itself to straightness. With greater loads, if any flexure is produced, however slight, the bar will not restore itself to straightness, but the bending will increase under the action of the load till the bar breaks.

Let E be the coefficient of elasticity of the material, A the area of section, and I the least moment of inertia about any axis through the centre of figure of the section (that is, I is the moment of inertia about an axis through the centre of figure of the section at right angles to the plane in which the column most easily bends). Let ρ be the least radius of gyration of the section, so that $\rho^2 = I/A$, and let λ be the length of an arc of the curved bar measured between two points of contrary flexure. Then the greatest load consistent with stability is

$$P = \pi^2 \frac{IE}{\lambda^2} = 10 \frac{IE}{\lambda^2} \text{ nearly} \quad . \quad . \quad . \quad (18)$$

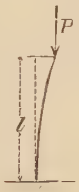
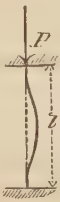
The greatest stress consistent with stability per unit of sectional area is

$$p = 10 \frac{\rho^2 E}{\lambda^2} \quad . \quad . \quad . \quad . \quad . \quad (18a)$$

The values of P and p must be divided by a suitable factor of safety to find the working load and safe working stress. Thus it is usual for wrought iron or steel to use a factor of safety 5 ; for cast iron 6, for wood 10. The following table gives values of λ, P, and p for the cases which arise in practice.

The formulæ in Table VIII. give the fracturing load of the long bar or column, supported in different ways, when E is the coefficient of elasticity. But if n E is substituted for E, we get the safe working load.

TABLE VIII.—*Strength of Long Compression Bars. Greatest Load consistent with Stability*

		Mode of fixing	Length of arc λ in terms of total length	Greatest load P consistent with stability	Greatest stress P per unit of area consistent with stability
I.		One end free, the other fixed.	$\lambda = 2l$	$\frac{\pi^2}{4}\frac{EI}{l^2}$	$2\frac{1}{2}\frac{E\rho^2}{l^2}$
II.		Both ends free, but guided in the direction of the load.	$\lambda = l$	$\pi^2\frac{EI}{l^2}$	$10\frac{E\rho^2}{l^2}$
III.		One end fixed, the other free, and guided in direction of the load.	$\lambda = \frac{l}{\sqrt{2}}$	$2\pi^2\frac{EI}{l^2}$	$20\frac{E\rho^2}{l^2}$
IV.		Both ends fixed in direction.	$\lambda = \frac{l}{2}$	$4\pi^2\frac{EI}{l^2}$	$40\frac{E\rho^2}{l^2}$

	E =	n E =
Wrought iron	29,000,000	5,800,000
Steel . .	30,000,000	6,000,000
Cast iron .	17,000,000	2,800,000
Wood (hard)	1,500,000	150,000
,, (soft) .	1,400,000	140,000

Values of the coefficient of elasticity E for some other materials are given in Table I., p. 40.

The following Table gives the most useful values of the least moment of inertia and radius of gyration of different forms of section.

	Least Moment of Inertia I	Square of least radius of gyration ρ^2
Circular section (diameter d) . .	$\cdot 05\, d^4$	$\frac{1}{16} d^2$
Annular section (diameters D, d) .	$\cdot 05\,(D^4 - d^4)$	$\frac{1}{16}(D^2 + d^2)$
Square section (side s) . . .	$\frac{1}{12} s^4$	$\frac{1}{12} s^2$
Rectangular section (longer side b, shorter h)	$\frac{1}{12} b h^3$	$\frac{1}{12} h^2$
Thin square cell (side s)	$\frac{1}{6} s^3$	$\frac{1}{6} s^2$
Thin rectangular cell (shorter side h, longer b) .		$\frac{h^2}{12} \cdot \frac{h + 3b}{h + b}$
Thin circular cell (diameter d)	$\frac{1}{8} d^3$	$\frac{1}{8} d^2$
Equal angle iron (width b)		$\frac{1}{24} b^2$
Equal armed cross (width of arms h) . . .		$\frac{1}{24} h^2$

39. In applying the above rules, it is assumed that the length is so great that the working stress is less than the safe crushing pressure. For bars of moderate length, when—

λ is less than $24\,d$ or $28\,h$ for wrought iron ;
,, ,, $10\,d$ or $11\frac{1}{2}\,h$ for cast iron ;
,, ,, $11\frac{1}{2}\,d$ or $13\,h$ for wood ;

the rules in Table VIII. are not applicable. The following empirical rules, suggested by Grashof, may then be used :—

For a bar free at both ends as in Case II.

$$P_3 = \frac{k_1 A I}{\frac{A l^2}{C_1} + I} \text{ or } = \frac{k_2 A I}{\frac{A l^2}{C_2} + I} \quad . \quad . \quad . \quad (21)$$

the lesser value given by these equations is to be taken. For a bar fixed at both ends as in Case IV

$$P_3 = \frac{4 k_1 A I}{\frac{A l^2}{C_1} + 4 I} \text{ or } = \frac{4 k_2 A I}{\frac{A l^2}{C_2} + 4 I} \quad . \quad (21a)$$

In these equations A is the sectional area of the bar perpendicular to the axis ; I is the moment of inertia of the section about an axis through the centre of gravity, and perpendicular to the plane in which bending is most likely to occur ; l is the length, and the other quantities are constants, the values of which are given in the following table. P_3 is the greatest safe load, the lesser of the two values being always taken.

	C_1	C_2	k_1	k_2
Steel . .	5000	5000	12000	12000
Wrought iron .	5600	5600	10000	10000
Cast iron . .	10000	2400	3000	12000
Wood . .	1500	2600	900	500

40. *Resistance of thin cylinders such as Boiler Flues. Resistance to collapse.*—Suppose a thin cylinder having rigid ends, which prevent deformation at those points, is subjected to a uniform external pressure. Then a compressive stress is induced in the material, the intensity of which is found by the same formula as that already given for bursting (§ 26). If d is the diameter of the cylinder, t its thickness, p the external pressure per unit area, and f the stress per unit of section of the cylinder,

$$p = \frac{2ft}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Under the action of an external pressure, however, the form of the cylinder is unstable, while, if the pressure is internal, it is stable. In the former case the pressure tends to increase any deviation from a truly circular form, in the latter case to diminish it. When the cylinder is sufficiently thin, long and large in diameter, it sometimes gives way quite suddenly under the action of the external pressure, being crushed up into the forms shown in fig. 26.

A tube which gives way thus is said to collapse, and the form of yielding is extremely characteristic. Re-examining Fairbairn's experiments some years since,[1] the author found

Fig. 26

that there was an absolutely clear rule for the whole of these experiments, connecting the number of lobes formed in collapsing with the ratio of length to diameter of the tubes. Thus, calling *a* a curve of four segments, *b* a curve of six segments and so on

For $\frac{l}{d}$ = ...	15 to $7\frac{1}{2}$	$7\frac{1}{2}$ to $4\frac{3}{4}$	$3\frac{3}{4}$ to $2\frac{1}{2}$
The number of segments in the collapsed tube was .	4	6	8 or 10

It is obvious that a ring of the thin cylinder under compression is in a condition quite similar to that of the long columns in Table VIII. and the segments of the collapsed tube correspond to the arc lengths λ in that Table. By using the equations there given, the formula is obtained

$$p = \frac{E}{6} \frac{n^2 t^3}{d^3} \quad . \quad . \quad . \quad . \quad . \quad (23)$$

for the relation between the collapsing pressure and the dimensions of the tube. A comparison of this formula with Fairbairn's experimental results shows that, just as in the case of long columns, there is a considerable discrepancy between the theoretical and actual collapsing pressure. That is due, no doubt, chiefly to a small deviation of the

[1] Resistance of Boiler Flues to Collapse, *Proc. Inst. Civil Engineers*, vol. xlvi.

resultant pressure on sections of the tube from the centres of the sections, the deviation being due to imperfections of form of the cylinders. But there are other reasons, especially the unknown condition of longitudinal stress in the cylinder, which affect the result. In particular it is clear from the experiments that the collapsing pressure does not increase quite so fast as t^3. Hence we are driven to seek in the experimental results for an empirical modification of the rational formula. Further, it should be at once pointed out that there are limits in the proportions of the tube within which alone any collapse formula should be applied. If the tube is very short, it will crush, or at all events safe limits of crushing stress will be exceeded before collapse becomes possible. Again, if the tube is so long that it collapses into four segments, its minimum collapsing pressure will be reached. It cannot collapse into a less number of segments, and its strength will not diminish with any further increase of length.

41. *Empirical formulæ.*— Let t be the thickness, d the diameter, and l the length between the rigid ends of a tube which collapses with a uniform external pressure p lbs. per sq. in. Then Fairbairn originally obtained the formula—

$$p = 9{,}672{,}000 \frac{t^{2\cdot19}}{l\,d} \quad . \quad . \quad . \quad . \quad (24)$$

This can easily be solved by logarithms, and in a logarithmic form the equation becomes

$$\log p = 6\cdot9855 + 2\cdot19 \log t - \log(l\,d)$$

All the dimensions are in inches.

Thickness of plate t	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$
	·25	·3125	·375	·4375	·5
$t^{2\cdot19}$	·048	·078	·117	·164	·219

For plates not less than $\frac{3}{8}$ inch thick, it is generally accurate enough to take

$$p = 9{,}672{,}000 \frac{t^2}{l\,d} \quad . \quad . \quad . \quad . \quad (24a)$$

which is easy to use. The factor of safety in practice for long flues is often as low as 3 or 4.

In an examination of Fairbairn's results and such other data as were available, the author obtained the following formulæ, which agree better with the data, especially for the thicker tubes, than Fairbairn's formula.

For tubes with a longitudinal lap-joint,

$$p = 7{,}363{,}000 \frac{t^{2\cdot1}}{l^{0\cdot9}\, d^{1\cdot16}} \quad . \quad . \quad . \quad . \quad (1)$$

For tubes with a longitudinal butt-joint,

$$p = 9{,}614{,}000 \frac{t^{2\cdot21}}{l^{0\cdot9}\, d^{1\cdot16}} \quad . \quad . \quad . \quad . \quad (2)$$

For tubes with longitudinal and cross-joints like ordinary boiler flues,

$$p = 15{,}547{,}000 \frac{t^{2\cdot35}}{l^{0\cdot9}\, d^{1\cdot16}} \quad . \quad . \quad . \quad (3)$$

The working pressure should not exceed about one-fourth the values given by these equations. Although these equations differ in form from Fairbairn's, they do not give greatly different values of the collapsing pressure in the more usual cases. It would seem from the form of collapse, that when l is at least $10\,d$ or $12\,d$ the strength does not decrease with further increase of length.

42. *Collapse rings.*—When a flue is so long and thin that collapse may occur, it may be greatly strengthened by rigid rings riveted to it at intervals. Fig. 27 shows some forms of such collapse rings. At *a* is the T-iron ring, first used by Sir W. Fairbairn ; this is spaced off the flue by ferrules to prevent overheating of the plates. At *c* is another mode of applying a T-iron ring. At *b* is the collapse ring introduced by Mr. Adamson. Mr. Adamson's ring is a solid forged ring $\frac{3}{8}$ to $\frac{1}{2}$ in. thick. The flue plates are flanged out and riveted to this on each side. At *d* is an angle-iron ring spaced from the flue by ferrules through which the rivets pass. The rivets may be 6 or 7 inches apart.

The angle-iron ring is made in halves, and these are connected by wrappers at the joints. This form of collapse ring answers well. At *e* is the Bowling collapse ring which is rolled so as to be without a weld. It serves as an expansion joint also, springing a little if the flue expands more than the shell of the boiler. If a flue is more than double the minimum limit of length for which collapse can occur, it will be doubled in strength by a collapse ring at its centre. If more than three times the minimum limit in length, it is tripled in strength by two equidistant collapse rings. But

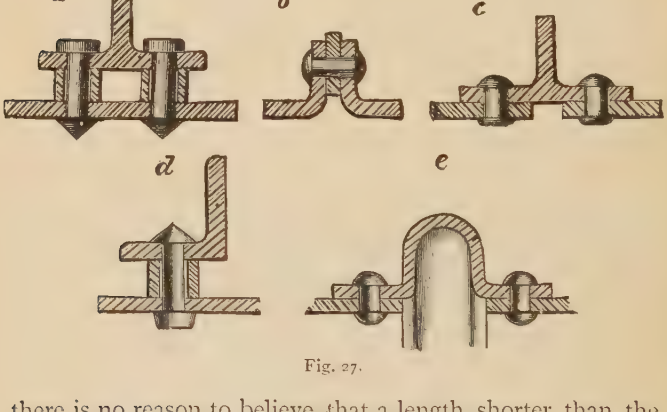


Fig. 27.

there is no reason to believe that a length shorter than the minimum length is increased in strength by a collapse ring, except so far as it may prevent accidental flattening of the flue during work, or strengthen the flue when overheated. When collapse rings are applied at short intervals of about 8 or 10 feet, the effective length of the flue may be taken to be the length between two collapse rings. Then, as will be seen presently, collapse need not be feared, and the crushing formula (22), p. 82, should be used to determine the working strength in all ordinary cases.

Practical Rules for Boiler Flues.—By introducing 'col-

lapse rings' strengthening the flue at intervals, the flue can be virtually shortened as much as we please, and therefore can be always brought to conditions in which collapse is impossible, or in which at least the safe limit of crushing pressure is reached long before there is any danger of collapse. Then the flue can be designed by using the simple rational formula (22). Hence the only practical question which needs to be solved is this : how short must the flue be in order that formula (22) may be safely used and collapse neglected? Now, whatever doubt there may be about the accuracy of the collapse formulæ above, they are at least reliable enough to fix the proportions of flues at which danger of collapse begins. For near those limits there is not much difference between the pressure determined by the collapse formula and that given by the crushing formula. Even if the limits are not very exactly determined it does not matter, for near the limits the application of the crushing formula will give nearly the same working pressure as if the collapse formula had been used.

First, then, as to the crushing formula. For ordinary riveted furnace flues the Board of Trade some years ago adopted the simple crushing formula (equation 22) taking $f = 4{,}000$ lbs. per sq. inch. As about half the metal is punched away at the riveted joint, and the section of the rivets is only about equal to the net section of the plates, this gives a stress of about 4 tons per sq. in. of net section of rivets and plates. This is a moderate working stress, and perhaps is not too low if the danger of a reduction of strength by corrosion or overheating is considered. An examination of a number of actual boiler flues of from 3 to 10 feet in length (or length between collapse rings) shows that the actual working stress in such cases is often less than that assumed in the Board of Trade rule. In most cases f is found to be 3,000 lbs. per sq. in., though in a few cases it is double this. Hence the simple rule for

the working pressure consistent with these values of the crushing becomes

$$\text{Working pressure} = p = c\,\frac{t}{d} \quad . \quad . \quad . \quad (a)$$

where c = 12,000 in extreme cases ; 8,000 according to the Board of Trade ; and 6,000 in most actual boiler flues.

Next, as to the collapse formula. Taking Fairbairn's empirical rule as the simplest, there are many actual boiler flues of about 30 feet length and 30 to 36 inches in diameter working with a pressure which would be given by taking the constant in Fairbairn's formula 3,500,000, so that

$$\text{Working pressure} = p = 3{,}500{,}000\,\frac{t^2}{l\,d} \quad . \quad . \quad (b)$$

The factor of safety is probably not greater than 3.

Now if equation (*a*) gives a less working pressure than equation (*b*), flues designed by (*a*) will obviously be safe against collapse. This is the case if

$$c\,\frac{t}{d} < 3{,}500{,}000\,\frac{t^2}{l\,d} ;$$

that is, if

$$\begin{aligned} l &< 580\,t \text{ for } c = 6{,}000 \\ &\quad 430\,t \quad \text{,,} \quad 8{,}000 \\ &\quad 290\,t \quad \text{,,} \quad 12000 \end{aligned}$$

Here l is the whole length of flue if there are no collapse rings, and the greatest length between collapse rings in other cases. Hence we arrive at this very simple practical result : If the flue be short, or shortened virtually by collapse rings, so that the length is less than the values given below, the flue ought to be designed by equation (*a*), and it is unnecessary to use any collapse formula. It is obvious that all actual flues can be brought within limits for which collapse is not to be feared, and can then be designed by equation (*a*).

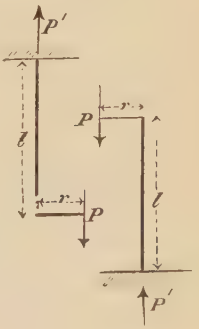
For a corrugated flue formula (*a*) may be used, putting

Thickness of Plates	Greatest length of flue in inches		
	c = 12,000	c = 8,000	c = 6,000
$\frac{3}{8}$	108	162	216
$\frac{7}{16}$	144	215	288
$\frac{1}{2}$	180	270	360
$\frac{9}{16}$	216	322	432

for t the virtual thickness, that is the actual thickness multiplied by the ratio of length along corrugations to length of flue.

Compound Stress

43. I. *Tension or pressure, combined with bending.*— When a force P, fig. 28, acts in a plane passing through the axis of the bar, and parallel to that axis, the stress on transverse sections of the bar is equivalent to that due to a direct tension or pressure P, and a bending moment Pr. The greatest stress is then



$$f = \frac{P}{A} + \frac{Pr}{Z} \quad . \quad . \quad . \quad (25)$$

where A is the transverse sectional area, and Z the modulus of the section. The greatest stress will be a tension or pressure, according as P tends to extend or to compress the bar.

II. When the force P acts as above, but not parallel to the axis of the bar, fig. 29, its direction will intersect that axis, at some distance l from the point of support. At that point, resolve the force P into its components. The component $P \cos \theta$ produces a simple tension, or compression; the other component, $P \sin \theta$, produces bending, the greatest bending moment being $P\, l \sin \theta$ Then

the greatest stress at the section nearest the point of support is

$$f = P\left(\frac{\cos\theta}{A} + \frac{l\sin\theta}{Z}\right) \quad . \quad . \quad . \quad (26)$$

44. III. *Combined twisting and bending.*—Let the force P act in a plane perpendicular to the axis of the bar, at a distance r from the axis, and at a distance l from the point of support. The force P will give rise to a parallel reaction

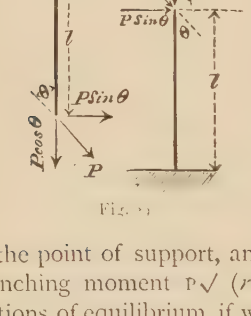


Fig. 29

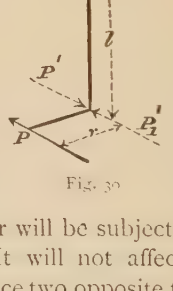


Fig. 30

P_1 at the point of support, and the bar will be subjected to a wrenching moment $P\sqrt{(r^2+l^2)}$. It will not affect the conditions of equilibrium, if we introduce two opposite forces P', P'_1, each equal to P or P_1. Then the wrenching moment will be seen to be equivalent to a simple twisting moment, due to P and P', and a bending action, due to P_1 and P'_1. The twisting moment is $T = P\,r$, and the greatest bending moment is $M = P\,l$.

Let M_e be a simple bending moment, which would produce an effect on the bar, equivalent to that due to the combined bending and twisting action. Then the theory of elasticity furnishes the two following values of M_e, according as we have regard to the greatest stress or the greatest strain induced in the bar :

$$M_e = \tfrac{1}{2}M + \tfrac{1}{2}\sqrt{(M^2+T^2)} \quad . \quad . \quad . \quad . \quad (27)$$

$$= \tfrac{3}{8}M + \tfrac{5}{8}\sqrt{(M^2+T^2)} \quad . \quad . \quad . \quad . \quad (27a)$$

The former value will be used in this Treatise. It can be put in the simpler approximate forms—

$$\left.\begin{aligned} M_e &= 0{\cdot}98\ M + 0{\cdot}2\ T \quad \text{if } l > r \\ &= 0{\cdot}7\ M + 0{\cdot}48\ T \quad \text{if } r > l \\ &\text{and with less close approximation,} \\ &= 0{\cdot}914\ M + 0{\cdot}414\ T \text{ if } r \text{ and } l \text{ are unknown} \end{aligned}\right\} \quad (28)$$

The greatest safe load in the above case is

$$P = \frac{2fZ}{l + \sqrt{(l^2 + r^2)}} = \frac{fZ}{a\,l + b\,r} \text{ nearly} \quad (29)$$

where a and b are the numerical values of the constants in the approximate formulæ (28) given above.

If an equivalent twisting moment is required instead of an equivalent bending moment, let T_e be that moment. Then

$$T_e = 2M_e = M + \sqrt{(M^2 + T^2)} \quad (29a)$$

45. *Stability of a shaft subjected to twisting and thrust.*— A long shaft like the screw shaft of a steamer becomes unstable in form, if the length between the bearings is too great, the action being similar to that which occurs with long columns as described above. Let l be the length of the shaft between the bearings; P the greatest load consistent with stability as in § 38; T the twisting couple on the shaft, which may be obtained from the H.P. as in § 36. Then, if the twisting moment is neglected, we know already that

$$\frac{\pi}{l^2} = \frac{P}{E\,I}; \text{ and hence } P = \pi^2 \frac{E\,I}{l^2}.$$

But if the twisting moment is taken into account,

$$\frac{\pi^2}{l^2} = \frac{P}{E\,I} + \frac{T^2}{4\,E^2\,I^2} \quad \text{and} \quad P = \pi^2 \frac{E\,I}{l^2} - \frac{T^2}{4\,E\,I},$$

which can be obtained in a similar manner. This formula is due to Prof. A. G. Greenhill, 'Proc. Inst. of Mech. Engineers,' 1883, p. 182.

Strength of Flat Plates

46. I. A flat plate, of thickness t, is supported, but not fixed, on a circular support of radius r, and is uniformly loaded with p lbs. per sq. in. (fig. 31). Then the greatest stress is

$$f = \tfrac{5}{6}\frac{r^2}{t^2}p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Suppose for instance the plate is of wrought iron with a tenacity of 48,000 lbs. per sq. in. Taking a factor of safety

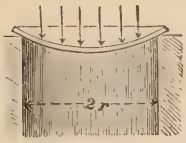


Fig. 31

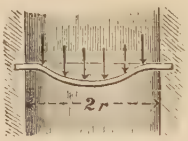


Fig. 32

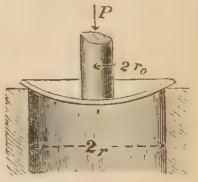


Fig. 33

of 8, the working stress f would be 6,000 lbs. per sq. in. Now let the radius be 10 inches and the thickness $\frac{1}{2}$ inch. Then the greatest distributed pressure p, if the working stress is not to be exceeded, is

$$p = 6{,}000 \times \frac{6}{5} \times \frac{0{\cdot}5^2}{10^2} = 18 \text{ lbs. per sq. in.}$$

II. A circular flat plate, of radius r and thickness t, is encastré at the edge, and is uniformly loaded with p lbs. per sq. in. (fig. 32). Then the greatest intensity of stress is

$$f = \tfrac{3}{8}\frac{r^2}{t^2}p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

III. A circular plate, of radius r and thickness t, is supported at the edge, and loaded with a concentrated load P, applied at a circumference, the radius of which is r_0 (fig. 33). The greatest stress is

$$f = \left(\tfrac{4}{3}\ \log.\ \frac{r}{r_0} + 1\right)\frac{P}{\pi t^2} \quad . \quad . \quad . \quad . \quad (32)$$

$\frac{r}{r_0} =$	10	20	30	40	50
$\frac{4}{3}\log.\frac{r}{r_0} + 1 =$	4·07	5·00	5·53	5·92	6·22

The above rules are due to Grashof.

IV. *Strength of stayed surfaces.*—A flat plate, of thickness t, is supported uniformly by stays arranged in lines (fig. 34). Distance of stays from centre to centre $= a$, uniform load $= p$ lbs. per sq. in. The greatest stress in the plate is

$$f = \tfrac{2}{9}\frac{a^2}{t^2}p \quad . \quad . \quad . \quad . \quad (33)$$

Fig. 34.

Each stay supports $p\,a^2$ lbs.

V. A rectangular plate, of thickness t, length l, and breadth b, is encastré at the edge, and loaded uniformly with p lbs. per sq. in. The greatest stress is

$$f = \tfrac{1}{2}\frac{l^4}{l^4 + b^4}\frac{b^2}{t^2}p \quad . \quad . \quad . \quad . \quad (34)$$

VI. A square plate, s inches in length of side, is similarly supported and loaded. The greatest stress is

$$f = \tfrac{1}{4}\frac{s^2}{t^2}p \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

If in these formulæ the working stress f is assumed, the working pressure p can be determined. The formulæ are applicable to thin flat plates not strained beyond the elastic limit. Any arching or dishing of the plates alters the conditions considerably.

The Board of Trade rule for the flat stayed surfaces of marine boilers is as follows :—

Let p be the safe working pressure in lbs. per sq. in. ; t the thickness of the plate in inches ; s the area of surface supported by one stay in sq. ins. ;

$$p = \frac{c\,(16t+1)^2}{s-6}.$$

The constant c has the following values :—

	Plates not exposed to fire	Plates with steam on one side and fire on the other	Plates with water on one side and fire on the other
Stays with nuts and washers three times the diameter of the stay, and $\frac{3}{8}$rds the plate thickness.	100	60	—
Stays with nuts only .	90	54	—
Stays screwed into plate and nutted . . .	—	—	80
Stays screwed into plate and riveted . . .	—	36	60

The strength of stays will be considered in § 75, p. 135.

CHAPTER IV

ON FASTENINGS

Riveted Joints

47. The simplest fastening is the rivet, employed to unite wrought iron, soft steel or copper plates. A rivet is virtually a bolt, with the head, body and nut in one piece. It is a permanent fastening, only removable by chipping off the head. Bolts are most often used with the straining force parallel to the axis, so that the bolt is in tension ; but rivets are almost always placed at right angles to the straining force, so as to be in shear. They are not reliable in tension.

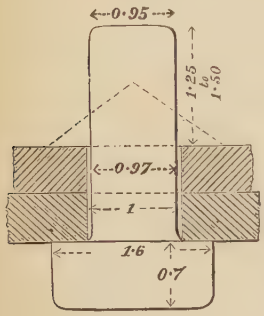


Fig. 35

A rivet is formed of round bar, and, when ready for use, has the form shown in fig. 35. It is parallel for about half its length, and very slightly tapers for the remainder. The head is cup-shaped, or, more often, pan-shaped, as shown. For iron plates, the rivets are of very soft uniform iron, and for steel plates they are of either iron or soft steel. They are made in rivet-making machines of various kinds, being pressed, while red hot, in suitable dies. When used, the rivets are again heated to red heat, placed in the rivet hole in the plates to be connected, and then the

second head is formed by hand, or by machine. In hand riveting, the tail of the rivet is held up, while the head is formed by two riveters working with hammers, and the head is either made conical by the hammers alone, or finished by the aid of a cup-shaped die, called a snap. In machine riveting, the rivet is pressed between two dies, actuated by a lever, or by steam or hydraulic pressure. In machine-riveting it is of importance that the plates should be well bolted together during riveting, or collars are formed on the rivet between the plates. Machine riveting causes the rivet to fill up the holes more perfectly than hand riveting, and makes a closer and stronger joint, especially if the plates are thick. Steel rivets at first gave trouble from the injury to the quality of the steel in reheating the rivets. Hence for a time iron rivets were used with steel plates, although this involved a sacrifice of part of the advantage of using steel plates. Now steel rivets are generally used with steel plates. The rivet steel, however, is of a very soft quality, and its shearing resistance is not very much greater than that of rivet iron. Very good rivet iron or rivet steel may be riveted up cold. This process is often used when the plates are less than $\frac{3}{8}$ inch thick, as, for instance, in constructing gasometers.

Punching and drilling rivet holes.—Rivet holes are most commonly made by punching. This somewhat rough process is objectionable on two grounds. The spacing of the rivet holes is not perfectly accurate, so that when two plates are brought together, the holes are not in perfect register. Next the metal round the hole is injured by lateral flow of the metal under the pressure of the punch. With very rigid material the metal between the rivet holes may be cracked through.

The intensity of the pressure on the punch increases as the plates are thicker, and hence the pressure of fluidity of the material is more likely to be reached and the consequent lateral flow is more likely to occur the thicker the plates

which are punched. The injury done by punching to the material may, however, be entirely removed in two ways. If the hole is punched $\frac{1}{16}$ inch smaller than is required and rymered out to size, the strained material is removed and the plate is then found to be uninjured. Or, if the plate is annealed after punching, the injury is obviated. On the whole steel plates, especially thick steel plates, have been found to be more injured by punching than wrought-iron plates. The practice with steel plates at the present time is this : Plates less than $\frac{1}{2}$-inch thick are punched, and are not generally annealed after punching. Plates more than $\frac{1}{2}$-inch thick should be annealed after punching, or the holes punched small and rymered out. Plates of an inch or more in thickness are better drilled. In boiler work it is desirable to remove any burr from the edges of the holes, by a very slight countersinking. Punched holes are slightly taper. Hence two punched plates should be placed so that the smaller ends of the rivet holes come together.

The hole made by punching is slightly conical because the diameter of the hole in the die block or bolster is slightly larger than the punch, a clearance space of about $\frac{1}{32}$nd of the rivet diameter being left all round. Sellers makes the hole in die block equal to diameter of rivet + one-fifth thickness of plate. Sometimes this conicity is entirely removed by rymering out the holes before riveting. If this rymering is done after the plates to be riveted are brought together, it insures the perfect agreement of the corresponding holes. The old plan of driving a conical drift into the rivet holes is an objectionable method of insuring agreement, as it very imperfectly smoothes the rivet holes and injures the plates. In boiler and other work required to be staunch, the conicity of the holes is rather advantageous. The two plates are so punched that when brought together the smaller ends of the conical holes are inside the joint and the larger ends outside. The rivet then takes a form which holds the plates together. In

boilers where one end of a plate is over and the other under the adjoining plate, some edges must be punched from one side of the plate, and the other edges from the other side. Sometimes the arris at the edge of rivet holes is removed by a countersink tool, and occasionally the countersink is of sensible depth, as in fig. 38. Riveting of this kind is sometimes used in shipbuilding, and it has the advantage that the rivet head is less likely to break off.

In drilling rivet holes the metal is removed by cutting, the material round the hole is uninjured, and the holes are more accurate in size and spacing. The arris formed at the edge of the drilled hole should be removed. On the other hand, the sharp, square edge of a drilled hole appears to be unfavourable to the resistance of the rivet. Sir W. Fairbairn showed that the resistance of the rivet was increased by slightly rounding the edges of the hole.

When the riveting is done at red heat, the contraction of the rivet, in cooling, nips the plates powerfully, and causes considerable tension on the rivet. In very long rivets, this may cause fracture of the rivet, and to prevent this the tail end is cooled before placing it in the rivet hole. In ordinary riveting, the contraction is advantageous in securing staunchness of the joint. Further, the contraction creates frictional resistance to slipping between the plates, which enables the joint to sustain a considerable force, even when the rivets do not fit the holes. The tension in the rivet may be estimated at 21,000 lbs. per sq. in. of its section, and the friction due to this would be about 7,000 lbs. per inch of rivet section. Experiments show a still greater friction, but if the tension in the rivet exceeds the elastic limit, its permanence cannot be relied on. English engineers entirely neglect the friction, in estimating the strength of the joint, the reasons assigned being that the amount of tension in the rivet is not ascertainable, and that vibrations and other causes, tending to slightly elongate the rivet, may, in course of time, destroy it altogether.

If the thickness of the plates, through which a rivet passes, is 6 inches or more, it is better to use bolts instead of rivets.

The staunchness of the joint, or its power of resisting the tendency to leak, when subjected to steam or water pressure, depends on the nearness of the rivets to the edge of the plate, and their nearness together. The metal between two rivets is in the position of a beam subjected to uniform pressure, and tending to deflect. If the joint is not naturally staunch, it may be rendered so by caulking, that is, burring down a narrow strip at the edge of the plate by a chisel (fig. 36 *c*). In fig. 36 the condition of the plates before caulking is shown at *a*. At *b* is a fullering tool used to close up the plates; at *c*, a caulking tool used to burr down the edge of the plate. Mr. Webb does not use a caulking tool at all, but only a fullering tool like, *b* but

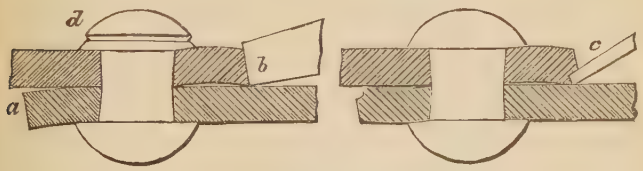


Fig. 36

having a small projection, which indents the plate at the middle of its thickness. In the best boiler work the plates are planed on the edges with a slight bevel, before riveting, and this much facilitates the closing of the joints by fullering or caulking. It is a point of importance in boiler work to arrange the joints so that they can be caulked. The rivet heads are sometimes caulked also, as shown in fig. 36, *d*.

Mr. Webb has found that the magnetic oxide on the surface of plates as they come from the rolling mill diminishes the staunchness of the joint. He has adopted the plan of sponging the surfaces of the joint with a solution of sal ammoniac before riveting. The plates then adhere

more closely and require less caulking. He afterwards sponges the whole interior of the boiler with sal ammoniac to prevent irregular corrosion in working.

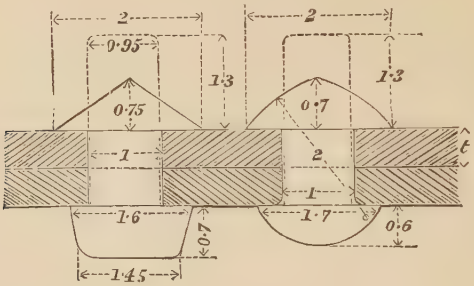


Fig. 37

48. *Forms of rivets.*—Fig. 37 gives the proportions of the rivets commonly used in hand-riveting, the heads formed by the riveter being of conical form. Fig. 38 gives proportions for the rivets generally used in machine riveting. In

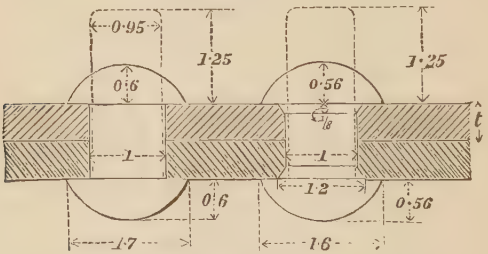


Fig. 38

boiler work the rivet head is rather larger than for girder work. Fig. 39 shows a countersunk rivet, which is only used when the surface of the plate must be fair and without projections. Countersunk rivets weaken the plate more, and are less reliable than ordinary rivets. The proportions of the head vary from 1·5 in diameter and 0·5 in height to 1·6

in diameter and 0·4 in height. The conical heads shown in fig. 37 are formed entirely by hand hammers, and are not finished with a snap. They are most used where there is restricted space for hammering, and are less reliable than cup or spherical heads. The cup-shaped head may be formed in hand riveting by a die or snap, which requires the use of a sledge hammer.

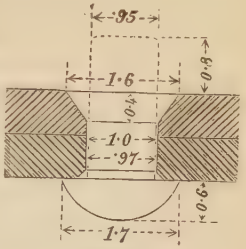


Fig. 39

The numbers on the figures are proportional to the diameter of the rivet, and give good ordinary proportions, although it must be remembered that the sizes used by different engineers vary more or less. To fill the rivet hole and form the head, a length equal to about $\frac{3}{4}$ of the diameter is required in countersunk riveting, and 1·3 to 1·7 times the diameter in ordinary riveting.

Lap and butt riveting.—When one plate is made to overlap the other, and one or more lines of rivets are put through the two, the riveting is lap riveting (figs. 42, 44). When the plates are kept in the same plane, and a cover plate, or butt strap, is put over the joint and riveted to each, the riveting is butt riveting (figs. 43, 45).

Single and double riveting.—If there is one line of rivets in lap riveting, or one line on each side of the joint in butt riveting, the joint is single riveted (figs. 42, 43). If there are two lines in lap, or two lines on each side of the joint in butt riveting, the joint is double riveted (figs. 44, 45).

Single and double shear joints.—When the plates are so arranged that they tend to shear the rivets in a single plane (fig. 42), the joint is a single shear joint. If the plates tend to shear the rivet in two planes, the joint is a double shear joint; such a joint is shown in fig. 47.

Combined lap and butt joint.—A form of joint intermediate between a lap and a butt joint has recently come

into use for locomotive boilers. It is shown in fig. 40. It consists of a lap joint with a cover plate outside the joint. There are three rows of rivets, the middle row having twice

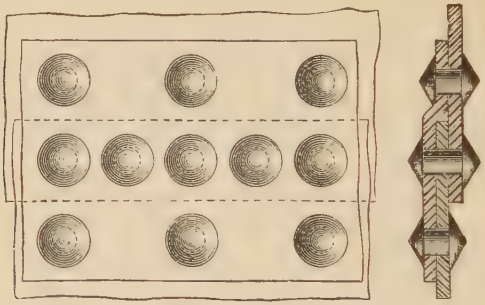


Fig. 40

as many rivets as the outside rows. The ordinary proportions for $\frac{3}{8}$-inch plates are : rivets $\frac{13}{16}$-inch diameter; pitch of middle row 2 inches; pitch of outside rows 4 inches.

49. *Size of rivets for plates of different thickness.*—Let t = thickness of plate, d = diameter of rivet, f_s = resistance of plate to shearing, f_c = resistance of punch to crushing. The area sheared by the punch is $\pi\, d\, t$, and the resistance to shearing is $\pi\, d\, t\, f_s$. The strength of the punch is $\frac{\pi}{4} d^2 f_c$. Hence, if

$$\pi\, d\, t f_s \text{ is greater than } \frac{\pi}{4} d^2 f_c,$$

$$\text{or if } d \text{ is less than } 4\, t \frac{f_s}{f_c},$$

the punch will crush before the plate shears. If $f_c = 4 f_s$, d must not be less than t, or the plate cannot be punched. To allow a margin of safety for the punch, the rivet diameter is rarely less than one and a half times the thickness of the

plate, except when the plates are drilled. The diameter of rivets in practice ranges from

$$d = \tfrac{3}{4}\,t + \tfrac{3}{8} \text{ to } \tfrac{7}{8}\,t + \tfrac{3}{8}$$

and a very simple and convenient rule is

$$d = 1{\cdot}2\sqrt{t} \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

This rule will be adopted in the following calculations.

In practice the real diameter of the rivet in the joint is somewhat greater than its nominal diameter. The rivet hole is made about 4 per cent. larger in diameter than the rivet. In riveting up, the rivet is compressed so as to fill the rivet hole. Hence, in calculations on the strength of the joint, the rivets should be taken about 4 per cent. larger than their nominal diameters. The following table gives the diameters of rivets calculated by this rule, the size to the nearest sixteenth of an inch, and the probable real diameter of the rivet after riveting.

Diameters of Rivets for different thicknesses of Plates

Thickness of Plates *t*.	Diameter of Rivets *d*.		Diameter of rivets after riveting 1·04 *d*.
$\frac{1}{4}$	0·60	$\frac{9}{16}$	0·59
$\frac{5}{16}$	0·67	$\frac{11}{16}$	0·72
$\frac{3}{8}$	0·73	$\frac{3}{4}$	0·78
$\frac{7}{16}$	0·79	$\frac{13}{16}$	0·85
$\frac{1}{2}$	0·85	$\frac{7}{8}$	0·91
$\frac{9}{16}$	0·90	$\frac{7}{8}$	0·01
$\frac{5}{8}$	0·95	$\frac{15}{16}$	0·97
$\frac{3}{4}$	1·04	$1\frac{1}{16}$	1·10
$\frac{7}{8}$	1·12	$1\frac{1}{8}$	1·17
1	1·20	$1\frac{3}{16}$	1·24

50. *Overlap of plates and pitch of rivets.*– If the rivet hole is too near the edge of the plate, the latter is liable to be burst through in punching ; and if the *pitch* of the rivets, or distance from centre to centre, is too small, a crack extends between them. A practical limit is thus fixed for the

minimum overlap and pitch. A good rule is, that the distance from the edge of the rivet hole to the edge of the plate, or to the edge of the next rivet hole, should in no case be less than the diameter of the rivet. We thus get the proportions shown in fig. 41, as the *minimum* proportions of overlap and pitch. The unit is the diameter of the rivet. Very often in practice the minimum proportions shown in fig. 41 are a little increased. Thus the distance from centre of rivet to edge of plate is made $1{\cdot}5\ d + \frac{1}{16}$. The pitch in almost all cases, as will be shown presently, is determined so as to insure the greatest strength ; always, however, the rivet diameter must be so chosen that the pitch is not less than $2\ d$.

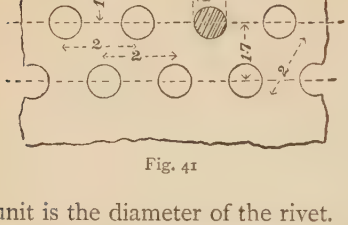


Fig. 41

51. *Proportions of riveted joints.*—Fig. 42 shows an

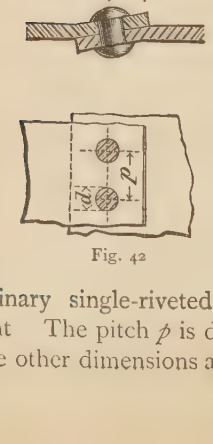
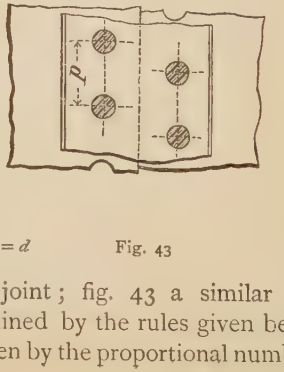


Fig. 42



Unit = d Fig. 43

ordinary single-riveted lap joint ; fig. 43 a similar butt joint The pitch p is determined by the rules given below. The other dimensions are given by the proportional numbers,

the unit for which is the diameter of the rivet. The objection to a lap joint is that the straining force in one plate is not directly opposed to that in the other, but forms with it a couple tending to bend the joint each time the stress is applied, and rendering it weaker in consequence of the bending. Grooving of boiler plates is in some cases indirectly due to this bending. One of the sections in fig. 42 shows the plates bent before riveting so as to diminish the tendency of the joint to deform when strained. The butt joint, fig. 43, with a single butt-strap is subject to nearly the same action of bending as the lap joint. The butt joint with two cover plates is free from bending action, and consequently is stronger. Butt joints are preferable to lap joints for the longitudinal seams of boilers, and in that case should have double cover plates. The cross joints are also sometimes made with a cover strip welded into a ring and shrunk on.

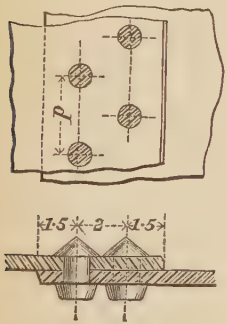


Fig. 44

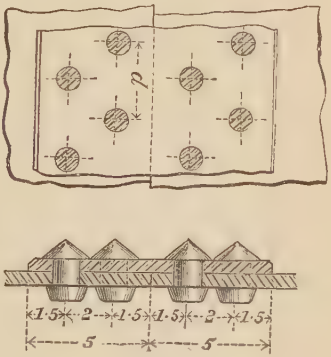


Fig. 45

Figs. 44 and 45 show double riveted lap and butt joints. Another form of lap joint is shown in fig. 46, and the corresponding butt joint in fig. 47. In these the pitch in the back row of rivets is double that in the front row. Such

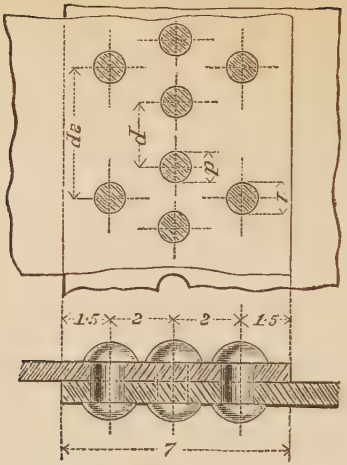


Fig. 46

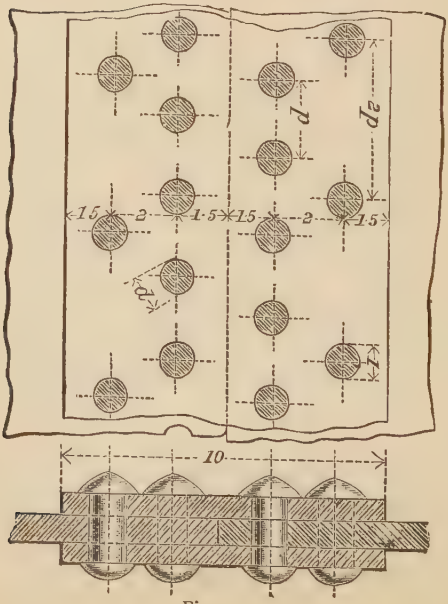


Fig. 47

joints can be made staunch and at the same time stronger than the ordinary form.

It would at first sight appear sufficient to make a single cover plate or butt strap equal in thickness to the plates, and a double butt strap half the thickness of the plates. Some experiments with joints thus made indicated that the butt strap was weaker than the plates. Hence it is preferable to make a single butt strap $1\frac{1}{8}$ of the plate thickness and a double butt strap $\frac{5}{8}$ of the plate thickness, at least in boiler work where strength is of the greatest importance. Butt straps of wrought iron are better cut from plates than made of rolled bars, which are very weak across the fibres.

Strength of Riveted Joints

52. Let d=diameter of rivets after riveting.

p=pitch of rivets.

t=thickness of plates.

l=semi-overlap, or distance from centre of rivet to edge of plate.

f_t=tenacity of material of plates.

f_c=resistance to crushing of plates or rivets.

f_s=shearing resistance of rivets.

T=resistance of a strip of the joint of width p.

Modes of fracture of riveted joints.—Consider, for simplicity, a simple, single-riveted lap joint, subjected to tension.

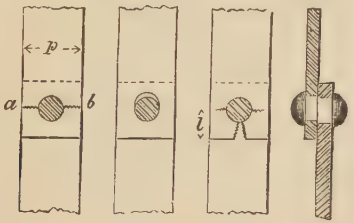


Fig. 48 Fig. 49 Fig. 50 Fig. 51

Since each rivet supports a strip of plate, whose width is p we may consider such a strip, independently of the rest.

Such a strip, subjected to tension, might fracture in four ways.

(1.) The plate may tear across, along the line of minimum section $a\,b$ (fig. 48). The area of either plate at $a\,b$ is $(p-d)\,t$, and the resistance to tension is $f_t\,(p-d)\,t$.

(2.) The plate and rivet may be crushed, as shown in fig. 49, and this will render the joint loose and insecure. The area of the plate or rivet supporting the pressure, estimated normally to the pressure, is $d\,t$, and this is called the bearing area. The resistance to crushing is $f_c\,d\,t$.

(3.) The plate may break across in front of the rivet (fig. 50), the action being similar to the transverse fracture of a bar, fixed at the ends, and loaded at the centre. The bending moment is about $\frac{1}{8}\,\mathrm{T}\,d$. Equating this to the moment of resistance of the section of the plate

$$\mathrm{T}=\tfrac{1}{3}\frac{(2\,l-d)^2\,t\,f_t}{d}.$$

(4.) The rivet may shear across (fig. 51). The area resisting shear is $\frac{\pi}{4}d^2$, and the resistance to shearing is $\frac{\pi}{4}d^2 f_s$.

53. *Condition that there may be the greatest economy of material in the joint.*—The condition which ensures the greatest economy of material is that the resistances of the strip to tearing, crushing, breaking or shearing, should be equal. Hence

$$\mathrm{T}=(p-d)\,t\,f_t \quad . \quad . \quad . \quad . \quad . \quad (2)$$
$$=d\,t\,f_c \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$
$$=\tfrac{1}{3}\,(2\,l-d)^2 t f_t \quad . \quad . \quad . \quad . \quad (4)$$
$$=\frac{\pi}{4}\,d^2 f_s \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For a multiple riveted joint with n rows of rivets, there are n rivets to a strip of the width p. Consequently $\frac{\mathrm{T}}{n}$ must be

substituted for T in equations (3), (4), and (5). If the rivets are in double shear, equation (5) becomes—

$$T = \frac{\pi}{2} d^2 f_s \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

54. *The tenacity of iron and steel used for riveted work.*—Specimens of iron and steel suitable for riveted work, of a length of 4 to 10 inches (exclusive of the ends held in the machine), break in the testing machine with the tensions given in the following table :—

	Tenacity in lbs. per sq. inch.
Iron plates	42,000 to 50,000
Steel plates	58,000 to 66,000
Rivet iron	62,000
Rivet steel	66,000

These tenacities will be termed the original tenacities to distinguish them from the tenacities after the plates have been drilled, punched, or riveted.

Tenacity of drilled plates.—When a row of holes is drilled in a plate and it is then tested, it is forced to break along the line of holes where the section is least. This line will be termed the line of fracture, and is marked *ff* in some of the woodcuts. At first sight it would appear that the tenacity (or stress per unit of section of metal along the line of fracture) should be identical with the original tenacity of the undrilled plate. The balance of experimental evidence is, however, that the tenacity of the drilled plate is 10 or 12 per cent. greater than this. In breaking an ordinary test bar the load causes fracture at the weakest section of a more or less considerable length of bar. In the case of a drilled bar no such selection of a weak section is possible. Besides this, it is probable that the form of the short portions of metal between the holes is such, that the flow of the material during the last stages of testing is more hindered than in an ordinary test bar, and that hence there may be a diminished contraction of area during fracture, and conse-

quently a real gain of strength to resist fracture in the drilled plate.

Tenacity of punched plates.—The experiments on the tenacity of punched plates are extremely discordant. A number of experiments show a loss of tenacity after punching varying from 5 per cent. up to 20 per cent. in iron plates, and from 8 per cent. to 35 per cent. in steel plates. With steel plates the loss increases with the thickness of the plate; with both steel and iron it is diminished by making the hole in the die block one-fourth greater in diameter than the punch; and the loss completely disappears, and the original tenacity is restored, if the plate is annealed after punching; or if a small ring, 0·04 to 0·08 inch in thickness, is rymered out of the punched hole.

There can be no doubt that, in punching, a portion of metal is squeezed laterally into the plate, and a condition of permanent stress is induced in the metal immediately surrounding the hole. It is this which is got rid of by annealing or rymering.

On the other hand, many of the experiments on the effect of punching have been made in an imperfect way. A narrow strip has been prepared for testing, and the hole then punched in it. In that case the material may be greatly injured by the lateral expansion of the unsupported material on each side of the hole. Some experiments on steel plates, in which a wide plate was punched and afterwards slotted into testing strips, showed a tenacity 5 per cent. less than that of similar drilled strips, but still 5 per cent. greater than the original tenacity of the plate.

There are a number of experiments on riveted joints which give so low a tenacity for the net section of the joint, that it is impossible not to suspect some error in the experiments. The Manchester Steam Users Association made some experiments on the bursting of an actual boiler by hydraulic pressure, and afterwards broke some test specimens of the joints in a testing machine. The stress on the boiler joints at the moment of bursting was 18 or 19 tons per sq.

inch, or very nearly the original tenacity of the material. The test specimens, on the other hand, broke with about 13 tons. There is no known reason why there should be so great a difference in the breaking weight in these two cases.

55. *Tenacity of the metal in riveted joints.*—When a portion of riveted joint is tested, further conditions affect the tenacity estimated on the section of fracture. The friction of the plates, if it exists, resists the breaking force and adds to the strength of the joint. But since a considerable relative movement of the plates occurs before fracture, and the rivets are greatly distorted, it is probable that the influence of friction is small or zero. Putting this aside, in most forms of joints, the resultant of the load does not pass through the middle of the plates, and hence the joint bends as well as stretches. The real stress is then greater than the quantity load divided by area of section, and the strength of the joint is diminished.

Let P be the breaking load of a riveted joint which has ω sq. ins. of section, through the weakest section or line of fracture. Then $\frac{P}{\omega}$ is the *apparent tenacity* of the joint, being less than the real tenacity because the stress due to bending is neglected. In general $\frac{P}{\omega}$ will be less than the original tenacity (§ 54) of the material of the joint f_t, although in some cases it may be greater, for the reasons mentioned in discussing the tenacity of plates with drilled holes. Let $\frac{P}{\omega} = kf_t$. Then k is a coefficient depending on the way in which the holes are made and the form of the joint. The following are values of k from various experiments on joints.

IRON PLATES.

	Values of k.
Single-riveted joints, drilled . . .	·88
,, ,, punched . . .	·77
Double-riveted joints, drilled . . .	·95
,, ,, punched . . .	·85

Steel Plates.	
Single-riveted, drilled	1·00
,, ,, punched	·90
Double-riveted, drilled	1·06
,, ,, punched	1·00
Treble-riveted, drilled	1·08

These numbers must be considered as approximations only, experiments giving more or less discordant values. It is to be understood, also, that a plate rymered or annealed after punching is equal in strength to a drilled plate.

Apparent tenacity of iron and steel in different kinds of joints.—Taking the average original tenacity of iron plates at 46,000 lbs. per sq. in., and that of steel plates at 62,000 lbs. per sq. in., the probable apparent tenacity of different joints will be as follows :—

	Apparent tenacity in lbs. per sq. in. f_t	
	Iron Plates.	Steel Plates.
Single-riveted, drilled	40,500	62,000
,, ,, punched	35,400	55,800
Doubled-riveted, drilled	43,700	65,700
,, ,, punched	39,000	62,000
Treble-riveted, drilled	45,000	67,000

56. *Shearing resistance of iron and steel.*—The shearing resistance of iron and steel bars, when sheared in a testing machine, is found to be very constantly $\frac{4}{5}$ths of their tenacity.

	Shearing resistance in lbs. per sq. in.
Rivet iron	49,600
Rivet steel	52,800

57. *Apparent shearing resistance of rivets in riveted joints.*—When a riveted joint gives way by shearing, the apparent shearing stress (load divided by sectional area of rivets) is different from the shearing resistance of the rivets in consequence of the friction and bending, and perhaps from the obliquity of the section at which the rivet shears. It also appears that the sharp edge of a drilled hole causes the rivet to shear with a rather less stress than the blunter edge of a punched hole. The shearing resistance of iron rivets is

given pretty definitely in experiments on riveted joints; but in experiments on steel joints with steel rivets, the shearing resistance varies very greatly. The following table gives values selected from the experiments which appear most reliable.

	Apparent shearing resistance of rivets in riveted joints f_s
Iron rivets, in punched holes . . .	46,000
,, ,, drilled holes . . .	43,000
Steel rivets, in punched holes . . .	53,000
,, ,, drilled holes . . .	49,000

58. *Ratio of apparent tenacity of plates to shearing resistance of rivets.*—Using the values of the apparent tenacity and shearing resistance given above, the following values of the ratio of resistance to tearing and shearing in different kinds of joints are obtained.

Ratio of Tearing and Shearing Resistance $\frac{f_t}{f_s}$, *in riveted joints.*

	Iron plates, Iron rivets		Steel plates, Steel rivets	
	Drilled, or punched and annealed or rymered	Punched	Drilled, or punched and annealed or rymered	Punched
Single-riveted .	0·94	0·77	1·26	1·05
Double-riveted .	1·02	0·85	1·34	1·17
Treble-riveted .	1·05	—	1·36	—

59. *Ratio of section of plates to section of rivets.*—In most tables of the pitch of rivets which have been published, a fixed ratio has been assumed for the tearing and shearing resistance in all forms of joints. In many of these, for instance, it has been assumed that the shearing and tearing resistances are equal. Any examination of experiments on riveted joints will, however, show that the ratio of the tearing to the shearing resistance varies considerably. Now a joint will be strongest when the areas of

plate in the line of fracture and of the rivets in the plane of shearing are such that the joint will give way by shearing or tearing indifferently. Putting ω_s ω_t for the shearing and tearing areas,

$$\omega_s f_s = \omega_t f_t$$

$$\frac{\omega_s}{\omega_t} = \frac{f_t}{f_s}.$$

For if the shearing section is in excess of the amount necessary to balance the tearing section, the rivets are larger than necessary, and the section of fracture is diminished; and if the tearing section is in excess, the rivets will give way before the whole strength of the plate is called into action. Consequently, the shearing and tearing sections should be in the inverse ratio of the apparent shearing and tearing resistances; and if these latter vary for different kinds of joints, so also should the ratios of the sections.

Experiment shows that the shearing and tearing resistances vary a good deal with the quality of the iron and steel plates, the care taken in punching or drilling the rivet holes, and the quality of the rivet iron and rivet steel. Especially, the ratio $\frac{f_t}{f_s}$ is greater for steel than iron, and sensibly greater with drilled than with punched holes. Judgment must be used in determining what that ratio is in given cases, but the values given above may be taken as probable average values. Where special experiments on the materials used cannot be made, the values of $\frac{f_t}{f_s}$ given above may be taken as the best values of the ratio $\frac{\omega_s}{\omega_t}$ in designing a joint.

60. *Theoretical overlap.*—Taking the diameter of a rivet at the value given in (1) (§ 49), and eliminating t in the value for the breaking resistance of the portion of plate in front of a rivet in (§ 52), we get

$$T = 0{\cdot}23(2l - d)^2 d f_t.$$

If the portion of plate is to be at least as strong as the rivet,

$$0{\cdot}23(2l-d)^2 d f_t \geqq 0{\cdot}785\, d^2 f_s$$

$$l \geqq 0{\cdot}925\sqrt{d}\sqrt{\frac{f_s}{f_t}} + \frac{d}{2}.$$

Now the largest value of $\frac{f_s}{f_t}$, from the table above, is for iron 1·3, and for steel 0·85. Inserting these values,

$$l \geqq 1{\cdot}05\sqrt{d} + \frac{d}{2} \text{ for iron}$$

$$\geqq 0{\cdot}85\sqrt{d} + \frac{d}{2} \text{ for steel}$$

$d=$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$
For iron $l=$	1·00	1·14	1·29	1·41	1·55	1·67	1·80
For steel $l=$	0·86	0·98	1·12	1·22	1 35	1·46	1·57
$1{\cdot}5d$	=0·75	0·94	1·12	1·31	1·50	1·69	1·88

The ordinary practical rule (§ 50) is to make $l=1{\cdot}5d$, and values of $1{\cdot}5d$ are given for comparison. It will be seen that for the smaller diameters of rivets it would be desirable to increase a little the overlap assigned by the ordinary practical rule. For the larger diameters of rivets an overlap of $1{\cdot}5d$ is amply large enough.

61. *Theoretical rivet diameter for equal crushing and shearing resistance.*—The value of the crushing pressure f_c at which a joint begins to yield is entirely unknown. With all joints broken in the testing machine, and especially with steel joints, the rivet holes are found to have become elliptical, and the rivets to be deformed before fracture. In some experiments, it appeared that this crushing action had the effect of diminishing the apparent tenacity of the plates or the apparent resistance of the rivets. By making the rivets small and numerous the crushing stress can be reduced. Hence, it has been proposed as desirable that the rivet diameter should be so determined that the crushing stress should be below the limit found to be in-

jurious. If f_c is that limit, then from equations (3) and (5) for rivets in single shear—

$$f_c\, d\, t \geqq \frac{\pi}{4} d^2 f_s$$

$$d \leqq 1{\cdot}27 \frac{f_c}{f_s} t.$$

Similarly for rivets in double shear—

$$d \leqq 0{\cdot}635 \frac{f_c}{f_s} t.$$

The value of the crushing stress f_c, which produces injury to the tenacity or shearing resistance of the joint, is very uncertain. In the case of steel joints there is no indication of injury with crushing pressures of 50 tons per sq. inch.

If $\frac{f_c}{f_s} = 2$, then

$d \leqq 2{\cdot}54\ t$ for rivets in single shear

$\leqq 1{\cdot}27\ t$ for rivets in double shear.

Then, with the ordinary proportions of rivets, the crushing action will in no case need to be considered in single-shear joints, and only in double-shear joints when the plates are less than $\frac{7}{8}$ inch thick. On the whole the author is inclined to believe that the importance of crushing action has been exaggerated, and it will be assumed in the following calculations that crushing action is sufficiently provided for with the ordinary proportions of diameter of rivet to thickness of plate, and that no special attention needs to be paid to it.

62. *Pitch of single-riveted joints.*—Equating the tearing and shearing resistance (§ 52),

$$T = (p - d)\, t f_t = 0{\cdot}785\, d^2 f_s$$

$$p = 0{\cdot}785 \frac{d^2}{t} \cdot \frac{f_s}{f_t} + d. \qquad (6).$$

From this formula values of the pitch for single riveted joints have been calculated, for values of $\frac{f_t}{f_s}$ ranging from 0·75 to 1·35. The table above (§ 58) will be a guide as to the most probable value of $\frac{f_t}{f_s}$ in differently made joints, and the headings of the following tables have been made to agree with that table.

Single Riveting.

Thickness of plates	Nominal diam. of rivets	Real diam. of rivets	Iron Rivets				Steel Rivets			
			Iron punched plates		Iron drilled or punched and annealed or rymered plates		Steel punched plates		Steel punched and annealed or rymered plates	
			Pitch of rivets for values of $\frac{f_t}{f_s}$ =							
			·75	·85	·95	1·0	1·05	1·15	1·25	1·35
$\frac{5}{16}$	$\frac{11}{16}$	·72	2·46	2·25	2·09	2·02	1·96	1·85	1·77	1·69
$\frac{3}{8}$	$\frac{3}{4}$	·78	2·48	2·28	2·12	2·06	1·99	1·89	1·81	1·72
$\frac{13}{32}$	$\frac{13}{16}$	·85	2·58	2·38	2·22	2·15	2·09	1·98	1·90	1·81
$\frac{7}{16}$	$\frac{7}{8}$	·92	2·69	2·48	2·32	2·25	2·19	2·08	2·00	1·90
$\frac{15}{32}$	$\frac{15}{16}$	·98	2·59	2·40	2·25	2·19	2·13	2·03	1·95	1·87
$\frac{1}{2}$	$1\frac{1}{16}$	1·10	2·79	2·59	2·43	2·37	2·31	2·20	2·12	2·04
$\frac{9}{16}$	$1\frac{1}{8}$	1·17	2·81	2·62	2·46	2·40	2·34	2·24	2·16	2·08
1	$1\frac{1}{4}$	1·30	3·07	2·86	2·70	2·63	2·56	2·45	2·36	2·28

63. *Pitch of double-riveted joints.*—For double-riveted joints, there are two rivets to each strip of a width equal to the pitch. Hence

$$T=(p-d)\,t f_t=\frac{\pi}{2} d^2 f_s.$$

$$p=1{\cdot}57\,\frac{d^2 f_s}{t f_t}+d. \quad . \quad . \quad . \quad (7)$$

From this formula the following table has been computed, for values of $\frac{f_t}{f_s}$ ranging from 0·85 to 1·35.

Double Riveting.

Thickness of plates	Nominal diam. of rivets	Real diam. of rivets	Iron Rivets			Steel Rivets	
			Iron punched plates	Iron drilled or punched and rymered plates		Steel punched plates	Steel drilled or punched and rymered plates
			Pitch of rivets for value $\frac{ft}{fs}=$				
			·85	1·00	1·10	1·20	1·35
$\frac{5}{16}$	$\frac{11}{16}$	·72	3·78	3·33	3·12	2·91	2·66
$\frac{3}{8}$	$\frac{3}{4}$	·78	3·78	3·33	3·12	2·91	2·66
$\frac{7}{16}$	$\frac{13}{16}$	·85	3·91	3·45	3·24	3·03	2·77
$\frac{1}{2}$	$\frac{7}{8}$	·92	4·05	3·58	3·37	3·16	2·88
$\frac{9}{16}$	$\frac{15}{16}$	·98	3·82	3·39	3·18	3·00	2·76
$\frac{5}{8}$	$1\frac{1}{16}$	1·10	4·08	3·63	3·42	3·22	2·98
$\frac{11}{16}$	$1\frac{1}{8}$	1·17	4·06	3·63	3·42	3·23	2·99
1	$1\frac{1}{4}$	1·30	4·42	3·95	3·74	3·52	3·26

64. *Graphic method of designing joints.*—Schwedler has introduced a mode of designing joints which is particularly useful for irregular or complicated joints. The width w of a strip of plate of a strength equivalent to that of one rivet is given by the equation

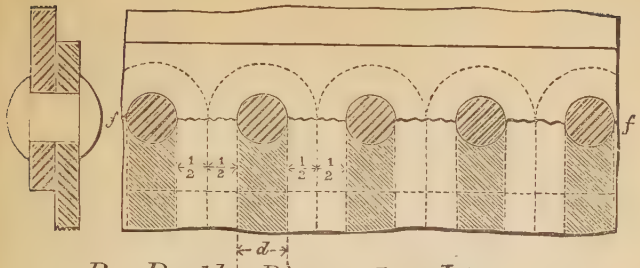
$$w\,t\,f_t = \frac{\pi}{4}\,d^2 f_s$$

the rivets being in single shear. Then

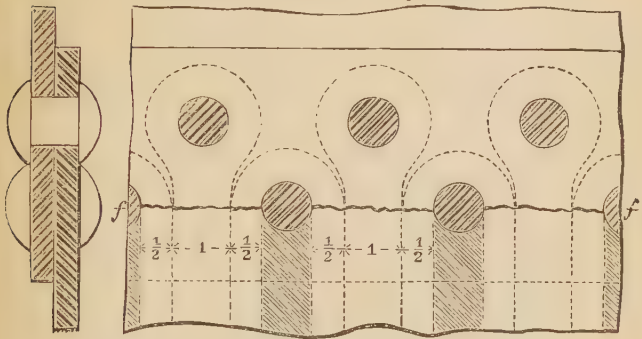
$$w = 0{\cdot}785\,\frac{d^2}{t}\cdot\frac{f_s}{f_t} \quad . \quad . \quad . \quad (8)$$

Suppose d and t given, and the value of $\frac{f_t}{f_s}$ also given or taken from the table above. Then w can be calculated. If round each rivet a circle is drawn of diameter $d+w$, and from these circles lines are drawn cutting up the plate into strips of width w, a portion of plate of sufficient strength will have been allotted to each rivet, and any redundant portions will indicate useless material in the joint. The following figures show the application of this method to some forms of joint,

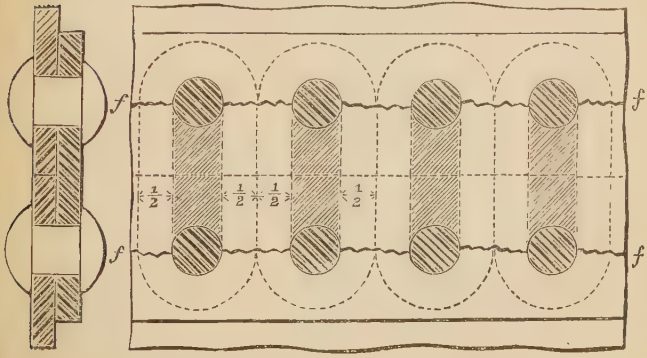
A. *Single Riveted Lap Joint.*



B. *Double Riveted Lap Joint.*



C. *Cover Plate Single Riveted Butt Joint.*



Unit w. ff, Line of fracture.

Fig. 52.

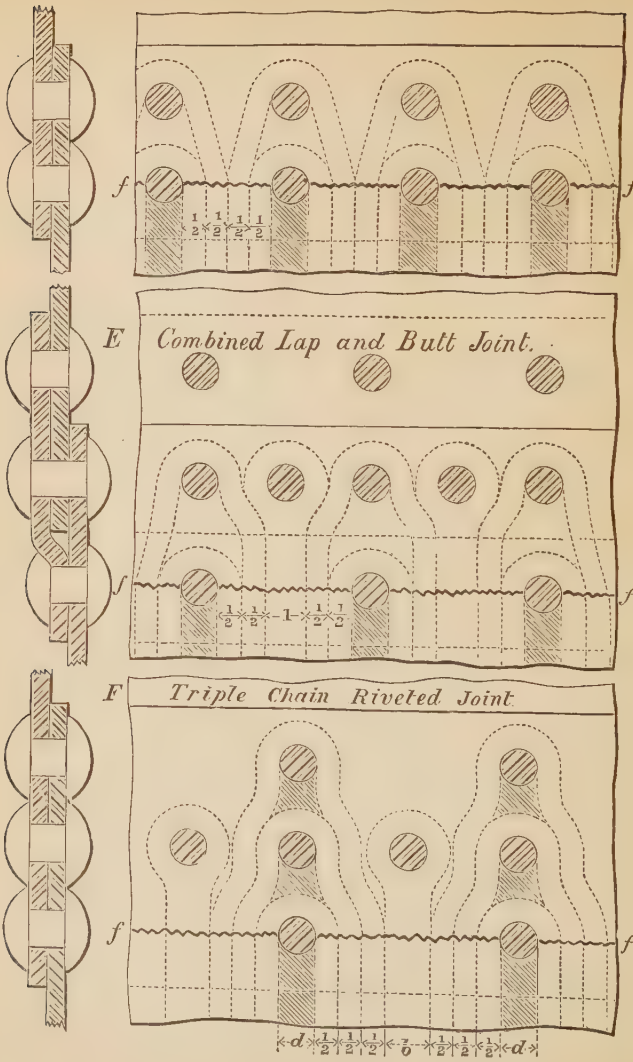

D. Double Chain Riveted Lap Joint.
f
f
1/2 1/2 1/2 1/2
E
Combined Lap and Butt Joint.
f
f
1/2 1/2 1 1/2 1/2
F
Triple Chain Riveted Joint
f
f
d 1/2 1/2 1/2 7/8 1/2 1/2 1/2 d
Unit w.
ff Line of fracture.

Fig. 53.

the shaded portions being parts of the plate which do not add to the strength of the joint.

In fig. 52, A is a single-riveted and B a double-riveted joint so arranged that the shearing and tearing resistances are equal. The shaded portions represent metal unavoidably wasted, because the joint will give way at the line of fracture *ff*. C is the cover strip of a butt joint similarly designed.

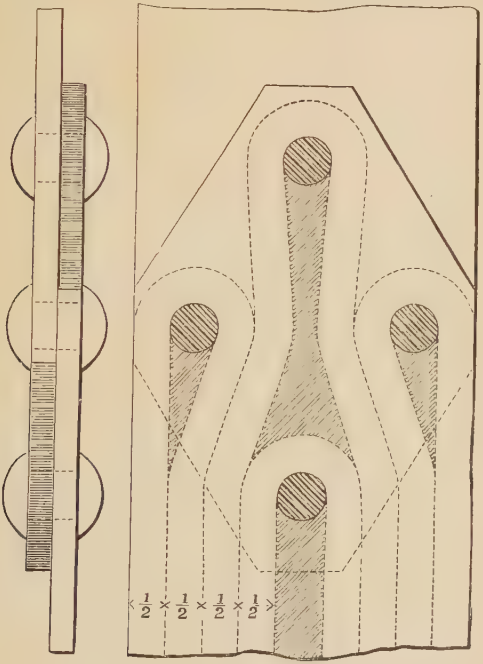


Fig. 54. Unit = *w*.

Fig. 53 shows joints of a more complicated construction. Fig. 54 shows the same method applied in designing a joint in a tie bar. As to the distance apart of the rows of rivets, no specific rule can be given, but it is rational to suppose that the strips carrying the load due to each rivet should not be too sharply bent.

65. *Efficiency of riveted joints.*—The efficiency of a joint is the ratio of the strength of the joint to the strength of an equal width of solid plate. Suppose the joint is properly designed, so that the shearing strength of the rivets is not less than the tearing resistance of the plates. Then, if there were no alteration of the tenacity of the plates by punching or drilling in making the joint, or by bending or crushing action under the load, the efficiency would be simply

$$\eta = \frac{p-d}{p}.$$

But since the apparent tenacity of the joint differs from the original tenacity of the plate in most cases, the true efficiency is

$$\eta = k\frac{p-d}{p}, \quad . \quad . \quad . \quad (9)$$

where k has the values given in the table above.

The values of η for a series of single and double riveted joints are given in the following table. They exactly correspond to the tables of pitches of joints given above, and the values of k are those already given for iron and steel plates. The efficiencies here given are slightly greater than those found in the most reliable experiments. But then in most experiments the joints have not had precisely the best proportion of shearing and tearing area.

Single Riveting.

Thickness of plates	Nominal diam. of rivets	Real diam. of rivets	Iron punched plates		Iron drilled or punched and annealed or rymered plates		Steel punched plates		Steel punched and annealed or rymered plates	
			Efficiency η of joints for values of k=							
			·77		·88		0·9		1·0	
$\frac{5}{16}$	$\frac{11}{16}$	·72	·55	·52	·58	·56	·57	·55	·59	·57
$\frac{3}{8}$	$\frac{3}{4}$	·78	·53	·51	·55	·54	·55	·53	·57	·55
$\frac{7}{16}$	$\frac{13}{16}$	·85	·52	·49	·55	·54	·53	·51	·55	·53
$\frac{1}{2}$	$\frac{7}{8}$	·92	·51	·49	·52	·52	·52	·50	·54	·52
$\frac{9}{16}$	$\frac{15}{16}$	·98	·48	·45	·49	·48	·49	·47	·50	·48
$\frac{5}{8}$	$1\frac{1}{16}$	1·10	·47	·44	·48	·47	·47	·45	·48	·46
$\frac{11}{16}$	$1\frac{1}{8}$	1·17	·45	·42	·46	·45	·45	·43	·46	·44
1	$1\frac{1}{4}$	1·30	·42	·40	·46	·45	·45	·43	·45	·43

Double Riveting.

Thickness of plates	Nominal diam. of rivets	Real diam. of rivets	Iron punched plates	Iron drilled or punched and rymered plates		Steel punched plates	Steel drilled or punched and rymered plates
			Efficiency η of join's for values of k=				
			·85	·95	1·00	1·00	1·06
$\frac{5}{16}$	$\frac{11}{16}$	·72	·69	·74	·77	·75	·77
$\frac{3}{8}$	$\frac{3}{4}$	·78	·68	·73	·75	·73	·75
$\frac{7}{16}$	$\frac{13}{16}$	·85	·66	·71	·74	·72	·74
$\frac{1}{2}$	$\frac{7}{8}$	·92	·65	·70	·73	·71	·73
$\frac{9}{16}$	$\frac{15}{16}$	·98	·63	·67	·69	·67	·69
$\frac{5}{8}$	$1\frac{1}{16}$	1·10	·62	·66	·68	·66	·68
$\frac{3}{4}$	$1\frac{1}{8}$	1·17	·60	·65	·66	·64	·65
1	$1\frac{1}{4}$	1·30	·60	·63	·65	·63	·64

66. *Breaking strength of a riveted joint.*—The breaking strength of any riveted joint designed with the proportions given above is easily calculated from the efficiency. Let f_t be the breaking stress of the solid plate per sq. in. of gross area. Then the strength of a structure with riveted joints is the same as if it were formed of solid plate of a tenacity ηf_t. Or if t is the thickness of the plates, then the riveted structure is as strong as if it were composed of solid plate of a thickness ηt.

Working strength of a riveted structure.—In bridge work and boiler work, the working stress is usually $\frac{1}{4}$th to $\frac{1}{5}$th of the breaking stress.

67. *Cold riveting for thin plates.*—For thin plates the rivets are hammered up cold. With plates $\frac{1}{8}$ inch to $\frac{3}{16}$ inch, the rivets are $\frac{1}{4}$ inch to $\frac{5}{16}$ inch diameter and $\frac{7}{8}$ inch pitch. The half width of overlap is about $\frac{1}{2}$ inch, and the staunchness of the joint is secured by laying between the plates, in a zigzag direction round the rivets, a string smeared with red lead.

68. *Plates with thickened edges.*—To obviate the loss of strength at the riveted joints of boilers, plates with thickened

edges have been used. Let t=thickness of plate, t_1=thickness of edge. Then if $t_1 = t\,\frac{p}{p-d}$ the joint will be as strong as the solid plate. The joint must be designed, as if the plate were t_1 inches thick. Only two edges of the plate are usually thickened, and these are placed, so as to form the longitudinal joint, which is subjected to the greatest strain.

69. *Junctions of three plates.*—In boiler work where the riveted seams must be watertight a difficulty arises where the cross joints and longitudinal joints meet, because there, three plates overlap. At such places one or more plates are thinned out by forging so that the joint may be solid throughout.

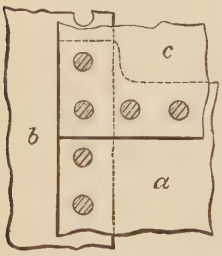


Fig. 55

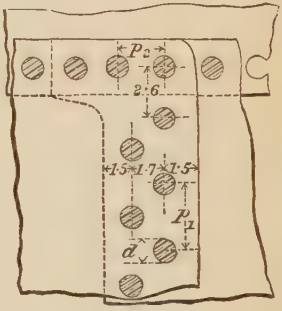


Fig. 56

Fig. 55 shows three plates *a*, *b*, *c*, overlapping and single riveted. Fig. 56 a similar joint where the longitudinal seam is double riveted and the cross seam single riveted.

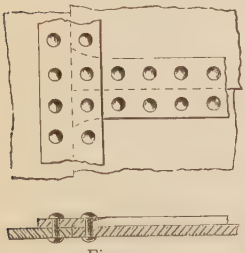


Fig. 57

Fig. 57 shows a junction of three plates with butt straps, the longitudinal butt strap being planed down and tucked under the cross butt strap.

In fig. 58 the cross seam is overlapped and single riveted, but the longitudinal seam has

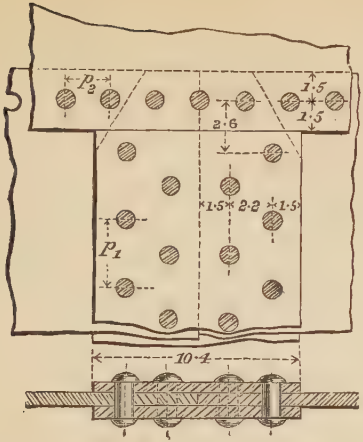


Fig. 58

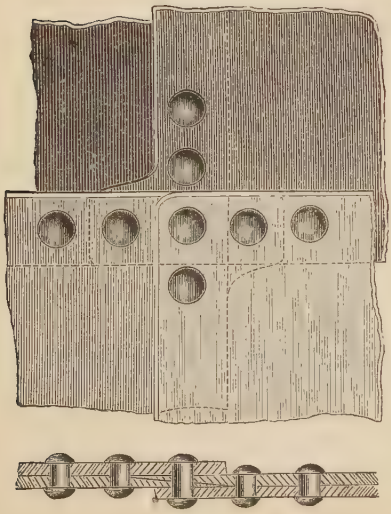


Fig. 59

double butt straps and is double riveted. The upper butt strap is planed at the end so that it can be better caulked where it abuts against the plate. The proportional unit is the rivet diameter.

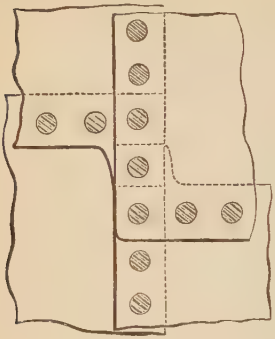


Fig. 60

Junctions of four plates.—Figs. 59, 60 show four overlapping plates, single riveted. Each of the two interior plates is thinned out at the junction. It will be seen that the forged part is lengthened out so as to be gripped by an additional rivet in the thin part.

70. *Connection of plates not in one plane.*—This is commonly effected by the use of a kind of angular joint strip, called an *angle iron*. These angle irons are rolled of a great variety of sizes, and are of very great service in all descriptions of wrought-iron work.

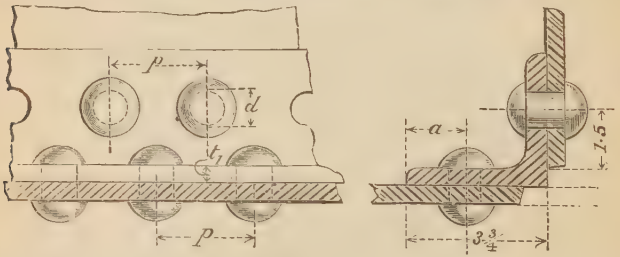


Fig. 61

Fig. 61 shows an angle-iron joint. No very definite rule can be given for the size of angle iron to be used, but generally the mean thickness of the angle iron is about equal to, or a little greater than, that of the plates to be connected. If the mean thickness of the angle iron is $t_1 = t + \frac{1}{16}$, then the width w of each limb of the angle iron may be $3\frac{3}{4}t$; the diameter d of the rivets $= 2t$; $a = \frac{1}{2}(w - t_1)$. The angle

iron usually tapers so that it is rather thicker at the root than at the point. In bridge work the angle irons are often heavier. Care must be taken in arranging the rivets that the heads in one row do not prevent the proper riveting of those in the other row.

Fig. 62 shows a T iron joint, the object being to stiffen the plates against flexure.

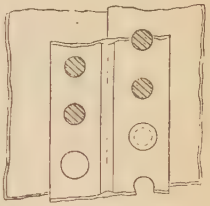


Fig. 62

Fig. 63 shows methods of connecting plates by flanging the plates themselves, instead of using angle irons. This is more expensive, and is impracticable when the plates are not of good quality. The curvature should not be too sharp. The inside radius may be, at least, four times the thickness of the plates. The width of

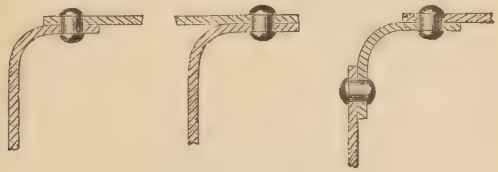


Fig 63

overlap must be, at least, three times the diameter of the rivet.

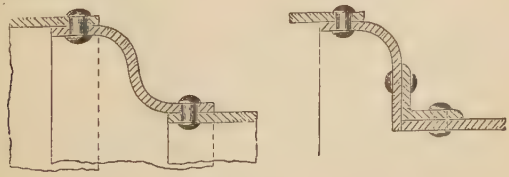


Fig. 64

Fig. 64 shows joints used at the junction of the cylindrical barrel of locomotive boilers with the external fire box.

71. *Connection of parallel plates.*—A case which frequently occurs is where two plates, near together, require to be connected. For instance, at the bottom of the fire-box of locomotives, a connection has to be made between the inner and outer fire-box. The following sketches show how this may be effected.

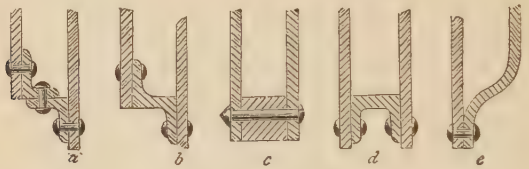


Fig 65.

In fig. 65*a* there are two angle irons. This is rather complicated, and there are inside joints, which cannot be caulked. Fig. 65*b* is simpler, but has an inside joint, which cannot be caulked. Fig. 65*d* is an admirable joint, and is formed by what is termed a channel iron. But it is difficult to bend the channel iron round the corners of the fire-box. Fig. 65*e* is simple, but forms a corner for the lodgment of sediment. Fig. 65*c* is the form most commonly used.

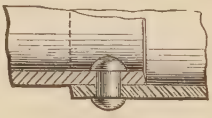


Fig. 66

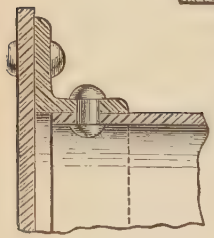


Fig. 67

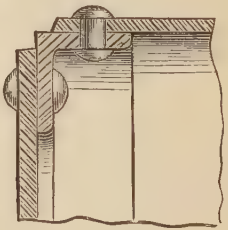


Fig. 68

Connections in cylindrical boilers.—Fig. 66 shows the ordinary way of overlapping the cross joints in shells and

flues. Figs. 67 to 70 show arrangements used for connecting the boiler shell and end plates. Figs. 71, 72, show modes of connecting the furnace flues and end plates. When the

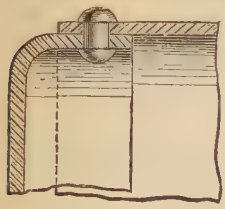


Fig. 69

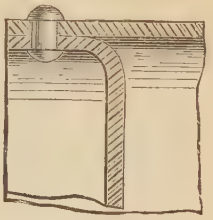


Fig. 70

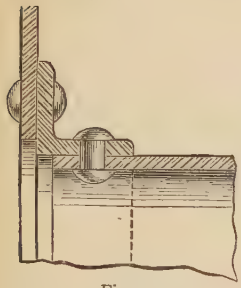


Fig. 71

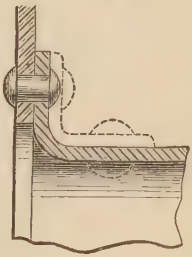


Fig. 72

joint is made as in fig. 72, a half ring cover is sometimes used like that shown dotted to protect the upper half of the

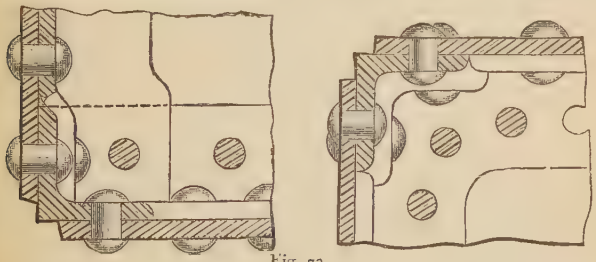


Fig. 73

joint from grooving. Collapse rings which also form flue joints have been shown in fig. 27 (p. 86)

Corner connections.—Figs. 73, 74 show the connection of

three plates in different planes by means of angle irons. The proportions of the angle irons may be the same as in fig. 61.

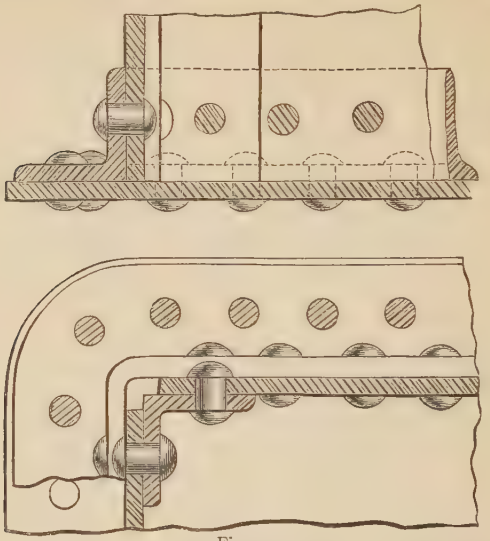


Fig. 74.

Elliptical rivets.—Since the efficiency of the joint is the ratio $(p-d)/p$ of the distance between the holes to the pitch,[1] we may increase the efficiency by using rivets of elliptical section. With such rivets, placed with their least breadth in the line of fracture of the plates, the quantity $p-d$ would be greater, while the shearing section remained the same. Such rivets have been used by Mr. Webb. By adopting the elliptical form, two variables, the axes of the ellipse, take the place of the single variable d, in the equations. It would thus be possible to satisfy the conditions of equal bearing, tearing, and shearing resistance, for rivets of any desired section.

72. *Position of rivets in tie bars and struts.*—When a bar, subjected to a longitudinal straining force, is attached at each end by a single rivet or pin, the rivets should be

[1] Or more strictly $k\,(p-d)/p$. See § 65.

placed on the centre line of the bar. It is a fair assumption, and must be nearly true, that the straining force acts through the centre of the rivet. Hence, if the rivets are in the centre line of the bar, the resultant straining force passes through the axis of the bar, and the stress on each transverse section is uniform. If the rivets are not so placed, one side of the bar is more strained than the other, and gives way before the other has fully exerted its powers of resistance. When there are several rivets at each end of a bar, they should, for the same reason, be placed symmetrically on either side of the axis, and as uniformly distributed as possible over the area in which they are placed. If they cannot be placed symmetrically, an approximation is made to the best conditions, by arranging them, so that their common centre of gravity falls on the axis of the bar. In that case, if each rivet supports the same fraction of the load, the resultant force will still pass through the axis of the bar.

73. *Cylindrical riveted structures.*—A cylindrical vessel made of numerous plates may be formed of a series of cylindrical rings alternately larger and smaller, so that each alternate ring can be slipped inside the others. Then if D is the diameter of the smaller rings, $D+2t$ is that of the larger ones. A second plan is to make the rings of equal diameter, and to use a butt strap over the joints. A third plan, common in boiler flues, is to make the rings conical, the diameters being D at one end, and $D+2t$ at the other. The rings are then slipped into each other, and the joints should be so placed that the flame does not directly strike the edges of the plate.

The strength of a cylindrical vessel to resist bursting has been given in eq. 2, § 26. Let D be the diameter of the boiler shell, t the thickness of the plates, f the tenacity of the plates, η the efficiency of the riveted joints. Then the pressure required to burst the boiler is

$$p=\frac{2\eta tf}{D} \quad . \quad . \quad . \quad (10)$$

and the working pressure should not exceed one-fifth of the bursting pressure. The rules for cylinders resisting an external collapsing pressure are given in §§ 40, 41.

Working strength of solid wrought-iron cylinders to resist internal pressure.

Thickness of cylinders 1 inch. Working stress taken at 7,600 lbs. per sq. in. for wrought iron, and 10,600 lbs. per sq. in. for steel

Diameter Inches	Working Pressure in lbs. per sq. in. (Excess of internal over external pressure)	
	Iron	Steel
12	1267	1767
18	845	1177
24	633	884
30	507	707
36	422	589
42	362	505
48	317	463
54	282	393
60	253	354
66	230	321
72	211	294
78	195	272
84	181	252
90	169	235
96	158	221
102	149	208
108	141	196

For any other thickness of plate multiply the pressures in this table by that thickness. If the cylinder is riveted, multiply also by the value of η in the tables above, corresponding to the kind of joint used.

Thus suppose a boiler is 6 feet 6 ins. in diameter and constructed of $\frac{3}{8}''$ iron plates. From the table, a solid or welded iron shell of that thickness should carry $195 \times \frac{3}{8} = 73$ lbs. per sq. in. pressure. If, however, the shell is single-riveted, with punched rivet holes, $\eta = \cdot 51$ to $\cdot 53$, say $\cdot 52$. Then the safe working pressure is $0 \cdot 52 \times 73 = 38$ lbs. per sq. in. With steel plates and drilled holes, single-riveted $\eta = \cdot 55$ to $\cdot 57$, say $\cdot 56$. Then the safe working pressure would be

$272 \times \frac{3}{8} \times 0{\cdot}56 = 57$ lbs. per sq. in. With double-riveted joints, the working pressure would be $195 \times \frac{3}{8} \times 0{\cdot}68 = 50$ lbs. for iron punched plates; and $272 \times \frac{3}{8} \times {\cdot}75 = 76$ lbs. per sq. in. with steel drilled plates.

74. *Taper and curvature of boiler plates.*—When a boiler, boiler flue, or other cylindrical structure, is made up of

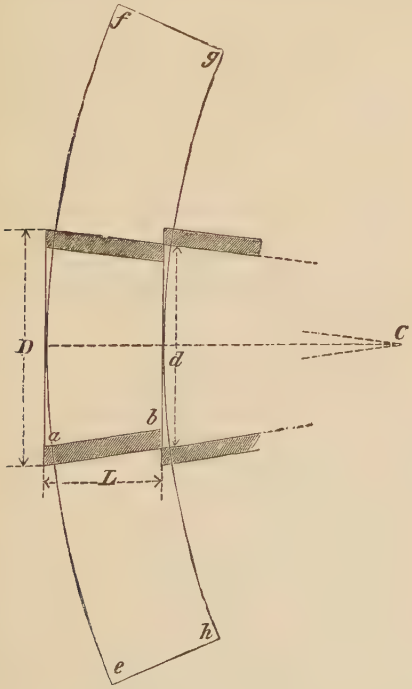


Fig. 75.

slightly conical rings, which are slipped over each other to form the overlap, fig. 75, the joints being what are technically termed 'following' joints, the plates, instead of being rectangular, must be portions of the development of a cone

Let D be the greater, and d the less, diameter of the conical frustum, and L its length ; t, the thickness of the plates. Then $d = D - 2t$ very nearly. The development of the frustum is an annular segment $efgh$, drawn with radii, $R = C\,a = C\,e$, and $r = C\,b = C\,h$, and whose lengths, measured along the arcs ef and hg, are πD and πd. Since the incli-

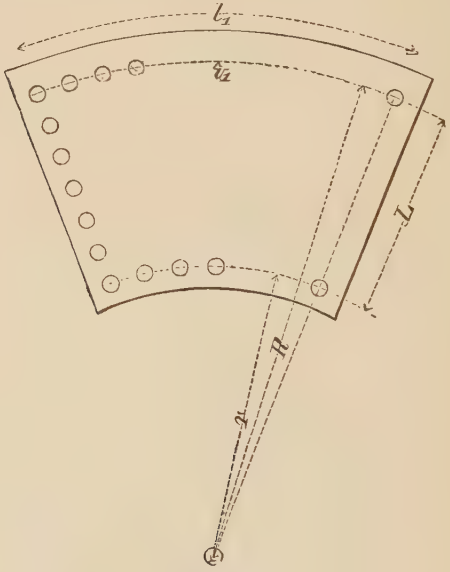


Fig. 76.

nation of the cone's sides is small, $R = \frac{D\,L}{D-d} = \frac{D\,L}{2\,t}$ nearly, and $r = \frac{d\,L}{D-d} = \frac{d\,L}{2\,t}$ nearly.

For a boiler plate, let l_1, fig. 76, be the distance between longitudinal seams, measured at the larger end of cone, so that, if there are n plates in each ring, $l_1 = \frac{\pi D}{n}$. Let L be the

distance between the cross seams ; v_1 and v_2, the versed sines of the arcs, formed by the rivets when developed.

$$R = \frac{D\,L}{2\,t} \qquad r = \frac{D\,L}{2\,t} - L$$

$$v_1 = \frac{l_1{}^2 t}{4\,D\,L}$$

$$v_2 = \frac{l_1{}^2 (D - 2t) t}{4\,D^2 L}$$

With these dimensions the centre lines of the rivets can be set out, and if, then, the width of overlap is added all round, the size of the plate is determined.

Boiler Stays

75. Boiler stays are fastenings which support flat surfaces, and, although they are sometimes more complex than simple fixed fastenings, they are conveniently treated here.

Locomotive or water space stays.—The flat surfaces of locomotive fire-boxes are supported by stays screwed into

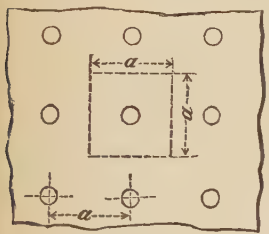


Fig. 77

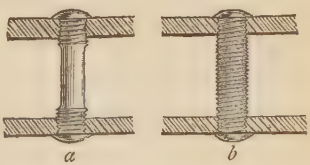


Fig. 78

both the inner and outer fire-box plates and riveted over. Sometimes the stays are screwed over their whole length (*a*, fig. 78). Sometimes the middle part is turned down to the diameter at the bottom of the screw thread (*b*, fig. 78) and there is then less liability to fracture. Occasionally, especially in stays near the level of the fire-bars, a small hole

is drilled into the stay (fig. 79) the leakage from which gives warning if fracture has occurred.

Stays are almost always of copper in copper fire-boxes, and of iron in iron or steel fire-boxes. Steel stays have also been used. In marine boilers the stays are put in with a nut at each end or a nut at one end and a head at the other. Where the nuts are not exposed to excessive heat, so that they burn away, they are preferable to riveting and support the plates better. Copper stays are more easily riveted cold than iron or steel stays. The stays vary in diameter from $\frac{3}{4}$ inch to one inch, and are pitched uniformly at a distance a so that each stay supports the pressure on an area of $a \times a$ square inches (see fig. 77). The working stress may be 4,000 to 5,000 lbs. per sq. in. for copper, 5,000 to 6,000 lbs. for iron, and 6,000 to 7,500 for steel.

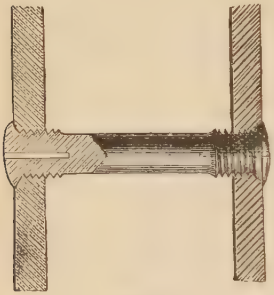


Fig. 79

The following rules give ordinary proportions for stays :— Let a be the pitch of the stays, d their gross diameter, d_1 the net diameter at bottom of screw thread, t=thickness of plate, p=excess of internal over external pressure. The thickness of the plates is determined by their resistance to bending and rules are given in § 46, p. 93. Usually

$$t = 0{\cdot}011\ a\sqrt{p} \text{ for copper plates;}$$
$$= 0{\cdot}009\ a\sqrt{p} \text{ for iron or steel plates.}$$

Hence, $a = \frac{91\ t}{\sqrt{p}}$ for copper plates.

$$= \frac{111\ t}{\sqrt{p}} \text{ for iron or steel plates;}$$

With the limiting stresses given above we get :

$$d_1 = k_1 a \sqrt{p}$$

where $k_1 = 0{\cdot}0178$ to $0{\cdot}0159$ for copper stays ;
$0{\cdot}0159$ to $0{\cdot}0146$ for iron stays ;
$0{\cdot}0146$ to $0{\cdot}0130$ for steel stays.

The gross diameter d is obtained approximately enough from the equation

$$d = k a \sqrt{p} + 0{\cdot}055,$$

where $k = 0{\cdot}0198$ to $0{\cdot}0177$ for copper stays ;
$= 0{\cdot}0177$ to $0{\cdot}0162$ for iron stays ;
$= 0{\cdot}0162$ to $0{\cdot}0144$ for steel stays.

The strength of stayed surfaces is discussed at page 93. Longitudinal stays of great length will be treated in the next chapter.

76. *Diagonal Stays.*—The simplest form of diagonal stay is shown in fig. 80. Here it is connected by pins to angle irons riveted to the boiler shell. There is then no tendency to produce bending stresses in the stay. Very often such

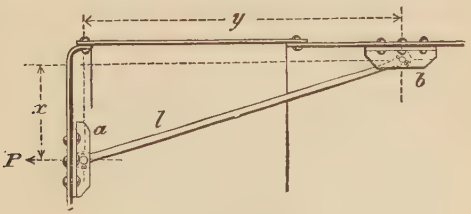


Fig. 80

stays are forged out into palms at the ends and these are riveted to the boiler shell, but there will then be a bending couple acting on the stay of indeterminate amount. Now let A be the area of the end plate supported by the stay, and p the pressure on it. Then the force P, acting at the end of the stay, which is the horizontal component of the tension in the stay, is p A. Consequently, the tension in the stay is p A l/y. The safe limit of stress for such stays is about

6,000 lbs. for iron and 7,500 for steel. Hence the area of section a of the stay may be

$$a = \frac{p \text{ A } l}{6{,}000\, y} \text{ or } \frac{p \text{ A } l}{7{,}500\, y}.$$

The horizontal shear on the rivets at b is p A ; the vertical shear on the rivets at a is p A x/y ; the horizontal pull on the rivet heads at a is p A ; and the vertical pull on the rivet heads at b is p A x/y.

Gusset Stays.—Fig. 81 shows the arrangement commonly adopted for staying the ends of Cornish and Lancashire boilers. The dotted lines in the end view show the areas which may be taken to be supported by each stay. The distance x between the end rivets of the stays and the rivets of the flues is ordinarily 9 inches, and the unsupported space round the flues of this breadth is called the breathing space. If the gusset stays are brought too near the flues, so that the boiler ends are too rigid and cannot spring, considerable stresses are induced by the expansion and contraction of the flue tubes. There is then danger of leakage at the joints of the flues or grooving round the edges of the flue angle irons. Below the flues the gusset stays must be brought nearer to the flues, say about 7 inches instead of 9 inches. The least section of the gusset stays may be calculated as if they were simple diagonal stays by the rule above. The length of the stay may be taken as the distance l between the centres of the end attachments, and its breadth b may be measured at the first rivet at the narrow end.

76*a*. *Bridge or Girder Staying.*—In many cases it is in convenient to support the roofs of fire-boxes by ordinary stays. Then wrought-iron bars of considerable strength are placed over the plate to be supported, and rest at the ends on the side plates. To these bars the stays are attached. Fig. 82 shows the ordinary arrangement of bridge and stays. The bridge is sometimes a solid bar, as shown at a and in plan at b. A cheaper form is shown at c, the bridge consisting

of two flat plates connected by rivets with spacing ferrules and having room between the plates for the stays. The

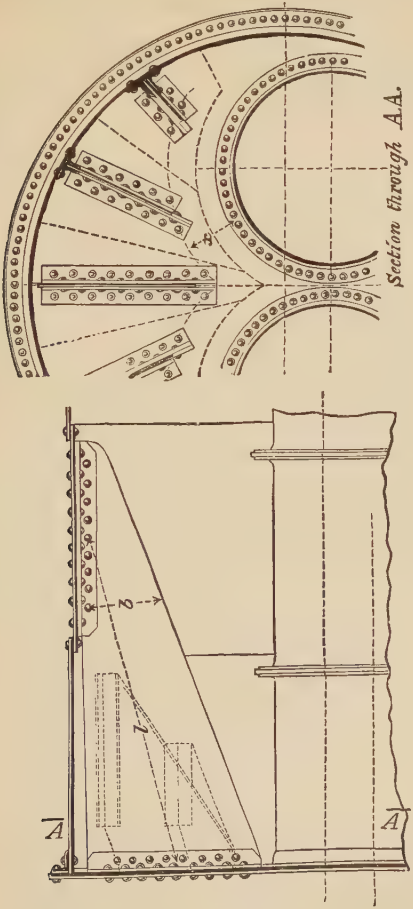


Fig. 81

water space between girder and roof of fire-box should be as open as possible and at least $1\frac{1}{2}$ inch in depth. The

girders should have bearing enough at the ends to prevent crushing of the plates on which they rest. The pitch of the

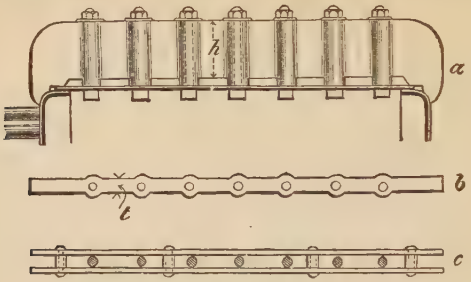


Fig. 82.

stays, when girders are used, is rarely less than four inches.

The girder or bridge is subjected to simple bending with a nearly uniformly distributed load. Let p be the steam pressure (above atmosphere), a the pitch of the stays which is the distance centre to centre of the girders, l the length of the bridge. Then the uniformly distributed load on each girder is $p\,a\,l$ pounds Let h be the height and t the thickness of the girder ; t is the sum of the thicknesses of the two plates, if the girder is constructed as shown at c. The bending moment at the centre of the girder is $\frac{1}{8}\,p\,a\,l^2$ and the modulus of the rectangular section of the girder is $\frac{1}{6}\,t\,h^2$. Hence, if f is the working stress in lbs. per sq. in.

$$\tfrac{1}{8}\,p\,a\,l^2 = \tfrac{1}{6}\,f\,t\,h^2,$$

$$t\,h^2 = \tfrac{3}{4}\frac{p\,a\,l^2}{f}.$$

$$h = \sqrt{\left(\tfrac{3}{4}\frac{p\,a\,l^2}{f\,t}\right)} \text{ if } t \text{ is assumed ;}$$

$$t = \tfrac{3}{4}\frac{p\,a\,l^2}{f\,h^2} \text{ if } h \text{ is assumed ;}$$

or, if $t = \frac{1}{4}\,h$ to $\frac{1}{5}\,h$ and f is taken at 12,000 for wrought iron and 15,000 for steel, we get

$$h = \sqrt{\left(\frac{p\,a\,l^2}{c}\right)}$$

	Values of c		
	Iron		Steel
For $t = 0{\cdot}2\,h$.	2,400	.	3,000
$= 0{\cdot}25\,h$.	3,000	.	3,750

The stays are designed by the rules already given for ordinary stays.

Fig. 83 shows a modified form of bridge stay, used by Mr. Webb of Crewe, with the stays tapped into the bridge. Sling stays are also shown supporting the bridge from the outer casing. Although these sling stays, when acting properly, diminish the bending moment on the bridge, they

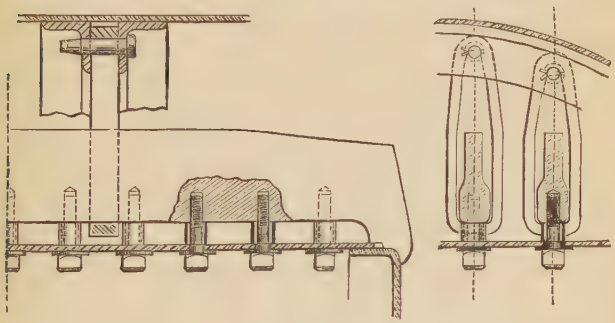


Fig. 83

may be slack. Generally the bridge is made as strong as if there were no sling stays, and the sling stays are put in as an additional precaution.

CHAPTER V

ON FASTENINGS

Bolts, Nuts, Keys, and Cotters

77. A *screw* is a cylindrical bar on which has been formed a helical projection or *thread*. The screw fits accurately into a hollow corresponding form, termed its *nut*. Pairs of elements thus formed are used in machinery (*a*) as fastenings, in which case they are commonly termed bolts; (*b*) for adjusting the relative position of two pieces; (*c*) for transmitting energy. It is chiefly as fastenings that they will be treated in the present chapter.

Fig. 84

Bolts or fastening screws are chiefly used to resist straining forces which act parallel to the axis of the bolt and normal to the surfaces connected together. The bolt is then in tension. When the straining force acts perpendicularly to the axis of the bolt and parallel to the surfaces connected, the bolt is in shear and is then equivalent to a rivet, and may be proportioned by the same rules. A bolt differs from a rivet in this, that it permits the connected pieces to be easily disconnected again when necessary.

For manufacturing reasons it is important that a common agreement should be come to as to the form of screw threads and the pitch of the screw thread for each size of screw bolt. Sir J. Whitworth first proposed a standard

system of screw threads, of the form shown in fig. 84, which is universally adopted in this country, for all the more important parts of machines. For wrought-iron gas tubes and the cheaper kinds of metal work, a screw thread of finer pitch, cutting less deeply into the metal, is used. In America, Mr. Sellers has introduced a uniform system with a thread differing in some important respects from Whitworth's. In Germany a system almost exactly like that of Sellers has been proposed for adoption.

78. *Pitch and form of screw threads.*—The Whitworth thread is triangular in section, the angle being 55° (fig. 84). One sixth of the depth of the thread is rounded off at the top and bottom to facilitate the cutting of the thread and to render it less liable to injury.

Fig. 85 shows other forms of screw threads. That shown

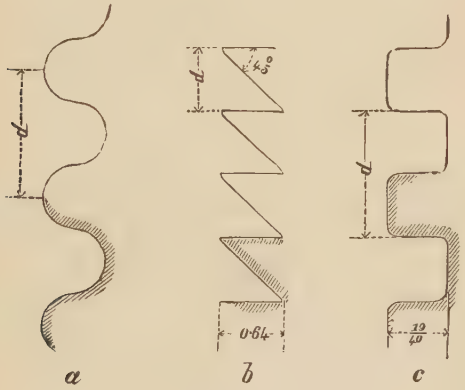


Fig. 85

at *c* is rectangular in section, and is often called the square thread. The surface of the thread is nearly normal to the axial force against which the screw acts, and hence there is no oblique or bursting pressure on the nut. There is also less friction and less wear with threads of this form, but they

are much more expensive to cut than triangular threads. Square threaded screws are chiefly used to transmit motion. When a screw has to resist a force acting always in one sense, the modified triangular screw thread, shown at *b*, may be used. This is termed the buttress thread. It has one surface normal to the axis of the screw like the square thread, and is as cheaply cut as the triangular thread. The breech screw of large guns is made of this form. Fig. 85*a* is a modified square thread which is used where the screw is liable to rough usage.[1] It may be noted that the shearing section of a triangular thread for a given length of nut is twice as great as that of a square thread. The triangular thread also permits a finer pitch.

The pitch of screws is fixed by practical experience, so as to be suitable for cast and wrought iron. The pitch and number of threads per inch, as arranged by Whitworth for the different diameters of screws, are given in a table below.

The following formulæ give values nearly the same as those in the tables :—

Whitworth and square threads.

$$\text{Pitch} = p = \begin{matrix}\text{Triangular Threads} \\ 0{\cdot}08d + 0{\cdot}04\end{matrix} \qquad \begin{matrix}\text{Square Threads} \\ 0{\cdot}16d + 0{\cdot}08\end{matrix} \quad . \quad (1)$$

$$\text{Number of threads per inch} = n = 1/p.$$

Diameter at bottom of thread

$$= d_1 = d - \frac{1{\cdot}28}{n} = 0{\cdot}9d - 0{\cdot}05, \text{ for triangular threads} \quad . \quad \Big\}$$

$$= d - \frac{3{\cdot}8}{n} = 0{\cdot}85d - 0{\cdot}075, \text{ for square threads} \quad . \quad \Big\} (2)$$

A square-threaded bolt is, therefore, slightly weaker, in tension, than a triangular-threaded bolt.

[1] Rods such as valve rods which are subjected to constant variation of stress and which have screwed ends are very liable to break. It is not unlikely that this is due to the acute angle of the Whitworth thread, the stress being very unequal on sections across the rod at the bottom of the thread. Steel appears to be still more seriously affected than iron. Hence in such cases it is better to adopt the form of thread shown at *a*.

Fig. 84 shows the method of designing a Whitworth thread. Two parallel lines are drawn, $0{\cdot}96\,p$ apart. These

Table of Whitworth Triangular Screw Threads

Diameter in inches		No. of threads per inch	Pitch in inches	Diameter at bottom of thread in inches	Area of section at bottom of thread in square inches
$\frac{1}{4}$	0·25	20	0·05	·186	·027
$\frac{5}{16}$	·313	18	·0555	·241	·046
$\frac{3}{8}$	·375	16	·0625	·295	·068
$\frac{7}{16}$	·438	14	·0714	·346	·094
$\frac{1}{2}$	·500	12	·0833	·393	·121
$\frac{5}{8}$	·625	11	·0909	·509	·204
$\frac{3}{4}$	·750	10	·100	·622	·304
$\frac{7}{8}$	·875	9	·111	·733	·422
1	1·00	8	·125	·840	·554
$1\frac{1}{8}$	1·125	7	·143	·942	·697
$1\frac{1}{4}$	1·25	7	·143	1·067	·894
$1\frac{3}{8}$	1·375	6	·167	1·161	1·058
$1\frac{1}{2}$	1·500	6	·167	1·286	1·299
$1\frac{5}{8}$	1·625	5	·200	1·369	1·472
$1\frac{3}{4}$	1·750	5	·200	1·494	1·753
$1\frac{7}{8}$	1·875	$4\frac{1}{2}$	·222	1·590	1·986
2	2·00	$4\frac{1}{2}$	·222	1·715	2·311
$2\frac{1}{8}$	2·125	$4\frac{1}{2}$	·222	1·840	2·659
$2\frac{1}{4}$	2·25	4	·250	1·930	2·926
$2\frac{1}{2}$	2·50	4	·250	2·180	3·733
$2\frac{3}{4}$	2·75	$3\frac{1}{2}$	·286	2·384	4·464
3	3·00	$3\frac{1}{2}$	·286	2·634	5·450
$3\frac{1}{4}$	3·25	$3\frac{1}{4}$	·308	2·855	6·402
$3\frac{1}{2}$	3·50	$3\frac{1}{4}$	·308	3·105	7·563
$3\frac{3}{4}$	3·75	3	·333	3·323	8·673
4	4·00	3	·333	3·573	10·027
$4\frac{1}{4}$	4·25	$2\frac{7}{8}$	·348	3·804	11·365
$4\frac{1}{2}$	4·50	$2\frac{7}{8}$	·384	4·054	12·908
$4\frac{3}{4}$	4·75	$2\frac{3}{4}$	·364	4·284	14·404
5	5·0	$2\frac{3}{4}$	·364	4·534	16·146
$5\frac{1}{4}$	5·25	$2\frac{5}{8}$	·381	4·762	17·810
$5\frac{1}{2}$	5·5	$2\frac{5}{8}$	·381	5·012	19·72
$5\frac{3}{4}$	5·75	$2\frac{1}{2}$	·400	5·239	21·57
6	6·0	$2\frac{1}{2}$	·400	5·489	23·64

are intersected by lines, inclined at 55°. Lastly, $\frac{1}{6}$th of the depth of the triangular spaces so obtained is rounded off

both at top and at bottom. The square thread has usually twice the pitch of a triangular thread of the same diameter, and the depth of the thread is $\frac{19}{40}$ of the pitch. But as square threads are cut in the lathe, standard proportions for them are not so necessary as for ordinary threads.

79. *Gas threads.*—For the wrought-iron tubes used for conveying gas, the Whitworth screw thread is not suitable. For these a special system of threads has been adopted, finer in pitch, and cutting less deeply into the metal of the tube than the Whitworth thread would do.

Diameter in inches	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{2}$	2
Pitch in inches	·036	·053	·053	·071	·071	·091	·091	·091
No. of threads per inch	28	19	19	14	14	11	11	11

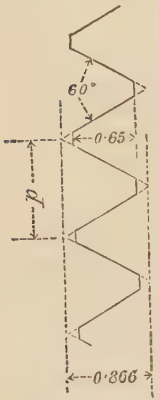


Fig. 86

80. *Sellers's screw threads.*—In the American or Sellers system of screw threads, fig. 86, the sides of the thread are inclined at 60°, and the angles at top and bottom are truncated to form a flat $\frac{1}{8}$th of the pitch in width. The number of threads per inch for different diameters of bolt is given in the table below. This thread system is now adopted by the Government and is termed the U.S. standard thread. The following equations give very approximately the pitch and diameter at bottom of thread in the Sellers system.

$$p = 0{\cdot}24\sqrt{d + 0{\cdot}625} - 0{\cdot}175 \quad . \quad (1a)$$

$$d^1 = d - 1{\cdot}3p = 0{\cdot}91d - 0{\cdot}08 \text{ nearly} \quad . \quad (2a)$$

Table of Sellers's U. S. Standard Triangular Threads

Diam. of bolt.	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
No. of threads per inch .	20	18	16	14	13	12	11	10	9	8
Diam. of bolt.	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$
No. of threads per inch .	7	7	6	6	$5\frac{1}{2}$	5	5	$4\frac{1}{2}$	$4\frac{1}{2}$	4
Diam. of bolts.	$2\frac{3}{4}$	3	$3\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{3}{4}$	4	$4\frac{1}{4}$	$4\frac{1}{2}$	$4\frac{3}{4}$	5
No. of threads per inch .	4	$3\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{4}$	3	3	$2\frac{7}{8}$	$2\frac{3}{4}$	$2\frac{5}{8}$	$2\frac{1}{2}$

81. *New Standard for small screws.*—A new standard for small screws such as are used by opticians and in telegraphic work has been adopted on the report of a Committee of the British Association. Let d = diameter of screw, and p = pitch in millimetres. Then for screws less than 6 mm. in diameter a series of pitches are assumed $0{\cdot}9^{0}$ $0{\cdot}9^{1}$, $0{\cdot}9^{2}$ . . . . and each screw pitch is characterised by a number which is the index of $0{\cdot}9$ in that series. For each

Table of New (British Association) Standard Small Screws

Number	Number of threads per inch	Dimensions in inches		Dimensions in millimetres	
		Diameter	Pitch	Diameter	Pitch
0	25·4	0·236	·0394	6·0	1·00
1	28·2	·209	·0354	5·3	·90
2	31·4	·185	·0319	4·7	·81
3	34·8	·161	·0287	4·1	·73
4	38·5	·142	·0260	3·6	·66
5	43·0	·126	·0232	3·2	·59
6	47·9	·110	·0209	2·8	·53
7	52·9	·098	·0189	2·5	·48
8	59·1	·086	·0169	2·2	·43
9	65·1	·075	·0154	1·9	·39
10	72·6	·067	·0138	1·7	·35
11	81·9	·059	·0122	1·5	·31
12	90·7	·051	·0110	1·3	·28

of these pitches a standard diameter is selected, given by the equation

$$d = 6\,p^{\frac{3}{5}}$$

The screw thread is similar to Whitworth's, but with an angle of $47\frac{1}{2}°$ instead of 55° and the rounding at top and bottom of thread is $\frac{2}{11}$ths of the pitch. Then the depth of thread is $\frac{3}{5}$ths of the pitch. The following table gives the pitch and diameter of a series of these screws.

Strength of Screw Bolts

82. *Strength of screw bolts subject to an axial tension only.*—Let P be the load on the bolt in lbs.; d = diameter of bolt at bottom of thread; f = working stress per unit of area. Then

$$P = \frac{\pi}{4} d_1^2 f \quad . \quad . \quad . \quad . \quad (3)$$

and for Whitworth threads replacing d_1 by d, by means of equation (2) above,

$$\left.\begin{aligned} d &= 0{\cdot}055 + 1{\cdot}127\sqrt{\left(\frac{P}{f}\right)} \text{ for triangular threads} \\ &= 0{\cdot}085 + 1{\cdot}32\sqrt{\left(\frac{P}{f}\right)} \text{ for square threads} \end{aligned}\right\} \quad . \quad (4)$$

This is a slight improvement of the formula commonly given for determining the strength of screw bolts. It neglects the twisting action due to screwing up. In consequence of the abrupt change of section the stress is not quite uniformly distributed at the section at the bottom of the threads, and this must be allowed for in fixing the working stress.

For wrought-iron bolts subject to a constant tension f = 8,000 to 10,000. If, as more commonly happens, the load may be regarded as varying constantly between 0 and its maximum value, f = 5,400 to 7,200 lbs. per sq. in. In this latter case, it is accurate enough to take in ordinary cases for Whitworth triangular threads—

$$d = 0{\cdot}0183\sqrt{P} \text{ to } 0{\cdot}0158\sqrt{P}$$
$$P = 3{,}000\,d^2 \text{ to } 4{,}000\,d^2$$

83. *Strength of bolts, taking torsion into account.*—Suppose that a load P is suspended from a square-threaded screw, and that it is screwed up, without, however, bringing the parts connected by the screw into actual contact, so as to develop a reaction between them additional to the load. In that case the friction of the nut on its support does not affect the stress in the bolt, and there is a definite relation

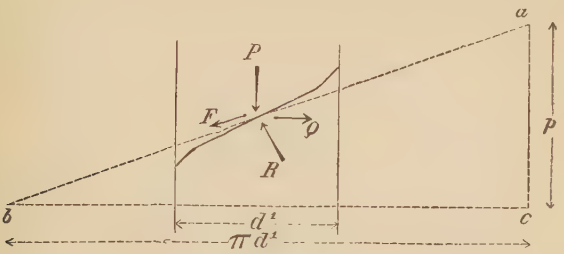


Fig. 87.

between the load P and the twisting force Q, applied to the bolt. Let fig. 87 represent a screw thread at its mean diameter d'. The forces, acting between any two elements of the bolt and nut, are a vertical force P, due to the load; a horizontal force Q, due to the pull on the spanner; a reaction R, normal to the thread; and a friction F, parallel to the thread.

The obliquity of the thread is the same as that of its development $a\,b$ on a plane surface, which makes, with the horizontal, an angle whose tangent is $p \div \pi d'$. Neglecting the small difference between the mean and outside diameters, d and d' of the thread we get

$$Q = P\,\frac{p + \mu\,\pi\,d}{\pi\,d - \mu\,p} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where μ is the co-efficient of friction. If the thread is

triangular instead of square, the normal reaction is greater, in the ratio of the slant length of a thread to its half thickness at the root. Hence, for triangular threads

$$Q = P\frac{p + 1{\cdot}15\,\pi\mu d}{\pi d - 1{\cdot}15\,\mu p} \quad . \quad . \quad . \quad . \quad (6)$$

The twisting moment of the force Q, acting at nearly $\frac{d}{2}$ from the axis, is $Q\frac{d}{2}$. Putting $\mu = 0{\cdot}15$, and using the previously found values for the pitch, we get

Twisting moment $= M = 0{\cdot}2\,P\,d$, nearly, in each case.

The greatest stress due to combined torsion and tension is then—

$$f = \frac{4P}{\pi d_1^2}\left\{\tfrac{1}{3} + \tfrac{2}{3}\sqrt{\left(\frac{2M}{Pd}\right)^2 + 1}\right\}$$

$$= 1{\cdot}339\frac{P}{d_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Putting for f the safe working stress, and replacing d_1 by d, we get

$$\left.\begin{aligned} d &= {\cdot}055 + 1{\cdot}285\sqrt{\frac{P}{f}}, \text{ for triangular threads} \\ &= {\cdot}085 + 1{\cdot}361\sqrt{\frac{P}{f}}, \text{ for square threads} \quad . \end{aligned}\right\} (8)$$

Comparing these with equations (4), in obtaining which the twisting moment was neglected, we see that the twisting moment adds about 15 per cent. to the diameter necessary for the bolt.

In ordinary cases, when bolts have to be tightened up while carrying the load, we may allow for the twisting action which is added to the tension by assuming a lower value for the working stress. Thus for wrought-iron bolts we may take in such cases

$$P = 2{,}400\,d^2 \text{ to } 3{,}000d^2$$
$$d = 0{\cdot}0204\sqrt{P} \text{ to } 0{\cdot}0183\sqrt{P}.$$

84. *Screws which are used to transmit motion.*—When a screw is used to transmit motion, by acting on a nut attached to the driven piece, the wear of the screw thread must be considered as well as its strength. In that case the pressure per unit of area of screw thread should be limited to some value K found by practical experience.

Let P be the axial force exerted by the screw in lbs.; d, d_1, the diameters of the screw at top and bottom of the screw threads; N the number of screw threads in contact with the nut. Then P should not exceed the value given by the equation

$$\begin{aligned} P &= K \frac{\pi}{4} (d^2 - d_1^2) N \\ &= \tfrac{1}{2} K \pi d (d - d_1) N \text{ nearly.} \end{aligned}$$

Now looking to equation (2) p. 144, if l is the length of nut in inches, and n the number of threads per inch, $N = n l$, and hence

$$\begin{aligned} P &= 2 K d l \text{ for triangular threads} \\ &= 6 K d l \text{ for square threads} \end{aligned}$$

of the proportions there stated. The pressure K on the projected area of screw thread may be taken not greater than 150 to 200 lbs. per sq. in.

Then P should not exceed 300 to 400 $d l$ for triangular threads, or 900 to 1,200 $d l$ for square threads. The advantage of the square thread due to its greater thread surface is obvious.

85. *Strength of bolts when the initial tension, due to screwing up, is taken into account.*—A nut is screwed up by means of a spanner, whose leverage is, on the average, $15 d$. Suppose that a nut is screwed up by a force Q, applied to the spanner at radius R; the tension P_1, produced in the bolt, being expended in compressing the pieces connected by the bolt. The friction of the nut, on its seat, balances part of the force Q. That friction acts

approximately at a radius $\frac{2}{3}d$, and its magnitude is $\mu_1 P_1$ lbs. Hence

$$P_1 = \frac{Q}{\frac{2}{3}\mu_1 + \frac{1}{2}\frac{p + \mu\pi d}{\pi d - \mu p}} \cdot \frac{R}{d} \quad . \quad . \quad . \quad . \quad (9)$$

Taking $\mu_1 = \mu = 0{\cdot}15$, for a turned nut on a turned washer; $R = 1{\cdot}5\,d$; $p = 0{\cdot}16\,d$, we get

$$P_1 = 82\,Q \quad . \quad . \quad . \quad . \quad . \quad . \quad (9a)$$

The force applied by a workman in screwing up a bolt will vary with the size of the bolt, and his experience teaches him in what case a heavy pressure may be applied with safety. Taking the heaviest ordinary pull of the workman to be $Q = 30$ lbs.—

$$P_1 = 82 \times 30 = 2{,}460 \text{ lbs.},$$

a force sufficient to break a $\frac{3}{8}$-in. bolt, and to seriously injure a $\frac{1}{2}$-in. bolt. Hence, bolts of less than $\frac{3}{4}$-in. diameter are not used for joints requiring to be tightly screwed up.

Suppose that bolts are used to connect together two parts of a machine, and that these are screwed up tightly before the effective load comes on the connected parts. Let P_1 be the initial tension on a bolt due to screwing up, and P_2 the additional load afterwards added. Then it can be shown that the greatest load on the bolt may either be little different from P_1 or P_2, according as the former or the latter is greater, or it may approach the value $P_1 + P_2$; and to determine what its value is, the relative rigidity of the bolts and the parts they connect must be known. Thus, suppose the pieces A and B, fig. 88, are connected by two bolts. To render the action of the elasticity of the connected parts

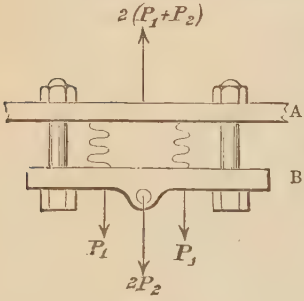


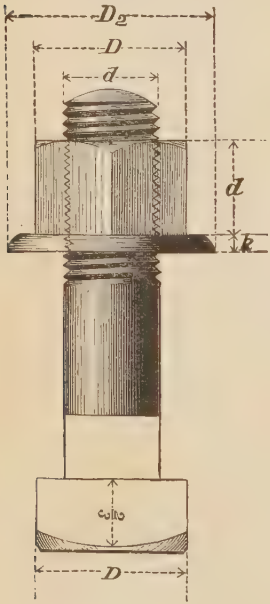
Fig. 88

more evident, suppose that springs are interposed between A and B. After screwing up till a tension of P_1 lbs. is created on each bolt, by the elastic resistance of the springs to compression, let an additional load of $2P_2$ lbs. be hung on the lower plate. This load may in part relieve the original tension if the extension of the bolts diminishes the compression of the springs, and in that case the ultimate tension on each bolt may be little greater than P_1 or P_2, according as P_2 or P_1 is greater. But if the extension of the bolts is small compared with the compression of the springs, then the ultimate load will approach the value $P_1 + P_2$, because the elastic thrust of the springs is little affected by the additional load. In ordinary cases the elasticity of the parts bolted together takes the place of the elasticity of the springs. Where rigid flanges are bolted together, metal to metal, it is probable that the extension of the bolts with any additional load relieves the initial tension, and the condition is similar to that in the former case described above. But in other cases, where the flanges are deflected by the bolts, or where elastic packing, such as india-rubber, is interposed, the extension of the bolts may very little affect the initial tension, and the condition is that of the latter of the two cases above.

Unfortunately in practice the strain likely to be put on a bolt by a workman varies greatly and cannot be definitely assigned. In the case of cylinder covers, and other similar cases where tight screwing up is to be expected, all that can be done is to assume that the greatest stress due to screwing up will bear some proportion to the steam or water pressure on the cover, and then to take a moderate value for the working stress so as to allow a margin of resistance for rough usage in screwing up. Looking to the proportionately greater straining action on small bolts due to screwing up, a smaller working stress may be adopted for small cylinders than for large cylinders. Thus the working stress may be taken :

	Diameter of cylinder in ins.	f lbs. per sq. in.
Less than	10	3,000
,,	15	4,000
,,	20	5,000
,,	60	6,000
Greater than	60	7,000

From these values and the net section at bottom of thread given in a table above, the diameter of the bolts can be determined. Or, if D is the diameter of the cylinder in ins., d_1 the diameter of bolt at bottom of thread, n the number of bolts, p the pressure on the cover in lbs. per sq. in.



$$\frac{\pi}{4} d_1^2 f n = \frac{\pi}{4} D^2 p$$

$$d_1 = D \sqrt{\left(\frac{p}{nf}\right)}$$

86. *Proportions of bolts and nuts.*—Fig. 89 shows the most ordinary type of bolt, nut, and washer. The bolt has a square head, and a square neck, to prevent the rotation of the bolt while the nut is being screwed up. The nut is hexagonal, and the washer

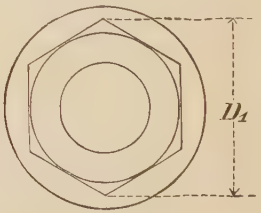


Fig. 89

circular. The washer is used when the bolt connects rough castings, and then forms a smooth seating, on which the nut

turns. It is sometimes used for appearance only. The following rules give good proportions :—

Bolt heads and nuts

Diameter across flats, rough, $D = 1\frac{1}{2}d + \frac{1}{8}$ } Hexagon
" " bright, $= 1\frac{1}{2}d + \frac{1}{16}$ } or square
Diameter across angles, rough $= D_1 = 1{\cdot}73\,d + 0{\cdot}14$ } Hexa-
" " " bright $= 1{\cdot}73\,d + 0{\cdot}07$ } gon
" " " rough $= D_1 = 2{\cdot}12\,d + 0{\cdot}18$ } square
" " " bright $= 2{\cdot}12\,d + 0{\cdot}09$ }

$D = 1{\cdot}5\ d + 0{\cdot}18$, and $D_1 = 1{\cdot}75\ d + 0{\cdot}16$ are very nearly Whitworth's standard sizes for finished nuts. In drawings on a small scale it is accurate enough to take $D_1 = 2d$.

Height of nut (rough) $= d$
(bright) $= d - \frac{1}{16}$
Height of lock nut $= \frac{1}{2}d$
Thickness of bolt head $= \frac{3}{4}d$ to d.

Length of spanner $= 15\,d$ to $18\,d$. The usual form of spanners is shown in fig. 90.

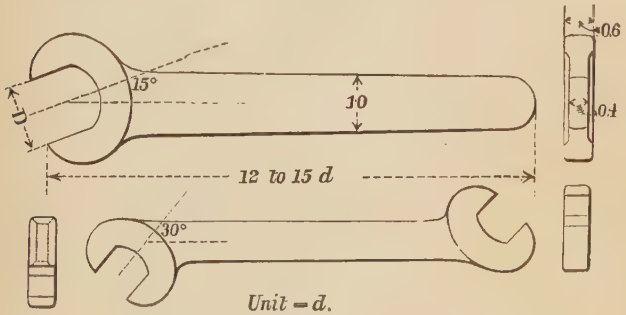


Fig. 90

Washers

Thickness $0{\cdot}15\,d$; diameter $\frac{9}{8}\,D_1$.

Small washers are usually 14 B.W.G., or 0·083 in. thick.

Washers, for wood, may be $3\,d$ in diameter, and $0{\cdot}3\,d$ in thickness.

	Diameter of Bolt											
	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Size of hole before tapping	$\frac{3}{16}$	$\frac{21}{64}$	$\frac{29}{64}$	$\frac{17}{32}$	$\frac{43}{64}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{31}{32}$	$1\frac{7}{64}$	$1\frac{5}{16}$	$1\frac{17}{32}$	$1\frac{3}{4}$
D	$\frac{17}{32}$	$\frac{11}{16}$	$\frac{29}{32}$	$1\frac{1}{8}$	$1\frac{5}{16}$	$1\frac{17}{32}$	$1\frac{23}{32}$	$1\frac{5}{16}$	$2\frac{3}{32}$	$2\frac{7}{16}$	$2\frac{25}{32}$	$3\frac{9}{64}$
D_1	$\frac{5}{8}$	$\frac{25}{32}$	$1\frac{1}{32}$	$1\frac{9}{32}$	$1\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{15}{16}$	$2\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{13}{16}$	$3\frac{5}{32}$	$3\frac{5}{8}$

Weight of Bolts and Nuts in Pounds.

	Diameter of Bolt											
	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Weight per inch of bolt .	·017	·034	·057	·087	·124	·17	·22	·28	·34	·49	·68	·88
Weight of square head .	·017	·044	·088	·126	·260	·397	·544	·694	·943	1·667	2·535	3·571
Weight of nut and part of bolt included . .	·022	·0530	·110	·172	·324	·494	·681	·873	1·188	2·088	2·954	4·387
Weight of washer and part of bolt included . .	·011	·0154	·039	·048	·096	·132	·161	·178	·273	·489	·723	1·014

SIZE OF WHITWORTH NUTS AND BOLTS.

The following are the sizes of bolts and nuts according to the Whitworth Standard, as revised some two years since. The exact sizes are given in decimals, and the nearest approximate sizes in sixty-fourths of an inch.

Diameter of bolts	Width of nuts across flats		Height of boltheads	
$\frac{1}{8}$	·338	$\frac{21}{64}$ *f*	·1093	$\frac{7}{64}$
$\frac{3}{16}$	·448	$\frac{29}{64}$ *b*	·1640	$\frac{5}{32}$
$\frac{1}{4}$	·525	$\frac{34}{64}$ *f*	·2187	$\frac{7}{32}$
$\frac{5}{16}$	·6014	$\frac{19}{32}$ *f*	·2734	$\frac{17}{64}$
$\frac{3}{8}$	·7094	$\frac{45}{64}$ *f*	·3281	$\frac{21}{64}$
$\frac{7}{16}$	·8204	$\frac{53}{64}$ *b*	·3828	$\frac{25}{64}$ *f*
$\frac{1}{2}$	·9191	$\frac{29}{32}$ *b*	·4375	$\frac{7}{16}$
$\frac{9}{16}$	1·011	$1\frac{1}{64}$ *b*	·4921	$\frac{31}{64}$ *f*
$\frac{5}{8}$	1·101	$1\frac{3}{32}$ *f*	·5468	$\frac{35}{64}$
$\frac{11}{16}$	1·2011	$1\frac{13}{64}$ *b*	·6015	$\frac{19}{32}$ *f*
$\frac{3}{4}$	1·3012	$1\frac{19}{64}$ *f*	·6562	$\frac{21}{32}$
$\frac{13}{16}$	1·39	$1\frac{25}{64}$ *b*	·7109	$\frac{45}{64}$ *f*
$\frac{7}{8}$	1·4788	$1\frac{31}{64}$ *b*	·7656	$\frac{49}{64}$
$\frac{15}{16}$	1·5745	$1\frac{37}{64}$ *b*	·8203	$\frac{13}{16}$ *f*
1	1·6701	$1\frac{43}{64}$ *b*	·875	$\frac{7}{8}$
$1\frac{1}{8}$	1·8605	$1\frac{55}{64}$ *f*	·9843	$\frac{63}{64}$
$1\frac{1}{4}$	2·0483	$2\frac{3}{64}$ *f*	1·0937	$1\frac{3}{32}$
$1\frac{3}{8}$	2·2146	$2\frac{7}{32}$ *b*	1·2031	$1\frac{13}{64}$
$1\frac{1}{2}$	2·4134	$2\frac{13}{32}$ *f*	1·3125	$1\frac{5}{16}$
$1\frac{5}{8}$	2·5763	$2\frac{37}{64}$ *b*	1·4128	$1\frac{27}{64}$
$1\frac{3}{4}$	2·7578	$2\frac{3}{4}$ *f*	1·5312	$1\frac{17}{32}$
$1\frac{7}{8}$	3·0183	$3\frac{1}{16}$ *f*	1·6406	$1\frac{41}{64}$
2	3·1491	$3\frac{5}{32}$ *b*	1·75	$1\frac{3}{4}$
$2\frac{1}{8}$	3·337	$3\frac{11}{32}$ *b*	1·8523	$1\frac{55}{64}$
$2\frac{1}{4}$	3·546	$3\frac{35}{64}$ *b*	1 9687	$1\frac{31}{32}$
$2\frac{3}{8}$	3·75	$3\frac{3}{4}$	2·0781	$2\frac{5}{64}$
$2\frac{1}{2}$	3·894	$3\frac{57}{64}$ *f*	2·1875	$2\frac{3}{16}$
$2\frac{5}{8}$	4·049	$4\frac{3}{64}$ *f*	2·2968	$2\frac{19}{64}$
$2\frac{3}{4}$	4·181	$4\frac{3}{16}$ *b*	2·4062	$2\frac{13}{32}$
$2\frac{7}{8}$	4·3456	$4\frac{11}{32}$ *f*	2·5156	$2\frac{33}{64}$
3	4·531	$4\frac{17}{32}$ *b*	2·625	$2\frac{5}{8}$

The thickness of the nuts is in every case the same as the diameter of the bolts : *f* = full, *b* = bare.

87. *Different forms of nuts.*—Ordinary nuts are chamfered off at an angle of 30° to 45°, as shown at *a*, fig. 91 ; or they are finished with a spherical bevil, struck with a radius of about 2 *d*, as shown at *b*. *Flange nuts*, *c*, are used when the hole, in which the bolt is placed, is considerably larger than the bolt itself. The flange covers and hides the hole. *Cap nuts*, *d*, are used where leakage along the screw thread

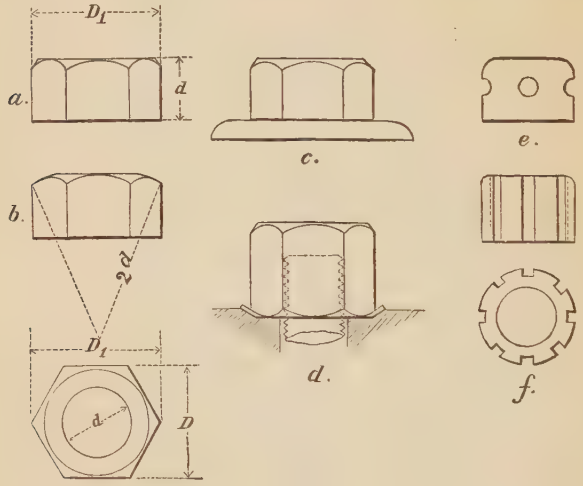


Fig. 91.

is feared. In the figure, a thin, soft copper washer is shown, which prevents leakage under the nut. *Circular nuts*, *e*, are occasionally used. They have holes, in which a bar, termed a 'Tommy,' is placed, for screwing them up. Sometimes grooves are cut, as shown at *f*. Steel nuts may be used, if great durability is required.

88. *Different forms of bolt heads.*—In fig. 92, *a* is a cup-shaped, *b* a countersunk, and *c* a square bolt-head. Rotation of the bolt is prevented in *a* by a square neck, in *b* by a set screw, in *c* by a snug forged on the bolt. Fig. 92, *d*, shows

a T-headed bolt in front and side elevation. Fig. 92, *e*, shows an eye bolt. Fig. 93 shows a spherical-headed bolt used.

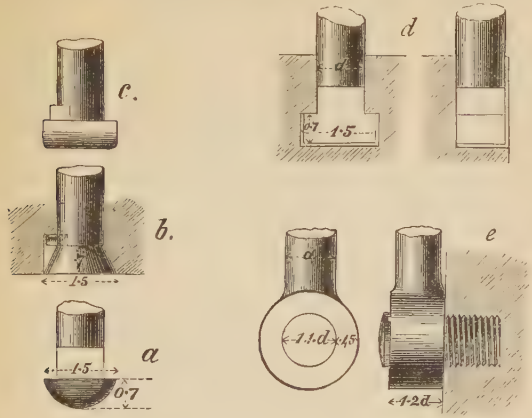


Fig. 92.

sometimes, for railway fastenings, with a square neck. The spherical head allows the bolt to take a fair bearing on the rail. The other figure shows a cup-head, with a snug forged

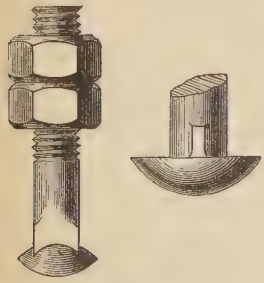


Fig. 93.

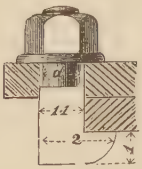


Fig. 94.

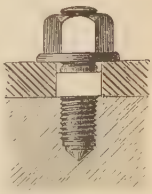


Fig. 95.

on the bolt, to prevent rotation when the bolt is screwed up. Proportional unit=*d*, in all these figures.

Fig. 94 is a *hook bolt*, which is used when one piece is too small to have a bolt hole through it, or when it is objectionable to weaken the piece by a bolt hole. Fig. 95

is a *stud*, which is screwed into one of the connected pieces, and remains in position when the nut is removed. Fig. 96 is a *set screw*, or bolt not requiring a nut. Fig. 97 shows,

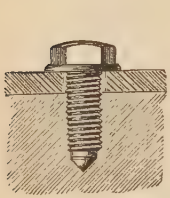


Fig. 96.

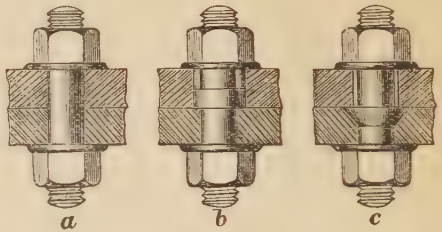


Fig. 97.

at *a*, a nut-headed bolt, or bolt having two loose nuts, instead of a nut and head ; at *b* and *c* similar bolts, with an intermediate head or flange. These bolts remain in place when the top nut is removed.

Fig. 98 is a bolt leaded into stone work. The tail of the bolt is rectangular, with jagged edges.

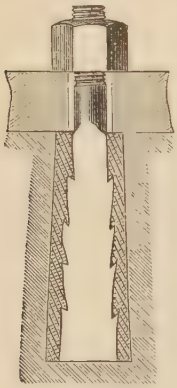


Fig. 98.

Fig. 99 is a fang bolt used for attaching ironwork to wood, and especially for attaching rails to sleepers. The fangs of the broad triangular plate, which forms the nut, bite into the wood, while the bolt is rotated by the

Fig. 99.

head, which bears on the ironwork. The large area of the nut prevents crushing of the wood.

89. *Locking arrangements for nuts* are intended to prevent the gradual unscrewing of nuts, subjected to vibration and frequent changes of load. No nut accurately fits its bolt; a certain amount of play, however minute, always exists. When a nut, having play, is subjected to vibration, it gradually slacks back. This is, to a great extent, prevented by double nuts, shown in fig. 100. One of the nuts is termed a lock nut, and is usually half as thick as the ordinary nut. When there are two nuts, the whole load may be thrown on the outer nut. The outer nut ought, therefore, to be the thicker nut. It is common in practice to put the thinner nut outside, the reason being, that ordinary spanners are sometimes too thick to hold the thin nut, when screwed home first. The more correct arrangement is that shown in the figure.

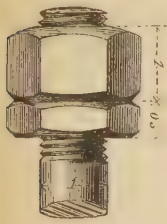


Fig. 100.

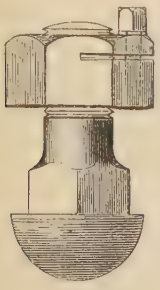
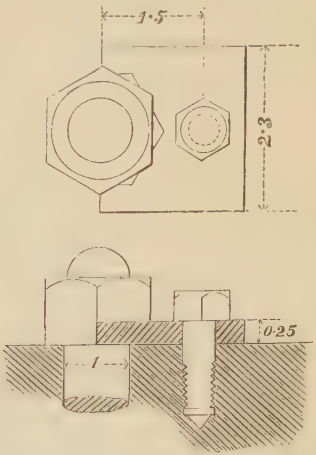


Fig. 101.



Unit = *d*.

Fig. 102.

Fig. 101 shows the form of Wiles's lock nut, which is now often used on quick-running machine parts subjected to a good deal of vibration. It is very simple and can be locked on any part of the bolt. The nut is half cut through

by a saw cut, and a small set screw is used to close slightly the jaws thus formed, after the nut is screwed home. The nut then grips the screw thread tightly. For nuts under one inch in diameter the set screw is omitted, and the jaws of the saw cut are slightly closed by a hammer blow, before the nut is put in place.

Another plan is to drill a hole through the top of the bolt above the nut, and drive a split pin or cotter through. The nut must always be in the same place when screwed up. A better plan is shown in fig. 102, a stop plate being used, fixed on one side of the nut. The set screw in the stop plate may have its diameter $=\frac{1}{4}d+\frac{1}{8}$.

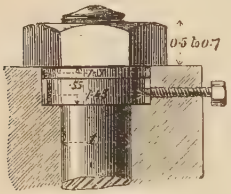


Fig. 103.

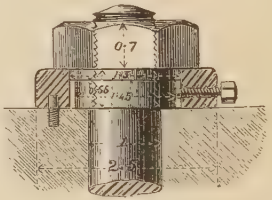


Fig. 104.

A very neat arrangement is shown in fig. 103; the lower part of the nut is turned circular, and fits in a recess in the piece connected by the bolt. A set screw is tapped through, and bears on the side of the nut. The diameter of the set screw may be $\frac{1}{8}d+\frac{1}{8}$. A stop ring is sometimes used (fig. 104), with a set screw tapped through it. The stop ring is of brass, or wrought iron, and it is prevented from turning by a stop pin of the same size as the set screw.

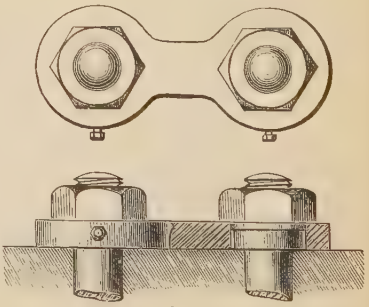


Fig. 105.

Fig. 105 shows a neat way of applying this last mode of locking to a pair of nuts. The locking plate embraces the turned part of both nuts, and the stop pin is dispensed with.

Elastic washers have been used as substitutes for lock nuts. Fig. 106 shows Grover's spring steel washer. When

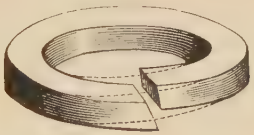


Fig. 106

the nut is tightened up, the washer becomes nearly, but not quite, flat, and its elasticity neutralises the play of the nut on the bolt.

90. *Bolting of cast-iron plates.*—Cast-iron plates are united by bolts; flanges, to receive the bolts, are cast

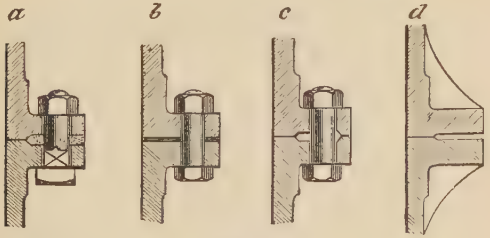


Fig. 107

on the plates, and these may be external or internal. The flanges are a little thicker than the plates, and the bolts in ordinary work are not less than $\frac{3}{4}$-inch diameter. Fig. 107 shows arrangements of bolts and flanges. *a* is an ordinary rust joint, the flanges fitting at a narrow chipping

strip and the rest of the space being filled with iron cement. *b* is a faced joint made with a layer of asbestos millboard or other packing. In *c* there are two faced parts so that the tension of the bolt cannot bend the flange. In *d* the flange is strengthened with feathers or brackets between the bolts.

Fig. 108 gives the ordinary proportions of bolt and flange.

Bolt diameter $= d = \frac{5}{4}t + \frac{1}{8}$ to $\frac{3}{4}t + \frac{1}{4}$

$t_1 = 0{\cdot}9\,t + \frac{3}{8}$

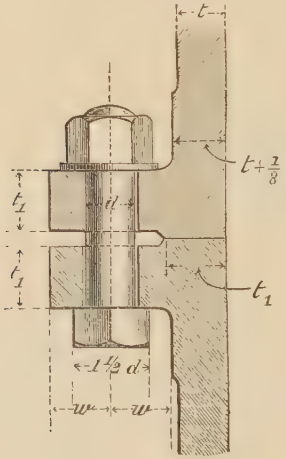


Fig. 108

Pitch of bolts about 6 *d*, or less if necessary for strength.

Width of chipping strip $= t_1$.

Width of flange $= 2\,w = 2\frac{1}{4}d + \frac{1}{4}$.

Some rather interesting experiments on the strength of flanges are given in a paper by Mr. Ellington, 'Proc. Inst. of Civil Engineers,' vol. xciv. p. 9.

91. *Stay bolts* for tanks. The flat surfaces of tanks are supported by stay bolts connecting opposite sides of the tank, and thus directly resisting the water pressure. Fig. 109 shows the form of such bolts for cast and wrought iron tanks.[1] The end of the bolt is enlarged in fig. 109*a*, so that the body of the bolt is of the same strength as the screw at the bottom of the thread. Then $d_1 = 0{\cdot}9d - 0{\cdot}05$. The cast-iron plate through which the bolt passes is

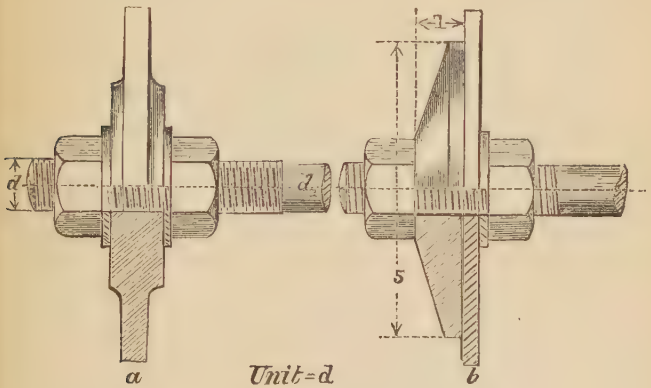


Fig. 109.

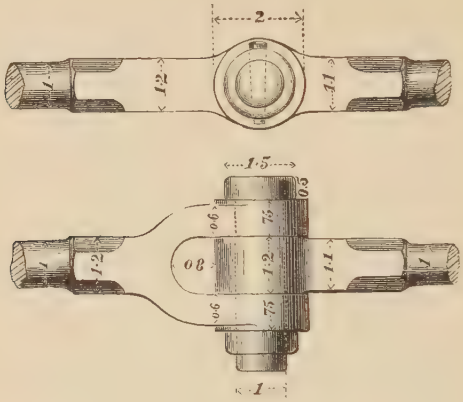
thickened to spread the pressure of the bolt. In fig. 109*b*. the wrought-iron tank plate cannot easily be thickened, but a large washer plate of cast iron is used to spread the bolt pressure.

Joint Pins. Knuckle Joint.

92. A joint pin is a kind of bolt, so placed as to be in shear. Fig. 110 shows an arrangement known as a knuckle joint. The proportions are empirical. If the joint pin were subjected to simple shear at two sections, it would be strong enough, when its diameter was equal to 0·7 of the diameter of the rods. But the pin wears, and is then subjected to

[1] Hydraulic Machinery, by W. Anderson.

bending, as well as shearing. When there is much motion at the joint, the width of the eyes of the rods, and the length of the pin, may be increased.



Unit = diam. of pin.
Fig. 110.

Keys.

93. Keys are small wedge-shaped pieces used to fix wheels, pulleys, cranks, and other pieces on shafts. The

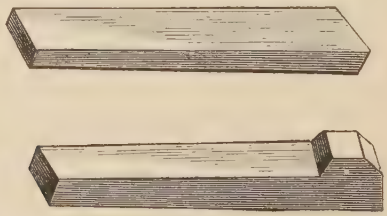


Fig. 111.

function of a key is to prevent the piece rotating relatively to the shaft; but, from the friction of the key in the keyways, it offers also some resistance to sliding along the shaft.

Fig. 111 shows the usual form of keys. The lateral sides are parallel, but the thickness tapers. The lower figure shows a key with a head which is necessary for slacking back the key in cases where the other end is inaccessible.

Keys may be saddle keys (fig. 112*A*); keys on flats (fig. 112*B*); or sunk keys (fig. 112*C*). Saddle keys are used for fixing light pulleys and other pieces. The rotation of

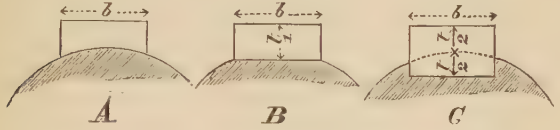


Fig. 112.

the pulley or other piece on the shaft is prevented only by the friction of the key. The taper of a key is about 1 in 64 to 1 in 100. Keys on flats are more secure than saddle keys. The flat is parallel to the axis of the shaft, and rather wider than the key. The taper may be the same as for a saddle key.

Sunk keys are much more satisfactory because slipping is entirely prevented unless the key shears. Keyways are slotted in the shaft and in the piece fixed on it, and the key is accurately fitted in the key ways It should fit at the sides even more accurately than on the top and bottom. A wood pattern of the key is usually first fitted in the key ways, and this serves as a guide in forging and planing the key. A sunk key with a saddle key placed at right angles to it, fig. 113, is a very good arrangement. If a wheel nave is accidentally bored out slightly larger than the boss on which it is fitted, it rocks if held by a sunk key only. This rocking is entirely prevented by the additional saddle key, which insures a bearing between the eye and shaft boss at three points in the circumference.

94. *Staking on.*—In the case of large pieces four keys are sometimes used, fig. 114, or, when the shaft boss is

square, eight keys, fig. 115. In the latter case the labour of planing the whole of the shaft boss and slotting out the square eye of the piece to be fixed to it is generally saved, only the key seats being faced. When four or eight keys

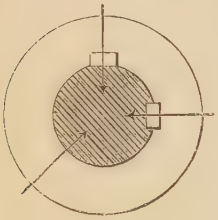


Fig. 113.

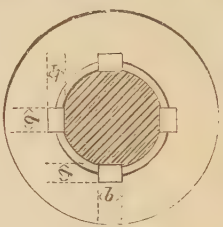


Fig. 114.

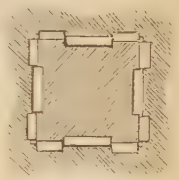


Fig. 115.

are used, there is a limited power of adjusting the piece keyed to the shaft so as to be coaxial with the shaft.

95. *Cone keys.*—When a wheel has to be bored out, to pass over a shaft boss, cone keys may be used to fix it on the shaft, fig. 116. These are of cast iron and are cast in a single piece, with three parting plates, nearly but not quite dividing it into three pieces. The casting is bored and turned, and afterwards split and the rough edges chipped away. There are thus obtained three cast iron slightly tapering conical or saddle keys, of the thickness necessary to fill the space between the eye of the pulley and the shaft. They resist the tendency to rotation of the piece keyed on by friction alone. The eye of the pulley must be bored slightly conical.

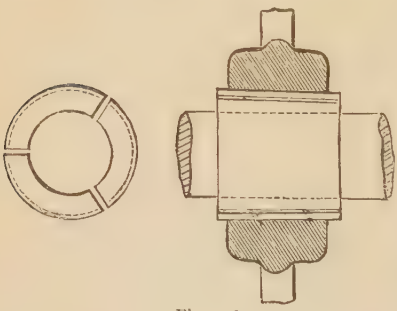


Fig. 116.

95*a*. *Pins.*—A taper pin (fig. 117) is sometimes used in place of a key. It is sunk half in the shaft, half in the piece keyed on.

Where very fast connection between the pieces is required, as in the case of cranks keyed on crank shafts, the crank is usually bored but slightly smaller than the crank shaft, expanded by heat and shrunk on. Rotation is then prevented by a key or pin.

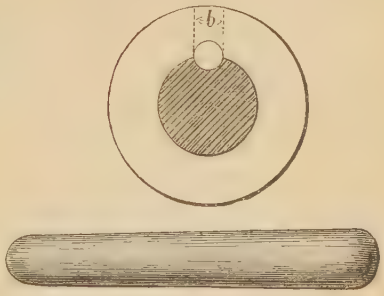


Fig. 117.

96. *Taper of keys.*—Let P be the pressure between the pieces acting through the key in the plane of rotation, μ the coefficient of friction, θ the angle of repose for the surfaces of contact so that $\mu = \tan \theta$.

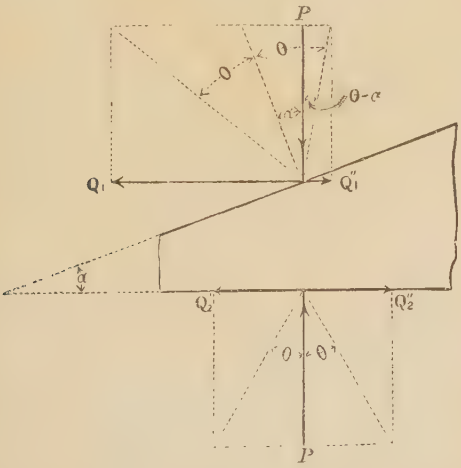


Fig. 118.

Then when the key is being driven home the force required is

$$Q_1' + Q_2' = P\{\tan(\theta + \alpha) + \tan\theta\}$$

and when the key is driven back the force required is

$$Q_1'' + Q_2'' = P\{\tan(\theta - \alpha) + \tan\theta\}$$

If in the latter case

$$\theta - \alpha = -\theta$$

or,

$$\alpha = 2\theta$$

no additional force is required to slack the key, beyond the horizontal component of the pressure on the sloping face. Hence $\alpha = 2\theta$ is a limiting value for the inclination of the key, and if this is exceeded the key will not remain in position after driving home. For metal on metal, somewhat oily, $\mu = 0{\cdot}08$ and $\theta = 4\frac{1}{2}°$, consequently the limiting angle for a key is about 9°, corresponding to a taper of about 1 in 7. The actual taper of keys is much less than this, being seldom greater than 1 in 64, and generally 1 in 100 to 1 in 150. The taper is smallest in the most accurate work.

97. *Strength of keys.*—For saddle keys no very exact rules can be given. They are only used where the stress between the connected pieces is small. For sunk keys, let b=width, l=length of key, t=mean thickness of key, f_s, f_c shearing and crushing resistance of material. The resistance to shearing is $b\,l\,f_s$. The bearing surface of the key on the sides of the key way is $\frac{1}{2}\,t\,l$, and the crushing resistance is $\frac{1}{2}\,t\,l\,f_c$.

If $f_c = 2f_s$, then $t = b$, or the key should be square in section. For practical reasons the key is generally wider than this, and then its shearing resistance is usually in excess.

98. *Common proportions of keys.*—The following rules are empirical, and in general give an excess of strength to the key:

Diameter of eye of wheel, or boss of shaft $= d$

Width of key $= b = \frac{1}{4}d + \frac{1}{8}$ . . . . . (11)

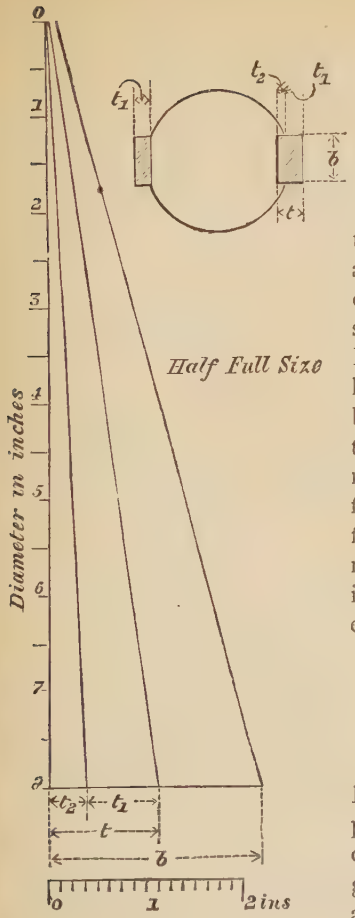


Fig. 119

Mean thickness of sunk key $= t = \frac{1}{2}\, b$

$t_1 = 0{\cdot}3\ b$

$t_2 = 0{\cdot}2\ b$

Mean thickness of key on flat $= \frac{1}{3}\, b$

When wheels or pulleys transmitting only a small amount of power are keyed on large shafts, the dimensions above are excessive. In that case let H.P. be the horse-power transmitted by the wheel or pulley, N the number of rotations per minute; or let P be the force acting at its circumference in lbs., and R its radius in inches. Then it is sufficient to take in the expressions above,

$$d = \sqrt[3]{\frac{100 \text{ H.P.}}{N}} \text{ or } \sqrt[3]{\frac{P\,R}{360}} \quad . \quad . \quad (13)$$

Fig. 119 gives a scale of proportions for keys. It is convenient to draw a diagram of this kind full size and to use it in place of formulæ or tables.

Cotters

99. A cotter is a tapered bar driven through two pieces which are to be connected, and prevents their separation by

resistance to shearing at two cross sections. The cotter should be so designed as to diminish as little as possible the strength of the connected pieces. When the cotter is long it serves to adjust the length of the pieces connected. When driven home it shortens the total length, and *vice versâ*.

The simplest forms of cotter are shown in fig. 120. In fig. 120 *a*, *b*, the fixing resists equally a thrust or tension. Fig. 120 *c* is an arrangement for resisting tension only, often used for foundation bolts. The enlarged or gib ends of the cotter prevent its displacement.

Fig. 120 *e*, *f*, shows another arrangement for resisting

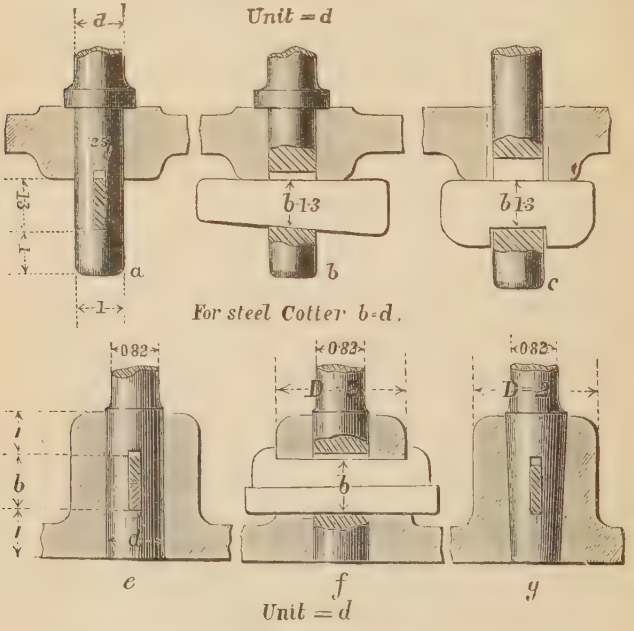


Fig. 120

only a tension, the cotter being divided into two parts termed respectively the gib and cotter. The cotter way is here

exactly parallel. The rod is diminished beyond the connected pieces to a section equivalent to that through the cotter way. Fig. 120,g shows an arrangement for both thrust and tension, often used for pistons and piston rods. Often part of the rod is enlarged to receive the cotter, so that the area through the cotter hole is equal to the section of the rest of the rod. If d_1 is diameter of the rod, d diameter of the enlarged part, and t thickness of the cotter,

$$\frac{\pi}{4} d_1^2 = \frac{\pi}{4} d_2 - d\,t$$

If $t = \frac{1}{4} d$, as is common,

$$d = 1{\cdot}21\, d_1, \text{ and } d_1 = 0{\cdot}82\, d.$$

The sides of the cotter slope at 1 in 30 to 1 in 60 with the axis of the rod, so that the total taper in simple cotters is 1 in 30 to 1 in 15. By driving the cotter the rod is drawn up to its seating and clearance is left to permit this.

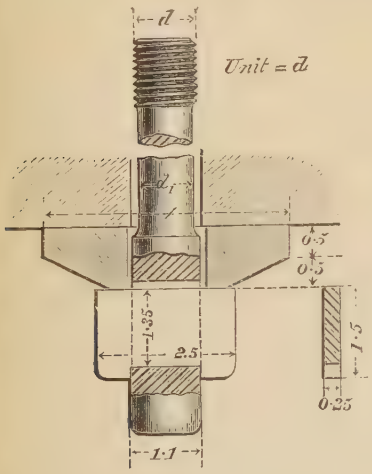


Fig. 121

Fig. 121 shows an ordinary foundation bolt and cast-iron washer. The body of the bolt when long may be reduced to $d_1 = 0{\cdot}9\, d - 0{\cdot}05$. It is then as strong as the screwed part at the bottom of the thread.

100. *Taper of cotter.*—It is easy to show for cotters as for keys that the horizontal force necessary to drive the cotter home is

$$Q_1' + Q_2' = P \left\{ \tan (\theta + \alpha_1) + \tan (\theta + \alpha_2) \right\}$$

where θ is the angle of repose of the materials. Similarly the force necessary to drive back the cotter is

$$Q_1'' + Q_2'' = P \left\{ \tan (\theta - \alpha_1) + \tan (\theta - \alpha_2) \right\}$$

and the cotter will slip back without any additional force if

$$Q_1'' + Q_2'' = 0, \text{ and then } \alpha_1 + \alpha_2 = 2\theta$$

Taking θ for slightly greasy metal at $4\frac{1}{2}°$,

$$\alpha_1 + \alpha_2 \text{ must not exceed } 9°$$

corresponding to a total taper of 1 in 7. The taper in practice is less than this for safety. For simple cotters a taper of 1 in 24 to 1 in 48 is usual. When a set screw or other means of preventing the slacking of the cotter is added, the taper may be 1 in 8 or 1 in 6.

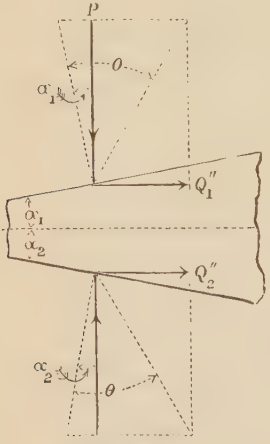


Fig. 122

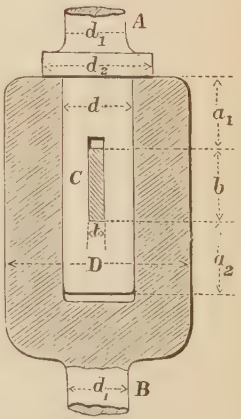


Fig. 123

101. *Strength and proportions of cotters.* – Let P be the longitudinal stress transmitted through two cottered rods; f_t and f_s the tearing and shearing resistance of the rods and cotters respectively. The values of f_s and f_t may be taken from Table II., p. 43, according to the kind of straining

action to which the fastening is subjected. Then taking a section through the cotter hole and the two sections of the cotter in shear

$$P = \left(\frac{\pi}{4} d^2 - d\,t\right) f_t \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$= \left\{\frac{\pi}{4} (D^2 - d^2) - (D - d)\,t\right\} f_t \quad . \quad . \quad . \quad (2)$$

$$= 2\,b\,t\,f_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The cotter may be subjected to a force in one direction only or to forces alternately in opposite directions. For these two cases the working stress may be taken.

	Wrought iron	Steel	Cast iron
Case 1. Stress in one direction only . . . . f_t =	10,000	13,200	2,800
f_s =	8,000	10,600	—
Case 2. Stress alternately in opposite directions . . f_t =	5,000	6,600	1,400
f_s =	4,000	5,300	—

Then taking $t = \frac{1}{4} d$, equations (1) and (3) give

$$b = 1{\cdot}06 \frac{f_t}{f_s} d$$

		In Case 1 or 2.
Rod, Wrought Iron.	Cotter, Wrought Iron	$b = 1{\cdot}33\,d$
„ „ „	„ Steel	$= d$
„ Steel.	„ Steel	$= 1{\cdot}32\,d$
„ Cast iron.	„ Wrought Iron	$= 0{\cdot}37\,d$

Similarly equations (1) and (2) give, putting f_t' for the tearing resistance of A and f_t'' for the tearing resistance of B,

$$(0{\cdot}78D^2 - 0{\cdot}25Dd - {\cdot}53d^2)\,f_t'' = {\cdot}53\,d^2 f_t'$$

A. Wrought Iron	B. Wrought Iron	$f_t' = f_t''$	$D = 1{\cdot}34\,d$
A. Steel	B. Wrought Iron	$f_t' = 1{\cdot}32 f_t''$	$D = 1{\cdot}41\,d$
A. Steel	B. Steel	$f_t' = f_t''$	$D = 1{\cdot}34\,d$
A. Wrought Iron	B. Cast Iron	$f_t' = 3{\cdot}6 f_t''$	$D = 1{\cdot}91\,d$

In the case where the rod is subjected to opposite forces, the pressure on the bearing surface of the cotter on the rod does not generally need to be specially considered. In other cases it is desirable to take care that this bearing pressure is not too great. That pressure is given by the equations

$$P = d\,t\,f_c = (D-d)\,t\,f_c \quad . \quad . \quad . \quad . \quad (4)$$

or if $t = \frac{1}{4}\,d$

$$P = \cdot 25\,d^2 f_c = \cdot 25(D - d)\,d\,f_c$$

where f_c is the pressure per unit of bearing surface. The pressure will be greatest on the rod B, and for this reason it is good and generally convenient to take $D = 2\,d$, so that the pressure on the bearing surface is the same for both rods.

Then from equations (1) and (4)

$$\cdot 25\,d^2 f_c = \cdot 53\,d^2 f_t$$

$$\frac{f_c}{f_t} = 2\cdot 1.$$

It would seem from experience that 20,000 lbs. per sq. in. is not too great a bearing pressure for a cotter tight in the rods. But if injury to rod or cotter is feared the rod may be enlarged a little and the cotter thickened. It is a very good plan to shape the cotter with cylindrical bearing surfaces to diminish the variation of stress due to the sudden variation of section of the rod.

The collar of rod A requires at least as much bearing surface on B as that of the cotter. Hence

$$\frac{\pi}{4}\left(d_2{}^2 - d^2\right) = d\,t = \cdot 25\,d^2$$

$$d_2 = 1\cdot 15\,d.$$

It may be useful to consider the cotter as subjected to bending instead of shearing. It is impossible then to determine exactly the bending moment, because the distribution of the pressure on the bearing surfaces is unknown. If the bending moment is taken at $\frac{1}{4}\,P\,d$, then

$$\tfrac{1}{4}\,\mathrm{P}\,d = \tfrac{1}{6}\,b\,d^2\,f_t$$

$$b = \tfrac{3}{2}\,\frac{\mathrm{P}}{d\,f_t}$$

or, with the proportions above, the cotter and rod being of the same material, $b = 0{\cdot}8\,d$, a less value than before.

The cotter is often made of steel because it is harder and not so easily set up by hammering. The dimensions $a_1 a_2$ might be calculated for shearing, but as the shearing resistance longitudinally, in wrought iron, is less than transversely, it is better to take

$$a_1 = a_2 = b$$

which allows a considerable margin of safety.

102. *Various arrangements of cotters.*—Fig. 124 shows the mode of cottering pump rods. The socket is conical.

Fig 124

Fig. 125

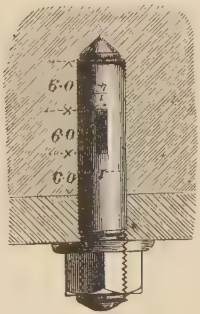


Fig. 126

The collar is provided merely to facilitate the disengagement of the parts after they have been for some time in use. The

cotter having been driven out, wedges can be driven between the collar and socket so as to force them apart.

Split pins are virtually very small cotters, driven home like ordinary cotters and the split end opened out to prevent slacking back (fig. 125).

Fig. 126 shows a bolt cottered into a casting. The effective diameter of the bolt is the diameter at the bottom of the screw thread, and that is the value to be taken for d in proportioning the cotter. The proportions marked on the figure have been so modified that the unit is the gross diameter d of the bolt.

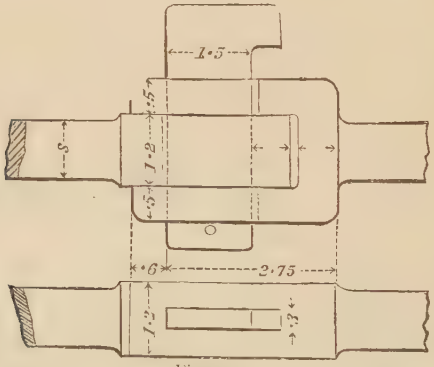


Fig. 127

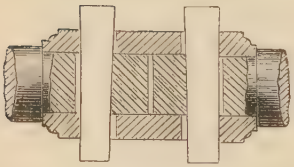


Fig. 128

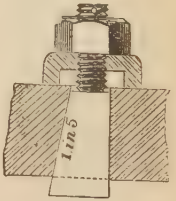


Fig. 129

Fig. 127 shows a simple cottered joint for two bars.

Fig. 128 is a cottered coupling suitable for rods transmitting a longitudinal thrust or pull.

If excessive taper must be given, to obtain sufficient

draught, the end of the cotter is screwed, as shown in fig. 129, and a nut, bearing on a recessed washer or short tube, holds it in place.

Gib and Cotter

103. When a cotter is used to connect strap-shaped parts to a more rigid rod, the cotter is divided into two parts, one acting as an ordinary cotter, the other having hooked ends, intended to prevent the spreading of the strap. It is convenient to make the outside of the gib parallel to the outside of the cotter, and to obtain the necessary draught by inclining the division plane between them. The taper is usually 1 in 24 to 1 in 48 for simple cotters, and 1 in 8 to 1 in 16, when the slacking of the cotter is prevented by a screw. The total breadth b and thickness t of the gib and cotter are the same as for a simple cotter.

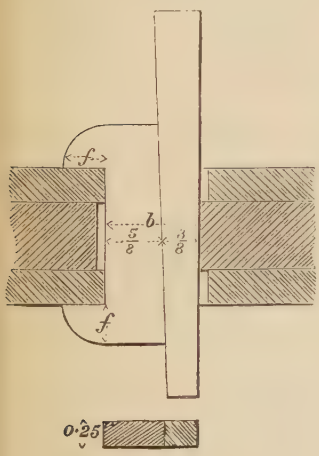


Fig. 130

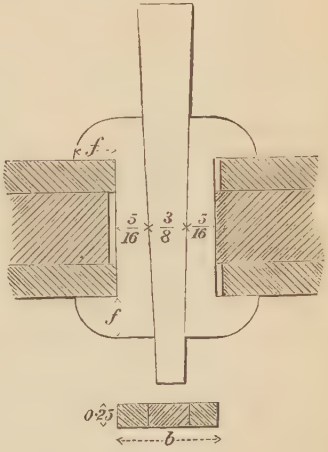


Fig. 131

Unit = b.

Figs. 130, 131 show ordinary proportions of gibs and cotters. The unit for the proportions is the breadth b.

The simplest way of securing a cotter is by a screwed

prolongation of the gib, as shown in fig. 132. A set screw, passed through one of the connected pieces, is sometimes used.

Fig. 133 shows another arrangement of gibs and cotter. In this case the space is restricted, and the draught required

Fig. 132

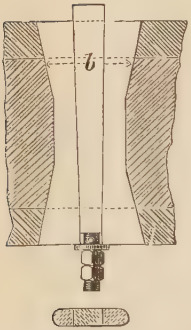


Fig. 133

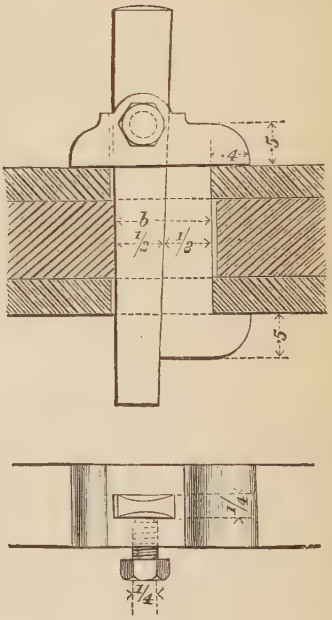


Fig. 134

is very small. The cotter is secured by a screwed end, nut and washer.

Fig. 134 shows another way of securing the cotter which is neat. The cotter passes through the head of the gib and is fixed by a set screw.

CHAPTER VI

JOURNALS, CRANK-PINS, PIVOT AND COLLAR-BEARINGS

Journals

104. Journals are those parts of rotating pieces which are supported by the frame of the machine. They are commonly cylindrical, but sometimes spherical or conical. Some journals run constantly, others support a piece which moves occasionally. In the latter case the strength of the journal is chiefly to be considered; in the former, durability and freedom from liability to heat are as important as strength. Some journals are subjected to straining forces in the plane of their axis only, which produce bending and shearing stress. Others are subjected to bending and torsion, and are calculated by the rules for combined stress. Lastly, some journals are supported at one end only; others, which may be termed neck journals, are supported at both ends.

105. *Form of journals.*—The ordinary form of journals is shown in fig. 135. The bearing part of the journal is turned accurately cylindrical, and is terminated by raised parts or collars, which bear against the ends of the brass steps in which the journal revolves and limit its end play. The length of the brass step is, in some cases, 0·9 of the length of the journal. This permits a limited longitudinal motion, which ensures uniform wear of the step. In other

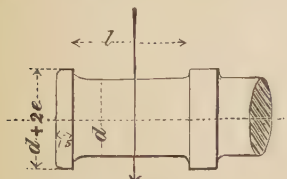


Fig. 135

cases, longitudinal play would interfere with the action of the machine, and it is made as small as possible. The proportional unit for fig. 134 is $e = \frac{d}{8} + \frac{1}{8}$ to $\frac{d}{10} + \frac{1}{8}$.

106. *General considerations on friction.*—It is a primary condition for the satisfactory working of a journal that it should run cool and not seize. In all cases the friction generates heat, and the temperature of the journal rises. But the rise of temperature may reach a limit, the heat being conducted away as fast as it is generated, and then the journal may work satisfactorily. To diminish the amount of friction the journal is supplied with a lubricant which forms a film between the journal and its bearing; and this lubricant not only diminishes the friction and the heat generated, but in favourable conditions it almost prevents wear of the journal or its bearing. The initial effect of a rise of temperature on a lubricated bearing is to diminish the friction. But at the same time the lubricant is rendered more fluid, and if the pressure between the surfaces is great enough the lubricant may be squeezed out, and then the friction suddenly increases and the journal seizes.

The so-called ordinary laws of friction were ascertained by Morin by means of experiments on plane sliding surfaces with very moderate velocities of sliding and very small intensities of pressure. According to these laws the frictional resistance to sliding is directly proportional to the total pressure between the surfaces, and is independent of the area of the surfaces and the velocity of sliding. Hence if R is the resistance to sliding, P the total pressure between the surface,

$$R = \mu P$$

where μ is a constant for given surfaces termed the *coefficient of friction*. This coefficient Morin found to be about 0·25 for wood sliding on wood, dry; about 0·15 for metal on metal, dry; and about 0·07 to 0·08 for metal on metal well lubricated.

In the case of a cylindrical journal resting in a bearing the total load being P, the frictional resistance to rotation, measured at the surface of the journal, would still be simply $R = \mu P$, if Morin's results were applicable, and if the radius of curvature of the bearing were just so much more than that of the journal that there was contact at a line. If the journal is initially exactly fitted to the bearing, or if, by wear, contact is established over a considerable area, the friction will depend on the way in which the pressure is distributed over the surfaces of contact. In that case it can be shown, on what seem reasonable limiting assumptions as to the distribution of the pressure, that the friction would probably lie between the values $\frac{\pi}{2}\mu P$ and $\frac{4}{\pi}\mu P$ or between $1{\cdot}57\,\mu P$ and $1{\cdot}28\,\mu P$. It is more convenient to take for journals $R = \mu P$ simply, treating μ as a special coefficient applicable to journals, which depends both on the frictional resistance to motion of the surfaces and on the undetermined effect of the distribution of the pressure between the journal and step. Supposing the journal is d inches in diameter, and l inches in length, dl is the projected area of the bearing on a diametral plane. Then $p = P/dl$ is called the pressure per unit of *bearing surface*. If a journal makes N revolutions per minute, its surface velocity is $\frac{\pi d N}{12 \times 60}$ feet per second, and the work expended in friction is

$$T = \mu P \times \frac{\pi d N}{12 \times 60}$$
$$= {\cdot}00436\,\mu P d N \text{ ft. lbs. per second.}$$

Practical experience has shown that the pressure on the bearing surface cannot exceed certain limits, differing in different classes of journals, without danger of seizing from the squeezing out of the lubricant. A certain bearing area dl is therefore necessary for a given load P.

107. To apply these results it is now necessary to dis-

cuss the value of the coefficient of friction μ. Under the conditions present in Morin's experiments on sliding friction μ appears to be really a constant, though for very small intensities of pressure, and for pressures much greater than in Morin's experiments its value increases, and for velocities higher than in Morin's experiments its value diminishes. Morin did, indeed, make some special experiments on journal friction, but again with very moderate intensities of pressure. In these it appeared that for dry journals $\mu = 0{\cdot}18$; for intermittently lubricated journals $\mu = {\cdot}07$ to ·08; and with continuously lubricated journals $\mu = {\cdot}03$ to ·05. Later experiments have shown that in conditions different from those in Morin's experiments, and in conditions more similar to ordinary practice, the coefficient μ is very much smaller than the values given above. Nor is this all. Not only is the friction numerically less than in Morin's experiments, but it follows quite different laws. In a remarkable paper of Hirn's in 1855, it was shown that for lubricated journals in ordinary normal conditions of running, the coefficient of friction or factor in the relation $R = \mu P$, so far from being constant, was more nearly proportional to the square root of the velocity of rubbing, and inversely proportional to the square root of the intensity of pressure, that is, $\mu = c\sqrt{\frac{v}{p}}$. In Beauchamp Tower's experiments, to which reference will be made presently, with still better lubrication it was found that $\mu = 0{\cdot}20\sqrt{\frac{v}{p}}$ where v is in feet per second, and p in pounds per sq. in. In either case it is obvious that the friction follows laws widely different from those which have been commonly assumed. Everything in designing journals depends on understanding the conditions which make the friction small. As these cannot be stated at present as definite rules some account of the latest experiments will be useful.

108. *Beauchamp Tower's experiments.* — These experi-

ments were carried out by the direction of the Institution of Mechanical Engineers, and they are the only experiments on a large journal with the full amount of pressure and velocity of rubbing usual in railway journals. Their accuracy is also entirely to be trusted. The journal was a steel horizontal journal 3 ins. × 6 ins. with a gun-metal step embracing the top semicircumference. In these conditions it was found impossible to get any regular results, till an oil bath, giving a continuous supply of lubricant, was fixed below the journal. It is the results thus obtained which will be first discussed. Under the conditions stated, it was found that the friction was extremely small; the largest values obtained being only from one-eighth to one-thirtieth of Morin's lowest values. In the next place the friction for a wide range of loads was practically independent of the load, so that the co-efficient of friction was inversely as the load. Further experiments by Mr. Tower, and an investigation by Professor Osborne Reynolds, proved that in the oil-bath experiments the condition of the journal and step was that shown in an exaggerated way in fig. 136. The step, either by wear or expansion, was of slightly greater radius than the journal, and the oil was carried in on the 'on' side between the journal and step, forming a complete and continuous film, and carried out again at the 'off' side. A pressure is generated in the fluid film which lifts and supports the whole load on the step, and the film is thinnest and the pressure greatest at a point on the 'off' side of the centre of the step.

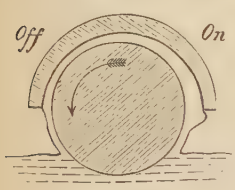


Fig. 136

Bearing these conditions in mind, the results of the experiments will be intelligible. Thus with olive oil, and with the loads on the 4 ins. × 6 ins. journal given below, the frictional resistance at the surface of the journal at 200 revs. per min. ($v=3{\cdot}49$) was found to be as follows :—

For convenience, values of μ in the relation $F = \mu P$ and of c in the relation $\mu = c\frac{\sqrt{v}}{p}$ have been calculated.

Friction of Journal with steady Load and Oil-bath Lubrication

Total load P on journal in lbs. . .	12,500	11,200	10,000	8,800	7,450	6,200	4,900	3.680	2,400
Pressure per sq. in. of journal p .	520	468	415	363	310	258	205	153	100
Friction at surface of journal in lbs. F .	12·5	14·6	13·9	13·9	12·6	12·3	12·3	12·8	13·2
μ . .	·0010	·0013	·0014	·0016	·0017	·0020	·0025	·0035	·0055
c . .	·278	·326	·311	·311	·282	·276	·274	·287	·294

Here, while μ varies greatly, c varies little and irregularly from its mean value 0·293. So that for cases like this the relation $\mu = c\frac{\sqrt{v}}{p}$ appears to hold, at least as far as variation of intensity of pressure is concerned. It remains to consider how the friction in such cases varies with the velocity and with different lubricants.

109. *Variation of friction with velocity of rubbing.*—The following are *mean values* of the friction on the 4″ × 6″ journal without bath lubrication and for pressures of from 100 lbs. per sq. in. of projected area of bearing up to the limiting pressures mentioned at the head of the columns. Beyond these limiting pressures the friction suddenly increased and soon the bearing seized. The pressures at which the bearing seized are also noted. Within the range of pressures from 100 lbs. to the limiting pressures, the friction may, as in the case discussed above, be taken to be independent of the load. Consequently we can use the mean values of the friction with different loads in examining the effect of the variation of speed and of different lubri-

cants. For this comparison the value of c in the relation $\mu = c\frac{\sqrt{v}}{p}$ is deduced for each of the experimental results.

Lubricant		Olive Oil	Lard Oil	Mineral Grease	Sperm Oil	Rape Oil	Mineral Oil
Limiting pressures between which mean values of friction are taken in lbs. per sq. in. of bearing surface	Lowest	100	100	100	100	100	100
	Highest	520	520	625	310	415	415
Pressures at which bearing seized in lbs. per sq. in. . . .		—	—	—	520	573	625
Friction of 4″ × 6″ bearing in lbs.							
At velocities in ft. per sec.							
1·75 . . .		8·64	8·40	12·72	6·00	6·24	8·40
3·49 . . .		13·20	12·24	19·44	7·44	9·36	12·24
5·23 . . .		16·08	15·84	23·76	11·52	12·00	15·60
6·99 . . .		18·72	18·96	28·80	13·20	14·16	18·00
Deduced values of c at velocities in ft. per sec.							
1·75 . . .		·273	·266	·402	·190	·197	·265
3·49 . . .		·295	·273	·434	·166	·209	·273
5·23 . . .		·293	·288	·433	·210	·219	·284
6·99 . . .		·296	·299	·455	·209	·224	·284
Mean values of c for each lubricant = .		·289	·281	·431	·194	·212	·276

With increase of temperature the friction diminished.

Here the values of c for each lubricant are quite satisfactorily constant, so that within the limits of pressure and velocity of Mr. Tower's experiments and with oil-bath lubrication the mean values of c may be used with confidence.

It appears, then, that with perfect oil-bath lubrication the friction is

$$R = \mu P = c \frac{\sqrt{v}}{p} P = c d l \sqrt{v}.$$

Where d and l are the diameter and length of the journal, P the total load, v the velocity of rubbing, and c is a constant for a given lubricant, the friction is therefore completely independent of the load on the journal, so long as the intensity of pressure p per sq. in. of projected area does not exceed a limit of about 300 to 600 lbs. per sq. in. according to the viscidity of the lubricant.

110. With the load on the top of the journal it was found impossible to lubricate the journal by a siphon feed at the top. The continuous film of oil between the step and journal was under a pressure which prevented the entrance of fresh oil. If in practice journals loaded on the upper circumference are lubricated by a siphon feed at the top, it must be partly due to the lubrication being so imperfect that a continuous film is not formed, and partly to vibration permitting an intermittent inflow of oil. By leading holes from the siphon feed at the top, through the step to points near its edges, imperfect lubrication was secured, and the bearing ran up to a load of 380 lbs. per sq. in. and then seized. With an oily pad under the journal, somewhat better results were obtained, and the bearing ran to a pressure of 582 lbs. per sq. in. before seizing.

With siphon lubrication the results were very irregular. The coefficient of friction was nearly inversely as the load per sq. in. of bearing surface and almost independent of the velocity. Hence for this case the frictional resistance

$$R = \mu P = \frac{c'}{p} P = c' d l$$

where the mean value of c' is 2·02 for rape oil With pad lubrication the coefficient of friction was nearly independent

of pressure on bearing surface and of velocity, so that in this case

$$R = \mu P = \mu p d l,$$

where $\mu = 0{\cdot}01$ for rape oil.

Hence, to resume Mr. Tower's experiments. With perfect or oil-bath lubrication $\mu = c \frac{\sqrt{v}}{p}$ where c ranges from $0{\cdot}21$ to $0{\cdot}43$ with different lubricants. With siphon lubrication $\mu = \frac{c'}{p}$, where $c' = 2{\cdot}02$ for rape oil. Lastly, with pad lubrication $\mu = 0{\cdot}01$ for rape oil. p is the pressure in lbs. per sq. in. of the projected area of the actual bearing surface.

As to the limiting pressures which could be carried without seizing, no doubt Mr. Tower's experiments seem to be contradicted by some cases in practice. There are cases in which pressures even of a ton or more per sq. in. are carried by lubricated journals without danger of seizing. In most of the cases where excessive pressures are carried, it appears that the pressure is intermittent or reverses in direction so as to come alternately on the two brasses. In such cases it is conceivable that the oil finds a way in during the relief of pressure, and is not squeezed out in the short period in which the pressure acts.

111. *Mr. Tower's experiments on collar friction.*—In a third report on experiments also carried out under the auspices of the Institution of Mechanical Engineers, results are given for a collar bearing, which are extremely interesting as showing the wide difference in friction when lubrication is less perfect than in a journal bearing. A ring of mild steel 12 ins. inside and 14 ins. outside diameter was placed between two gun-metal surfaces so as to form an annular bearing like the collar of a thrust bearing. The gun-metal surfaces rotated, while the steel ring was held fast by a lever which measured the friction. To secure lubrication four pipes led the mineral oil to holes in its circumfer-

ence which opened into grooves on the rubbing faces. The lubrication was adjusted to the minimum at which the bearing would run cool, varying from 60 to 120 drops per minute. The load on the bearing was put on by a powerful and carefully tested spring. With all except the lowest loads it was found impossible to keep the bearing cool without a little water running over it. Under these circumstances the friction was found to be almost absolutely independent of the speed, which ranged from 50 to 130 revs. per minute. The greatest pressure which could be carried on the bearing surface was far less than with a cylindrical journal, because of the greater speed of rubbing, and because a continuous oil film cannot be maintained between the surfaces. At the highest speeds the greatest pressure which could be carried was 75 lbs. per sq. in. and at the lowest speeds 90 lbs. per sq. in. It was stated in the discussion that Mr. Thorneycroft limits the pressure on the collar bearings of propeller shafts to 50 lbs. per sq. in. Mr. Ford Smith also stated that the pressure which could be carried depended partly on the hardness of the surfaces. Hardened steel working on dense cast iron would carry pressure with which other bearing surfaces failed.

Friction of a Collar Bearing

Under a pressure in lbs. per sq. in. of	The coefficient of friction μ was for speeds in revolutions per minute				
	50	70	90	110	130
15	0·045	·065	·043	·054	·064
30	·037	·048	·050	·049	·048
45	·036	·040	·036	·036	·037
60	·029	·038	·036	·037	·041
67	·035	·033	·035	·036	·038
75	·035	·034	·035	·035	·036
82	·034	·032	·035	—	—
90	·031	·044	—	—	—

Here μ is practically independent both of speed and pressure, except with the smallest pressures. For all but the smallest pressures its value is 0·036, and for the smallest pressures about one-third greater.

112. *Theory of journal proportions. Limit of bearing pressure.*—Since the friction of a journal produces heat, and the heating of the lubricant renders it less viscid and more liable to be squeezed out, it would seem that the journal proportions must in some way depend on the amount of heat produced per sq. in. of surface of journal. Further, it would seem rational at first sight to use the values of μ given by Mr. Tower's experiments to determine the amount of heat produced. No doubt those values should be used in all calculations on the friction of journals in normal conditions of running. But no relation can be traced between the proportions of journals adopted in practice in cases differing widely in size, intensity of pressure and speed, and the friction per sq. in. calculated by Mr. Tower's coefficients.

On the other hand, there does appear to be some relation, though not an exact one, if the friction is calculated on the old assumption that μ = constant. And perhaps this ought to have been expected. For the proportions of journals must mainly depend on the necessity of preventing a tendency to heat and seize. But such a tendency will arise just in those conditions in which lubrication has become for the moment less perfect than in normal conditions of running. Further, from the investigation above it appears that just as the lubrication becomes imperfect the assumption μ = constant becomes approximately true.

The work T expended in friction produces a quantity of heat,

$$H = \frac{T}{J} = \mu P \frac{\pi d N}{12 J} \quad . \quad . \quad . \quad . \quad (3)$$

where J is Joule's equivalent, or 772 ft. lbs. That heat should be dissipated as fast as it is generated, by conduction from the surface of the journal, and it appears to be a

reasonable assumption that the rate of dissipation is proportional to the area of the surface through which the heat is conducted, or to the product $d\,l$. Let h units of heat be dissipated per minute, for each unit of bearing surface, $d\,l$. Then

$$h\,d\,l = \frac{\mu\,\mathrm{P}\,\pi\,d\,\mathrm{N}}{12\,\mathrm{J}}$$

$$l = \frac{\mu\,\pi}{12\,\mathrm{J}\,h} \times \mathrm{P\,N} = \frac{\mathrm{P\,N}}{\beta} \quad . \quad . \quad . \quad (4)$$

where β is a constant, to be ascertained by experience in different cases.

This result implies that the limiting pressure per sq. in. of bearing surface is independent of the length of the journal for a given lubricant and given conditions of running. Putting $p = \mathrm{P}/d\,l$ we get

$$p = \frac{\beta}{\mathrm{N}\,d} \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

This implies that the limiting intensity of pressure varies inversely as the velocity of rubbing, if other conditions are the same.

This formula may be put in a more convenient form for crank pins. Let H.P. be the indicated horses' power transmitted to the crank pin, R the crank radius in inches; then the mean pressure on the crank pin is

$$\mathrm{P} = \frac{33000 \times \mathrm{H.P.} \times 12}{4\,\mathrm{R\,N}} \text{ nearly.}$$

Inserting this value in (4)

$$l = \gamma \frac{\mathrm{H.P.}}{\mathrm{R}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $\gamma = \frac{99000}{\beta}$ . . . . . (6)

113. *Comparison of the results of this theory with practical experience.*—Mr. Tower's experiments furnish some data

as to the limiting value of the constant β in the expressions just found. Taking the highest speeds and pressures at which the journal in those experiments ran without seizing when the lubricant was supplied by an oil bath, we get the following values of β. Let p be the greatest intensity of pressure on the bearing surface, N the number of revolutions per minute, d the diameter of the journal. Then $\beta = p\,\mathrm{N}\,d$.

	Greatest pressure at which the journal ran without seizing	$\beta =$	$\gamma =$
	lbs.		
Sperm oil	415	747,000	0·133
Mineral oil	625	875,000	0·114
Rape oil	573	916,000	0·109
Olive and lard oil . . .	520	936,000	0·106
Mineral grease . . .	625	1,125,000	0·088

When the lubrication was less perfect the values of β diminished thus:—

	Greatest pressure at which the journal ran without seizing	$\beta =$	$\gamma =$
	lbs.		
Rape oil fed by siphon . .	258	309,600	0·321
,, ,, with pad . . .	328	393,000	0·252

114. These are values of β for which the journal was just on the point of seizing. They must be divided by a 'factor of safety' to find values which allow a margin of safety against heating. We may expect to find therefore that the values of β deduced from actual working journals are lower than the values above by a varying amount depending on the varying margin of security allowed. It will be seen presently that this appears to be the case for journals loaded

like Mr. Tower's journals, by a load constant in direction. It appears, however, that in journals on which the direction of the load constantly changes the lubrication is more easily effected and the tendency to heating is less. For such cases it appears that journals are working for which the values of β are quite as high as the highest given above.

Proportions of Journals

115. *Strength of end journals.*—Let P be the load on a journal, assumed to be uniformly distributed; let d be the diameter, and l the length of the journal; f the working stress suitable for the material of the journal. Then the bending moment at the fixed end of the journal is $\frac{1}{2} P l$ and equating this to the moment of resistance of a circular section, §28, p. 52,

$$\tfrac{1}{2} P l = \frac{\pi}{32} f d^3 \quad . \quad . \quad . \quad (7)$$

$$P l = 0{\cdot}196\, f d^3.$$

For wrought iron f may be taken at 6,000 to 9,000 lbs. per sq. in.; the stress on each fibre being alternately a tension and a pressure (§25). For steel f = 9,000 to 13,500 lbs. per sq. in. For cast iron f = 3,000 to 4,500 lbs. per sq. in.

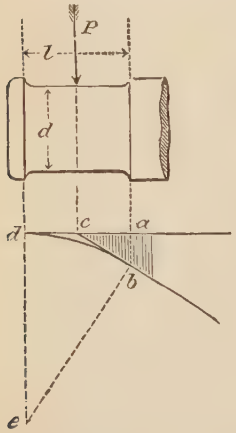


Fig. 127

116. The bending action may be shown graphically thus: Suppose first of all that the load on a journal (fig. 137) is P acting at the centre of its length. Then the bending moment diagram is the triangle cab, in which $ab = \frac{1}{2} P l$ is the bending moment at the root of the journal. If the load is uniformly distributed the bending moment curve is a parabola db, which may be approximated to by drawing be perpendicular to bc and describing

a circular arc $d\,b$ with centre e. The bending moment at the root of the journal is the same as before, and on this the diameter necessary for strength depends.

117. *Case I. Ratio of length to diameter of journal given or assumed.*—Most commonly the length l of the journal will not be given, and there are therefore two unknown quantities in equation (7), l and d. In a number of cases it is possible to fix beforehand the ratio l/d of length to diameter of bearing. Thus in the case of journals which work intermittently, or on which the pressure only acts for part of a rotation, $l/d = 1$; for very many journals in which the speed of rotation does not exceed about 150 revolutions per minute it has been customary to make $l/d = 1{\cdot}5$ if the step is of gun-metal, and $l/d = 1{\cdot}75$ if the step is cast iron. For journals at greater speeds of rotation it has been usual to make the proportion of length to diameter greater, and it will be seen presently that this is equivalent to reducing the pressure per unit of bearing surface as the speed increases.

Thus, for journals for which a suitable value of l/d has to be chosen, let,

Case a, $\frac{l}{d} = \frac{1}{8}\sqrt{\text{N}}$ if the direction of the load is constant.

Case b, $\frac{l}{d} = \frac{1}{11}\sqrt{\text{N}}$ if the load is in opposite directions alternately. Where N is the number of rotations per minute.

N =	100	150	200	250	400	750	1,000
Case *a*. l/d =	1·25	1·5	1·75	2·0	2·5	3·5	4·0
Case *b*. l/d =	0·9	1·12	1·25	1·5	1·8	2·5	3·0

118. When l/d is fixed, equation (7) above becomes

$$d = \sqrt{\frac{5{\cdot}1}{f}} \sqrt{\left(p\frac{l}{d}\right)} \quad . \quad . \quad (8)$$

where

$$\sqrt{\frac{5{\cdot}1}{f}} = 0{\cdot}0238 \text{ to } 0{\cdot}0291 \text{ for wrought iron.}$$
$$= 0{\cdot}0238 \text{ to } 0{\cdot}0194 \text{ for steel.}$$
$$= 0{\cdot}0413 \text{ to } 0{\cdot}0337 \text{ for cast iron.}$$

The following Table has been calculated from the rule suitable for wrought iron :—

$$d = 0{\cdot}0291 \sqrt{\left(p\frac{l}{d}\right)}$$

Diameter of Wrought-iron Crank-pin Journals in Inches

Load on Journal in lbs.	Ratio of Length to diameter						
	1·0	1·25	1·5	1·75	2	3	4
1,000	·84	·94	1·03	1·11	1·19	1·45	1·68
1,500	1·03	1·15	1·26	1·36	1·45	1·78	2·05
2,000	1·19	1·33	1·45	1·57	1·68	2·05	2·37
3,000	1·45	1·62	1·78	1·92	2·05	2·52	2·90
4,000	1·68	1·88	2·05	2·22	2·37	2·90	3·35
5,000	1·87	2·10	2·30	2·48	2·65	3·25	3·75
10,000	2·65	2·96	3·25	3·50	3·75	4·59	5·30
15,000	3·25	3·63	3·98	4·29	4·59	5·62	6·49
20,000	3·75	4·19	4·59	4·96	5·30	6·49	7·50
30,000	4·59	5·13	5·62	6·07	6·49	7·95	9·18
40,000	5·30	5·93	6·49	7·01	7·50	9·18	10·60
50,000	5·93	6·63	7·26	7·84	8·38	10·27	11·85

In this Table the safe stress f is taken at 7,250 lbs. per sq. in.

119. *Case II. The limiting intensity of pressure on bearing given.*—It has been shown above that if certain limits of pressure p per sq. in. of the projected area of the bearing are exceeded, the lubricant is driven out and

the bearing seizes. The more perfect the lubrication the higher the limiting pressure may be. On the other hand, the greater the velocity of rubbing of the journal on its step the lower the pressure which can be safely carried. Generally it appears that for bearings on which the load acts in a fixed direction, the safe limit of pressure is only half as great as in journals like crank-pins, on which, in each revolution, the direction of the load is alternately in opposite directions. While, on cross-head journals in which the direction of the load reverses and the direction of motion also reverses, still greater pressures are permissible. If p is the limiting intensity of pressure on the bearing,

$$p\,d\,l = \text{P},$$

and introducing the value of l thus given in the equation above, we get

$$d = \sqrt[4]{\frac{5{\cdot}1}{p\,f}}\sqrt{\text{P}} \quad . \quad . \quad (9)$$

and thence

$$l = \frac{\text{P}}{p\,d} \quad . \quad . \quad . \quad (9a)$$

The following are values of $\sqrt[4]{\frac{5{\cdot}1}{p\,f}}$ in different cases :—

For p =	200	250	300	400	500	750	1,000
Cast iron f = 3,000 .	·0540	·0511	·0488	·0454	·0429	·0388	·0361
Wrought iron f = 9,000 .	·0410	·0388	·0371	·0345	·0326	·0295	·0274
Steel f = 13,500 .	·0371	·0350	·0335	·0312	·0295	·0266	·0248

120. The following Table gives the diameters and lengths of wrought-iron journals, determined by these rules :—

Load on journal in lbs. P	p=200		250		300		400		750		1,000	
	d	l	d	l	d	l	d	l	d	l	d	l
1,000	1·29	3·88	1·23	3·26	1·17	2·85	1·09	2·30	·93	1·44	·87	1·15
2,000	1·83	5·47	1·74	4·60	1·66	4·01	1·54	3·25	1·32	2·02	1·23	1·63
3,000	2·25	6·67	2·13	5·63	2·03	4·93	1·89	3·97	1·62	2·47	1·50	2·00
4,000	2·59	7·73	2·45	6·53	2·35	5·68	2·18	4·59	1·87	2·86	1·73	2·32
5,000	2·90	8·62	2·74	7·30	2·62	6·36	2·44	5·12	2·09	3·19	1·94	2·58
10,000	4·10	12·2	3·88	10·3	3·71	8·98	3·45	7·24	2·95	4·52	2·74	3·65
20,000	5·80	17·3	5·49	14·6	5·25	12·7	4·88	10·2	4·17	6·40	3·87	5·17
30,000	7·10	21·2	6·72	17·9	6·42	15·6	5·98	12·6	5·11	7·83	4·75	6·32
40,000	8·20	24·4	7·76	20·6	7·42	18·0	6·90	14·5	5·90	9·04	5·48	7·30
50,000	9·17	27·3	8·68	23·0	8·29	20·1	7·71	16·3	6·60	10·1	6·13	8·16

121. *Limits of pressure on bearings in different practical cases.*—The following values of the pressure per sq. in. of projected area of bearing may be of use, partly in showing how great the variation of bearing pressure is in different cases, and partly as a guide in selecting a suitable value when designing a bearing :—

Pressure on Bearings and Slides

Pressure calculated in lbs. per sq. in. of bearing surface.	Intensity of pressure lbs. per sq. in.
Bearings on which the load is intermittent and the speed slow, such as crank pins of shearing machines . .	3,000
Cross head neck journals	1,200
Crank pins of large slow engines	800 to 900
Crank pins of marine engines usually	400 to 500
Main crank shaft bearings. Marine engines (slow). .	600
,, ,, ,, (fast) . .	400
Railway journals	300
Fly wheel shaft journals	150 to 250
Small engine crank pins	150 to 200
Slipper slide blocks, marine engines	100
Stationary engine slide blocks	25 to 125
,, ,, usually	30 to 60
Propeller thrust bearings	50 to 70
Lying shafts in cast-iron steps [1] (Sellers) . . .	15

[1] Quoted from Reuleaux.

122. *Case III. Journals for which both length and diameter are determined by rule.*—The reasoning in § 112 furnishes a very simple rule, by means of which both the diameter and length of a journal can be determined on principles which are reasonable. For the result there obtained fixes the pressure per unit of bearing surface at the following value :—

$$p = \frac{\text{Constant}}{v} = \frac{\beta}{d\,\text{N}}$$

where d is the diameter of the journal in inches and N the revolutions per minute. Without relying entirely on the reasoning on which this rule is based, it appears to agree with a wider range of cases than any other rule the author has tried. β is not a constant for all classes of journals, but it appears to have higher values exactly in those cases where greater perfection of workmanship and greater care in lubrication would lead one to expect greater values of the bearing pressure to be permissible.

The following are some values of β, from actual cases :—

	$\beta =$
Locomotive crank pins calculated from full pressure and speed	1,000,000 to 1,500,000
Marine engine crank pins	200,000 to 300,000
Stationary engine crank pins	60,000 to 200,000
Small portable engine crank pins	60,000
Railway carriage axles, calculated from full load and speed	300,000 to 400,000
Crank shaft bearings	30,000

Here the higher values correspond to cases in which the direction of the load reverses at each revolution of the lower values, to cases in which the direction of the load is constant.

123. *Calculated bearing pressures.*—The following Table gives some idea of the pressures permitted under this rule, excluding cases in which obviously practical considerations would lead to the adoption of other proportions :—

	Pressure per sq. in. of bearing when diameter of journal in ins.			
	3	6	12	24
Crank shaft bearings, $\beta = 30{,}000$				
At 25 revs.	400	200	100	50
,, 50 ,,	200	100	50	—
,, 100 ,,	100	50	—	—
Small crank pins, $\beta = 60{,}000$				
At 50 revs.	400	200	100	—
,, 100 ,,	200	100	50	—
,, 250 ,,	80	40	—	—
Large crank pins, $\beta = 120{,}000$				
At 50 revs.	800	400	200	100
,, 100 ,,	400	200	100	50
,, 250 ,,	160	80		
Large crank pins, $\beta = 240{,}000$				
At 25 revs.	—	—	800	400
,, 50 ,,		800	400	200
,, 100 ,,	800	400	200	100
Locomotive crank pins and crosshead pins, $\beta = 1{,}000{,}000$				
At 50 revs.	—	—	1,666	833
,, 100 ,,	—	1,666	833	416
,, 250 ,,	1,333	666	333	166
,, 1,000 ,,	333	166	83	—

124. *Rules giving length and diameter of journal.*—Combining the rules

$$p = \frac{P}{d\,l} = \frac{\beta}{N\,d}$$

and $P\,l = \dfrac{f\,d^3}{5{\cdot}1}$

we get

$$l = \frac{P\,N}{\beta} \qquad . \qquad . \qquad . \quad (10)$$

$$d = \sqrt[3]{\frac{5{\cdot}1}{\beta f}} \quad \sqrt[3]{P^2N} \qquad . \qquad . \quad (11)$$

Table of Values of $\sqrt[3]{\frac{5{\cdot}1}{\beta f}}$

Kind of journal	Crank shaft bearings	Small crank pins	Large crank pins	Large crank pins	Locomotive crank pins
Value of β	30,000	60,000	100,000	200,000	1,000,000
For cast iron $f = 3{,}000$.	·00384	·00305	·00257	·00204	·00119
For wrought iron $f = 9{,}000$.	·00266	·00211	·00178	·00142	·00083
For steel $f = 13{,}500$.	·00232	·00185	·00156	·00124	·00072

125. *Neck journals.*—The crosshead pins of engines are supported at each end and loaded uniformly. The bending moment is therefore $\frac{1}{8}$ P l. Hence the general equation which secures sufficient strength is

$$d = \sqrt[3]{\frac{1{\cdot}28}{f}} \quad \sqrt[3]{\text{P}\,l} \quad . \quad (12)$$

As far as strength is concerned, therefore, a neck journal may be 0·63 of the diameter of an end journal for the same load and of the same length.

Case Ia. Neck journal, l/d given or assumed.

$$d = \sqrt{\frac{1{\cdot}28}{f}} \quad \sqrt{\left(\text{P}\,\frac{l}{d}\right)} \quad . \quad (13)$$

Case IIa. Neck journal, bearing pressure p, given or assumed.

$$d = \sqrt[4]{\frac{1{\cdot}28}{pf}} \quad \sqrt{\text{P}} \quad . \quad . \quad (14)$$

$$l = \frac{\text{P}}{p\,d}. \quad . \quad . \quad . \quad (14a)$$

Case IIIa. Neck journal. Length and diameter determined by rule.

$$l = \frac{P\,N}{\beta} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$d = \sqrt[3]{\frac{1{\cdot}28}{\beta f}}\ \sqrt[3]{P^2 N} \quad . \quad (15a)$$

The values of f, p, and β are the same as for end journals.

126. *Case IV. Journals subjected to a transverse load and a twisting force.*—Crank-shaft journals. Let fig. 138

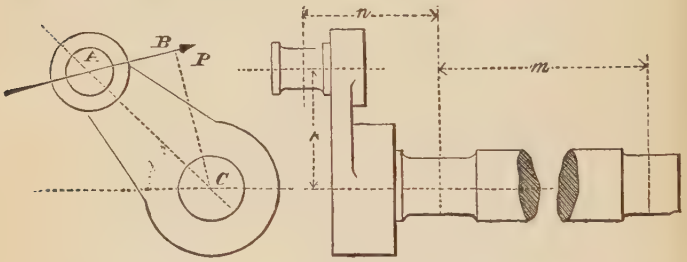


Fig. 138

represent a side and end elevation of a crank and crank-shaft, and let it be required to determine the dimensions of the shaft journal nearest the crank. Let P, acting in the direction A B, be the pressure transmitted to the crank-pin by the connecting rod, and let Q and R be the reactions due to P at the shaft journals. These forces may be taken to act at the centres of the journals.

$$P\,n = Q\,m \text{ and } P + Q = R.$$

Hence,

$$Q = P\,\frac{n}{m}; \quad R = P\left(1 + \frac{n}{m}\right) \quad . \quad . \quad . \quad . \quad (16)$$

R is the reaction at the journal nearest the crank.

The force P produces a bending moment $P\,n$, at the centre of the shaft journal, causing bending in a plane parallel to

its direction. At the same time the journal is subjected to a twisting moment P × C B = P $r \cos\theta$, where θ is the angle A C B, and r the crank radius. At the dead point, where the direction of P passes through C, $r \cos\theta = 0$, and the twisting moment vanishes. If P is constant, the twisting moment is greatest when the connecting rod is at right angles to the crank, and is then equal to P r.

By the rules for combining bending and twisting action, § 44, p. 90, the stress due to the combined moments is the same as that which would be produced by a simple bending moment,—

$$M = \tfrac{1}{2} P n + \tfrac{1}{2} \sqrt{\{(P n)^2 + (P r)^2\}}$$

$$= \frac{P}{2}\left\{ n + \sqrt{(n^2 + r^2)} \right\} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$= \frac{P}{2} (1{\cdot}84\, n + 0{\cdot}84\, r) \text{ nearly} \quad . \quad . \quad . \quad . \quad (18)$$

Equating this to the moment of resistance of a circular section,

$$M = \frac{f d^3}{10{\cdot}2}$$

$$d = \sqrt[3]{\frac{5{\cdot}1}{f}} \sqrt[3]{\left\{ P\left(n + \sqrt{n^2 + r^2}\right) \right\}} \quad . \quad . \quad (19)$$

$$\sqrt[3]{\frac{5{\cdot}1}{f}} \sqrt[3]{\left\{ P\,(1{\cdot}84\, n + {\cdot}84\, r) \right\}} \text{ nearly} \quad . \quad (20)$$

where $\sqrt[3]{\frac{5{\cdot}1}{f}}$ = 0·0876 to 0·0827 for wrought iron
= 0·0827 to 0·0722 for steel
= 0·1195 to 0·1043 for cast iron.

These equations exaggerate a little the straining action, because they neglect the distribution of the load on the surface of the journal. The length of the journal is to be calculated for the pressure R. Usually for crank-shafts $l = 1\frac{1}{2}$ to $1\frac{3}{4} d$, which allows a large margin for durability.

127. *Crank shaft journals when the load is uniformly distributed.*—The exact distribution of the load R on the crank shaft journal is not really known. Two considerations show that it must be distributed in a tolerably uniform

manner. If it were not, the intensity of pressure would squeeze out the lubricant and the journal would heat, and wear most rapidly at that part where the pressure was greatest. As the whole flexure of the journal due to the load is small, it is reasonable to suppose that the brasses soon wear to a form which is consistent with a uniform distribution of the pressure R over the whole length of the journal. If this is assumed, then the bending moment due to R distributed over the journal, if l is the length of the journal, is

$$R\frac{l}{8} = \frac{P\,l}{8}\left(1 + \frac{n}{m}\right) \quad . \quad . \quad . \quad (21)$$

and the true bending moment at the centre of the journal is

$$P\left\{n - \frac{l}{8}\left(1 + \frac{n}{m}\right)\right\} \quad . \quad . \quad (22)$$

which should be substituted for $P\,n$ in the equations above.

128. *Case V. Two forces at right angles acting at the crank pin.*—The following case sometimes occurs. Two forces, P_1, P_2, act at the crank pin at right angles, but at different distances from the section at which the bending moment is to be estimated. If two forces not at right angles are given, they can be resolved into two forces at right angles. Further, it is convenient in such cases to so resolve the forces that one is in the direction of the crank arm and the other at right angles to it.

Let P_1 be a force acting on the crank pin at n_1 from the centre of the shaft bearing, as in the figure above, and P_2 a force acting at a distance n_2. Further, let P_1 be at right angles to the crank arm and P_2 parallel to it. Let r be the radius of the crank and m the distance between the shaft journals.

The bending moments on the crank shaft due to P_1 and P_2 are, if the distribution of the load on the journal is neglected, $M_1 = P_1 n_1$, and $M_2 = P_2 n_2$, and the resultant bending moment is

$$M = \sqrt{M_1^2 + M_2^2} = \sqrt{P_1^2 n_1^2 + P_2^2 n_2^2}.$$

If the distribution of the load along the length l of the shaft journal is allowed for,

$$M_1 = P_1 n_1 - \frac{P_1 l}{8}\left(1 + \frac{n_1}{m}\right)$$
$$M_2 = P_2 n_2 - \frac{P_2 l}{8}\left(1 + \frac{n_2}{m}\right). \quad . \quad (23)$$

and the resultant bending moment is

$$M = \sqrt{(M_1^2 + M_2^2)},$$

as before.

129. If P_1 is at right angles to the plane through the crank arm and crank shaft, the twisting moment is

$$T = P_1 r.$$

The bending moment equivalent to the combined bending and twisting moments is

$$M_e = \tfrac{1}{2}M + \tfrac{1}{2}\sqrt{(M^2 + T^2)}$$
$$= \tfrac{1}{2}\sqrt{(M_1^2 + M_2^2)} + \tfrac{1}{2}\sqrt{(M_1^2 + M_2^2 + T^2)}. \quad (24)$$

From this the diameter of the shaft can be found by the equation

$$d = \sqrt[3]{\frac{10{\cdot}2}{f}}\sqrt[3]{M_e} \quad . \quad . \quad (25)$$

Pivot and Collar Bearings

130. At first sight the limits of pressure met with in practice in the case of pivot and collar bearings seem widely discrepant. On the pivots of the upright shafts of mills the pressure sometimes reaches half a ton per sq. in. On the collars of screw propeller shafts it seldom exceeds 50 to 70 lbs. per sq. in. The discrepancies largely disappear if the limit of bearing pressure is taken to vary inversely as the velocity of rubbing. This leads to a rule identical in form with that already applied in the case of journals. If for

pivot and collar bearings we assume for the greatest working bearing pressure

$$p = \frac{\beta}{N d} \quad . \quad . \quad . \quad . \quad . \quad (26)$$

precisely as for journals, then the values of β found by examining actual cases do not differ from those already deduced for journals, especially if it is remembered that a flat pivot or collar tends to exclude the lubricant by its centrifugal action. For the collar bearing with which Mr. Tower experimented, having a mean diameter of $13\frac{1}{2}$ inches, seizing occurred with values of β ranging from 60,000 to 120,000. Some actual pivots are working with $\beta = 40{,}000$.

To take a very extreme case, Mr. Adamson has described a pivot of an upright shaft, 5 inches in diameter, running at 150 to 200 revs. per min., and carrying a load of a ton per sq. in. In this case oil was supplied to the centre of the pivot by a force pump. This pivot gives $\beta = 2{,}000{,}000$, a value perhaps not surprising when the very exceptionally good lubrication is taken into account.

131. If P is the total axial load carried by the pivot of diameter d, running at N revs. per minute,

Fig. 139

$$\frac{\pi}{4} d^2 p = P \quad . \quad . \quad . \quad . \quad (27)$$

or if the bearing is a collar bearing with n collars, the inside and outside diameters of which are d_2 and d_1,

$$\frac{\pi}{4} (d_1^2 - d_2^2)\, n p = P \quad . \quad . \quad . \quad (28)$$

Introducing in these equations the value of p in equation (26)

$$d = \frac{4}{\pi \beta} P N \qquad \text{for a pivot} \quad . \quad . \quad (29)$$

$$d_1 - d_2 = \frac{2}{\pi \beta} \frac{P N}{n} \qquad \text{for a collar bearing} \quad (29\,a)$$

It appears that β should not be taken at more than 40,000 unless the lubrication is exceptionally good.

132. For cases where the speed is not great we may take as the maximum values of p in lbs. per sq. in. the following values given by Reuleaux:—

	$p =$
I. Wrought-iron pivot on gun-metal . . .	700
II. Cast-iron pivot on gun-metal	470
III. Wrought-iron pivot on lignum vitæ . . .	1,400

For these values of p, and a given value of P, the load on the pivot, we have

$$d = 1{\cdot}13\sqrt{\frac{P}{p}} \quad . \quad . \quad . \quad . \quad . \quad (29b)$$

or

$$d_1 = \sqrt{\left\{d_2^2 + 1{\cdot}27\frac{P}{p\,n}\right\}} \quad . \quad . \quad . \quad (29c)$$

Equations 29*b* or 29*c* are to be used in place of 29*a* or 29*b* when they give greater values of d or d_1. For $\beta = 40{,}000$, this will happen if Nd is less than 60 for Case I.; 85 for Case II.; or 30 for Case III., in the short table above.

CHAPTER VII

AXLES, SHAFTING, AND COUPLINGS

Axles and Shafts

133. The terms *axle* and *shaft* are applied rather indiscriminately to parts of machines which support rotating pieces, or which by their rotation convey and distribute motive power. They are usually cylindrical, but occasionally square or cross-shaped in section. They may be classified as follows :—

(1.) Axles loaded transversely, and subjected chiefly to bending action.

(2.) Transmissive shafting, subjected chiefly to torsion.

(3.) Crankshafts and other shafts subjected to combined torsion and bending.

General equations for the strength of axles and shafts.— If M is the bending moment, d the diameter of an axle, and f the working stress,

$$M = \frac{1}{10{\cdot}2} f d^3 = 0{\cdot}098 f d^3,$$

which is the form taken by equation 5 (p. 53) for a bar of circular section. If a shaft is subjected to a twisting moment T, then from equation 15 (p. 77)

$$T = \frac{1}{5{\cdot}1} f d^3 = 0{\cdot}196 f d^3,$$

and if a shaft is subjected to both twisting and bending, then an equivalent bending or twisting moment may be found by the rules on p. 90.

The determination of the greatest safe working stress is a matter of some difficulty. If the stress were due to a constant unvarying twisting or bending moment, or to a moment varying between known limits, regularly, the values given in Table II., p. 43, could be used. But this is not usually the case. In shafts such as those of turbines, the direction of the stress never reverses, and remains generally either at zero or its maximum value during the greatest part of its period of working. The frequency of variation is not great. In mill shafting there is greater frequency of variation, but the direction of the stress is constant. Lastly, in many other cases the stress not only varies constantly, but also constantly reverses in direction.

The following table of general values may serve as a guide :—

Working Stress in Shafting

	Working stress f		
	Steel	Wrought iron	Cast iron
Stress changing little during work and not reversing	13,500	9,000	3,600
Part of the stress reversing at each revolution	9,000	6,000	2,400
Stress constantly changing between equal and opposite values . .	4,500	3,000	1,200

134. *Axles loaded transversely.*—In designing axles of this kind, it is convenient to determine first the dimensions of the journals. If the axle is cylindrical, its diameter at any other point can be obtained from the journal diameter, if it is remembered that the diameters at any two points should be proportional to the cube roots of the bending moments at those points. If the section is not circular, it is still convenient to design a cylindrical axle, and then to replace the

cylindrical sections by equivalent sections of any other form. If the axle rotates, the cylindrical form is the only one which is of equal strength in all positions. The mode of designing axles is best explained by examples.

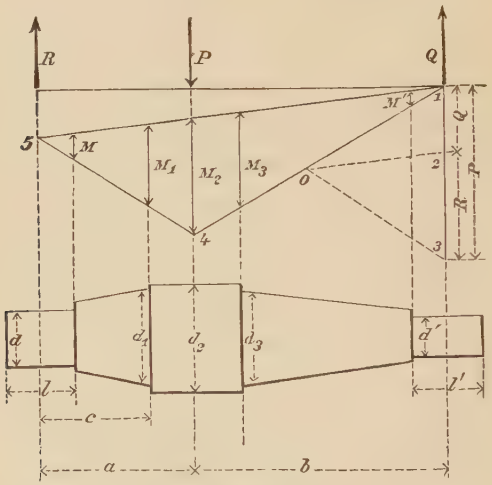


FIG. 140

135. *Example I.*—An axle is supported on two end journals, and carries a load, P, at a point between the journals. Fig. 140 shows the axle. The load P is in equilibrium with the reactions Q, R, acting at the centres of the journals.

$$Q = P\,\frac{a}{a+b}\,;\quad R = P\,\frac{b}{a+b}$$

These are the loads for which the journals are to be calculated. From the rules in §§ 115-124 will be determined d, d', l, l', and the projections which limit the end-play. The axle diameters which it is most necessary to determine are those marked d_1, d_2, and d_3. The bending moments at those points are $M_1 = R\,c$; $M_2 = R\,a$; $M_3 = Q\,e$. The bending

moments at the fixed ends of the journals are $M = R\frac{l}{2}$ and $M' = Q\frac{l'}{2}$.

Since at any section the diameter must be at least equal to $\sqrt[3]{\frac{10}{f}}\sqrt[3]{\text{(bending moment)}}$,

$$\left.\begin{aligned} \frac{d_1}{d} &= \sqrt[3]{\frac{M_1}{M}} = \sqrt[3]{\frac{2}{l}\frac{c}{}} \\ \frac{d_2}{d} &= \sqrt[3]{\frac{M_2}{M}} = \sqrt[3]{\frac{2}{l}\frac{a}{}} \\ \frac{d_3}{d'} &= \sqrt[3]{\frac{M_3}{M'}} = \sqrt[3]{\frac{2}{v}\frac{c}{}} \end{aligned}\right\} \quad . \quad . \quad . \quad (30)$$

The smallest values of the diameters consistent with the requirements of strength are, therefore, easily obtained from the journal diameters.

It is often convenient to measure the bending moments from the bending moment curve, which is easily drawn thus :—Take 13 on the direction of Q produced, and = P, on any scale ; choose any pole O, and draw 1 O 4, meeting the direction of P produced in 4. Join 30 and draw 45 parallel to 30, meeting the direction of R in 5. Join 51, and draw O 2 parallel to it. Then 12, 23 are the values of Q and R on the scale assumed for P. The vertical ordinates of the triangle 1 4 5 are proportional to the bending moments at the corresponding points of the axle. The values of M, M_1, M_2, &c., on any scale, measured on the diagram, may be used in the preceding equations, in determining the diameters of the axle.

The boss at the loaded part of the shaft is intended for keying on the wheel, or other part supported by the axle. Its projection must therefore be sufficient for cutting a key-way, § 98, even if it is then larger than is necessary for strength. If at any part the axle is not circular, it is only necessary to equate the modulus of a section of the required

form, to the modulus of the circular section previously determined. Thus, if the section is to be square, the equation

$$0{\cdot}118\, s^3 = {\cdot}0982\, d^3$$

will give the side of the square, the values of the moduli having been taken from Table V., p. 56. If the axle rotates, the value of the modulus must be that which corresponds to the position in which it is weakest.

136. *Example II.*—The axle supports two parallel loads between the journals. The bending moment curve is drawn thus : Let $a\,b$ be the centres of the journals, $c\,d$ the points at which the loads P Q are applied. At the points a and b the reactions R, S are produced by the action of P and Q. On the direction of S set off 12, 23, equal to Q and P on any scale. Choose a pole, O ; join O 1, intersecting the direction of Q in 4. Join O 2 and draw 45 parallel to it, intersecting the direction of P in 5. Join O 3, and draw 5 6 parallel to it, intersecting the direction of R in 6. If, now, O 7 is drawn parallel to the closing line 6 1, 3 7 and 7 1 will be equal to the reactions R and S. Also, 6 1 4 5 is the bending moment polygon, the breadth of which at any point, measured parallel to the forces, is proportional to the bending moment at the corresponding point of the axle. Having, therefore, the bending moments, the diameters of the axle may be obtained from the journal diameters, as before.

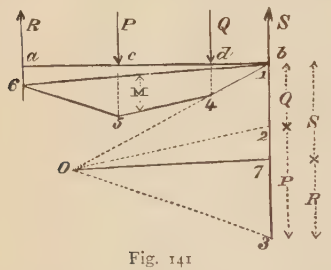


Fig. 141

When P=Q and $a\,c = d\,b$, the case is one which occurs very commonly in practice, in which it will be found that the bending moment is uniform from c to d. A railway carriage axle is in this position when the carriage is at rest, or moving along a straight portion of line. In passing

round curves, however, it is subjected to torsion as well as bending ; and in consequence of the pressure of the wheel flange against the rail, the forces are no longer parallel. Then the bending action between *c* and *d* is no longer uniform. It is for this reason that railway axles are tapered a little towards the centre.

The bending moment curve for forces not parallel is drawn in the same way as before, the only difference being that the lines 1 2, 2 3, 3 7, 7 1, parallel to the forces, no longer fall on a single line, but form a closed polygon.

137. *Shafts transmitting power, and subjected to torsion only.*—Rotating shafts are very extensively used, in transmitting the energy of prime-movers to the various parts of the factory or workshop in which it is applied to useful purposes. Such shafting was at one time of timber, then cast iron was adopted, and, later still, wrought iron has almost entirely superseded cast iron, except in a few cases, where the shafting is not subjected to much impulsive action. Shafts up to 4 inches in diameter are usually turned from rolled bars ; larger shafts from forged bars. In transmitting power shafts are subjected to torsion, but they are also subjected to bending action, due to their own weight, the weight of the wheels and pulleys they support, to the thrust of the gearing and the tension of the belting connected with them, and to other causes. This bending action is, to a great extent, indeterminate ; it will, therefore, be convenient to consider, first, the torsion due to the power transmitted, and then to examine how an allowance can be made for the other straining actions.

Let H. P. be the indicated horses' power transmitted.

N the number of revolutions of the shaft per minute.

P the twisting force in lbs., acting on a shaft at a radius R in ins.

f the greatest safe stress for the material of the shaft, in lbs. per sq. in.

d the diameter of the shaft in ins.

The mean twisting moment is in statical inch lbs. (§ 22)—

$$T = P\,R = 63{,}024\,\frac{H.\,P.}{N} \quad . \quad . \quad . \quad . \quad . \quad (31)$$

The moment of resistance of a circular section with respect to torsion is (§ 36 or § 133) $0{\cdot}196\,d^3 f$. Hence,

$$d = \sqrt[3]{\frac{5{\cdot}1}{f}}\sqrt[3]{P\,R} = \alpha\sqrt[3]{P\,R} \quad . \quad . \quad . \quad (32)$$

$$= \sqrt[3]{\frac{63{,}024}{0{\cdot}196 f}}\sqrt[3]{\frac{H.\,P.}{N}} = \beta\sqrt[3]{\frac{H.\,P}{N}} \quad . \quad . \quad (33)$$

The following are the values of the constants in these equations corresponding to the values of the working stress already given, p. 209 :—

	Values of α			Values of β		
	Steel	Wrought iron	Cast iron	Steel	Wrought iron	Cast iron
Stress changing little during work and not reversing . .	·0723	·0828	·1123	2·877	3·294	4·469
Part of the stress reversing at each revolution . . .	·0828	·0947	·1285	3·294	3·770	5·116
Stress constantly changing between equal and opposite values . . .	·1042	·1193	·1620	4·149	4·749	6·446

138. *Ratio of greatest to mean twisting moment in shafts driven by steam engines.*—In the case of a steam engine the twisting moment exerted on the crank shaft varies with the variation of the steam pressure and with the variation of the leverage of the crank as it rotates. For a single engine working with little expansion $T_{max} = 1{\cdot}3\ T_{mean}$, a rule which

may be used for locomotive cranks for instance. But if the engine works with much expansion the difference between the maximum and mean twisting moment is greater. Mr. Milton has found the ratio ρ of the greatest twisting moment to the mean twisting moment (that given by equation 31), from actual indicator diagrams, to be as follows in certain selected actual engines:—

	$\rho=$	$\sqrt[3]{\rho}=$
A. Single engine	2·1	1·28
B. Engine with equal cylinders, cranks at right angles	1·37	1·11
C. Compound engine cranks at right angles	1·48	1·14
D. Compound engine, cranks at 130° .	1·77	1·21
E. Compound cranks at 135°. Three bearings to crank shaft . . .	1·94	1·25

The diameters of a shaft directly connected with a steam engine, such as the propeller shaft of a ship, calculated from the indicated horses-power by the rules above, must be multiplied by $\sqrt[3]{\rho}$ to allow for the difference between the mean and maximum twisting moment.

139. *Shafts subjected to torsion and bending.*—Let T be the twisting moment (calculated by equation 31), and M the bending moment, at any section of a shaft. Then, the combined straining action is equivalent to that which would be produced by a twisting moment T_e, given by the following equation, which is eq. 29*a* of § 44.

$$T_e = M + \sqrt{(M^2 + T^2)} \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Let $M = k\,T$, in any given case, so that k is a known fraction. Then

$$T_e = (k + \sqrt{k^2 + 1})\,T \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

$$= (1{\cdot}83\,k + {\cdot}83)\,T \text{ approximately} \quad . \quad . \quad . \quad (35\,a)$$

Then the preceding formulæ may be used in designing the

shaft, if the equivalent twisting moment T_e is substituted for the actual twisting moment P R in eq. 32. Hence,

$$d=\sqrt[3]{(k+\sqrt{k^2+1})}\sqrt[3]{\frac{5{\cdot}1}{f}}\sqrt[3]{T} \quad . \quad . \quad . \quad (36)$$

Or, if d is the proper diameter of the shaft, calculated for the combined bending and twisting action, and d' is the diameter calculated for the twisting action alone, by eq. 32 or 33; then

$$d=n\,d' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

where n is equal to $\sqrt[3]{(k+\sqrt{k^2+1})}$, or to $\sqrt[3]{(1{\cdot}83\,k+0{\cdot}83)}$ nearly. The following Table gives some values of n for given values of k.

$k=$	0·25	0·50	0·75	1·0	1·25	1·50	1·75	2·0	3·0
$n=$	1·09	1·17	1·26	1·34	1·42	1·49	1·56	1·62	1·83

It appears, from some calculations of Prof. Rankine, that for such cases as the propeller shafts of steam-vessels, where the straining action, additional to the torsion transmitted, is chiefly due to the weight of the shaft itself, $k=0{\cdot}25$ to 0·5, and the diameter of the shaft should be 1·09 to 1·17 times the diameter, calculated from the torsion alone. For line shafting in mills, the bending action is often much greater, and the twisting moment is not constant, but rises above the mean value, calculated from the power transmitted. Practical experience appears to show, that for ordinary light shafting, k is 0·75 to 1, and the diameter of the shafting is 1·26 to 1·34 times the diameter, calculated from the mean torsion alone. For crank-shafts and heavy shafting subjected to shocks, $k=1$ to 1·5, and the diameter is 1·34 to 1·49 times that calculated from the torsion alone. Cases occur in which still greater allowance must be made.

Cranked shafts of marine engines.—Mr. Milton has calculated for the same engines mentioned above, the ratio

of the greatest equivalent twisting moment, due to twisting and bending, to the greatest simple twisting moment. From these we can easily obtain values of k; allowing for the variation of twisting moment mentioned above, the diameter of the cranked shaft should be greater than that calculated from the mean twisting moment (as given by equation 31), in the ratio

$$\sqrt[3]{\rho}\ \sqrt[3]{\{k+\sqrt{(k^2+1)}\}} \text{ to } 1.$$

Values of $\sqrt[3]{\rho}\ \sqrt[3]{\{k+\sqrt{(k^2+1)}\}}$ for these engines are given in the following table.

	Ratio of greatest to mean twisting moment ρ	Ratio of greatest equivalent twisting moment to greatest simple twisting moment	$k=$	$\sqrt[3]{\rho\{k+\sqrt{(k^2+1)}\}}$
Engine A	2·10	1·38	·33	1·43
,, B	1·37	1·15	·14	1·16
,, C	1·48	1·11	·10	1·15
,, D	1·77	1·05	·05	1·23
,, E	1·94	1·21	·19	1·33

Diameters of Wrought-iron Shafts for given Twisting Moments.

The following table is calculated for a stress of 9,000 lbs. per sq. in. which is suitable for wrought iron and in many cases for steel. For cast iron multiply the diameters in the table by 1·26. For steel multiply by 1 to 0·874 according to the strength and soundness assumed.

The shaft diameter is found from the known mean twisting moment in the column corresponding to the assumed ratio k of bending to twisting.

Twisting moment $\frac{P\,R}{}$ in inch lbs.	Diameter for twisting moment only d	Diameter for twisting and bending moment for $k=$ 0·5	0·75	1·0	1·5
125	·41	·48	·52	·55	·61
250	·52	·61	·66	·70	·73
500	·66	·77	·83	·88	·99
750	·75	·88	·94	1·00	1·12
1,000	·83	·97	1·05	1·11	1·24
1,500	·95	1·11	1·20	1·27	1·42
2,000	1·04	1 22	1·31	1·37	1·55
2,500	1·12	1·31	1·41	1·50	1·67

Twisting moment $P R$ in inch lbs.	Diameter for twisting moment only d	Diameter for twisting and bending moment for $k=$ 0·5	0·75	1·0	1·5
3,000	1·19	1·39	1·50	1·60	1·73
4,000	1·31	1·53	1·65	1·75	1·95
5,000	1·42	1·66	1·79	1·90	2·11
6,000	1·50	1·75	1·89	2·00	2·24
7,500	1·62	1·90	2·04	2·08	2·42
10,000	1·78	2·08	2·24	2·39	2·66
12,500	1·92	2·22	2·42	2·57	2·86
15,000	2·04	2·39	2·57	2·73	3·04
17,500	2·15	2·52	2·70	2·88	3·20
20,000	2·25	2·63	2·83	3·01	3·36
25,000	2·42	2·83	3·05	3·24	3·60
30,000	2·57	3·00	3·23	3·44	3·83
35,000	2·70	3·16	3·40	3·62	4·02
40,000	2·83	3·32	3·57	3·80	4·22
45,000	2·94	3·44	3·70	3·94	4·40
50,000	3·05	3·57	3·85	4·10	4·55
60,000	3·24	3·80	4·10	4·33	4·82
70,000	3·41	4·00	4·30	4·57	5·10
80,000	3·57	4·17	4·50	4·77	5·31
90,000	3·71	4·35	4·70	4·97	5·52
100,000	3·84	4·50	4·85	5·15	5·72
110,000	3·97	4·65	5·00	5·30	5·90
120,000	4·08	4·80	5·15	5·47	6·10
130,000	4·19	4·90	5·30	5·61	6·25
140,000	4·30	5·05	5·40	5·76	6·40
150,000	4·40	5·15	5·55	5·90	6·55
175,000	4·63	5·45	5·85	6·20	6·90
200,000	4·84	5·65	6·10	6·50	7·20
250,000	5·21	6·10	6·55	7·00	7·76
300,000	5·54	6·50	7·00	7·42	8·27
400,000	6·10	7·15	7·70	8·20	9·10
500,000	6·57	7·70	8·30	8·80	9·80
600,000	6·98	8·20	8·80	9·40	10·40
750,000	7·53	8·80	9·50	10·05	11·20
1,000,000	8·28	9·70	10·40	11·10	12·30
1,250,000	8·92	10·42	11·20	12·00	13·30
1,500,000	9·47	11·10	11·95	12·70	14·10
1,750,000	9·97	11·70	12·60	13·35	14·90
2,000,000	10·42	12·20	13·10	14·00	15·50
2,250,000	10·84	12·70	13·65	14·50	16·20
2,500,000	11·23	13·15	14·10	15·10	16·70
3,000,000	11·93	14·00	15·00	16·00	17 80
3,500,000	12·57	14·70	15·90	16·80	18·70
4,000,000	13·14	15·40	16·55	17·60	19·60
4,500,000	13·67	16·00	17·25	18·30	20·20
5,000,000	14·16	16·60	17·85	19·00	21·10

Diameters of Shafts, when the Horses Power and Revolutions per minute are given, calculated by Eq. 33.

Divide the H.P. by the number of revolutions per minute. The diameter will be found opposite the nearest number to the quotient in the following table :—

Horses power / Revolutions H.P. / N.	Diameter for twisting moment only d'	Diameter for twisting and bending moment for $k=$			
		0·5	0·75	1·0	1·5
·012	0·753	0·881	·948	1·009	1·121
·025	0·963	1·126	1·213	1·290	1·434
·050	1·213	1·428	1·528	1·625	1·806
·075	1·389	1·624	1·750	1·862	2·070
·1	1·529	1·788	1·926	2·050	2·277
·15	1·750	2·047	2·202	2·345	2·607
·2	1·926	2·255	2·425	2·58	2·87
·25	2·075	2·427	2·615	2·78	3·09
·3	2·205	2·58	2·78	2·95	3·28
·35	2·321	2·71	2·92	3·11	3·46
·4	2·428	2·84	3·06	3·25	3·62
·45	2·524	2·95	3·18	3·38	3·76
·5	2·614	3·06	3·29	3·50	3·90
·6	2·777	3·25	3·50	3·73	4·14
·7	2·925	3·42	3·68	3·92	4·36
·8	3·06	3·58	3·85	4·10	4·56
·9	3·18	3·72	4·01	4·26	4·74
1·0	3·29	3·85	4·14	4·41	4·90
1·25	3·55	4·15	4·47	4·76	5·29
1·5	3·77	4·41	4·75	5·05	5·62
1·75	3·97	4·64	5·00	5·32	5·92
2·0	4·15	4·86	5·23	5·56	6·18
2·25	4·32	5·05	5·44	5·79	6·44
2·5	4·47	5·23	5·63	6·00	6·66
2·75	4·61	5·39	5·81	6·18	6·87
3·0	4·75	5·56	5·98	6·37	7·08
3·25	4·88	5·71	6·15	6·54	7·27
3·5	5·00	5·85	6·30	6·70	7·45
3·75	5·12	5·99	6·45	6·86	7·63
4·00	5·23	6·12	6·59	7·01	7·80
4·25	5·34	6·25	6·73	7·16	7·96
4·5	5·44	6·36	6·85	7·30	8·11
4·75	5·54	6·48	6·98	7·43	8·26
5·0	5·63	6·59	7·10	7·55	8·39
5·5	5·82	6·81	7·33	7·80	8·67
6·0	5·99	7·01	7·55	8·03	8·93
6·5	6·15	7·20	7·75	8·25	9·17

Horse power / Revolutions H.P./N.	Diameter for twisting moment only d	Diameter for twisting and bending moment for $k=$ 0·5	0·75	1·0	1·5
7·0	6·30	7·37	7·94	8·45	9·39
7·5	6·45	7·55	8·13	8·65	9·61
8·0	6·59	7·71	8·31	8·84	9·82
9·0	6·85	8·02	8·63	9·19	10·20
10	7·10	8·31	8·95	9·52	10·58
11	7·33	8·58	9·24	9·83	10·91
12	7·53	8·81	9·50	10·10	11·21
13	7·75	9·07	9·77	10·40	11·54
14	7·94	9·29	10·00	10·65	11·82
15	8·12	9·50	10·23	10·90	12·10
16	8·30	9·71	10·46	11·13	12·37
17	8·47	9·91	10·67	11·36	12·62
18	8·63	10·09	10·87	11·57	12·86
19	8·79	10·28	11·08	11·80	13·10
20	8·93	10·45	11·25	11·98	13·30
21	9·08	10·62	11·44	12·18	13·52
22	9·23	10·80	11·63	12·38	13·74
23	9·38	10·97	11·81	12·58	13·97
24	9·51	11·12	11·98	12·75	14·17
25	9·66	11·30	12·16	12·95	14·39
26	9·76	11·41	12·29	13·08	14·54
27	9·88	11·55	12·44	13·25	14·71
28	10·00	11·70	12·60	13·40	14·90
29	10·12	11·84	12·75	13·56	15·08
30	10·23	11·95	12·89	13·70	15·24

If the shaft is of cast iron or steel, or if bending action is to be allowed for, proceed as indicated for the preceding table.

140. *Mill shafting.*—Ordinary mill shafting for textile manufactures is calculated for $k=1$; that is, the diameter is 1·34 times that which would be necessary, if there were no bending. When as usual it is of wrought iron, we have from eq. 33—

$$d = 1{\cdot}34\,\beta \sqrt[3]{\frac{\text{H.P.}}{\text{N}}}$$

$$= 4{\cdot}414 \sqrt[3]{\frac{\text{H.P.}}{\text{N}}} \quad . \quad . \quad (38)$$

$$\frac{\text{H.P.}}{\text{N}} = 0{\cdot}01163\, d^3. \quad . \quad . \quad (38\,a)$$

From this the following table has been computed, giving the values of $\frac{\text{H.P.}}{\text{N}}$ for the most ordinary sizes of shaft.

Horses Power transmitted by Wrought-iron Mill Shafting.

Multiply the tabular number in the column $\frac{\text{H.P.}}{\text{N}}$ by the number N of revolutions per minute, the result is the horses power which the shaft will transmit.

Diameter of shaft in ins.	$\frac{\text{H.P.}}{\text{N.}}$	Diameter of shaft in ins.	$\frac{\text{H.P.}}{\text{N.}}$
$1\frac{3}{4}$	0·0623	5	1·4536
2	0·0930	$5\frac{1}{2}$	1·9344
$2\frac{1}{4}$	0·1325	6	2·5112
$2\frac{1}{2}$	0·1817	$6\frac{1}{2}$	3·1944
$2\frac{3}{4}$	0·2418	7	3·9888
3	0·3139	$7\frac{1}{2}$	4·9056
$3\frac{1}{4}$	0·3993	8	5·9536
$3\frac{1}{2}$	0·4986	$8\frac{1}{2}$	7·1440
$3\frac{3}{4}$	0·6132	9	8·4800
4	0·7442	10	11·6288
$4\frac{1}{4}$	0·8930	11	15·4752
$4\frac{1}{2}$	1·0600	12	20·0896
$4\frac{3}{4}$	1·2470		

141. *Hollow shafts.*—At the present time the use of compressed steel permits shafts to be made hollow; in this way, since the least effective portion of the section of the shaft is removed, the weight is diminished in a much greater ratio than the strength. Let d be the diameter of a solid shaft, and d_1 d_2 the external and internal diameters of a hollow shaft of the same material. Then the shafts will be of equal strength when the moduli of the two sections with respect to torsion are equal, that is when (§ 36)—

$$d^3 = \frac{d_1^4 - d_2^4}{d_1}$$

Let $d_2 = xd_1$

$$d_1 = \sqrt[3]{\left\{\frac{d^3}{1-x^4}\right\}} \quad . \quad . \quad (39)$$

which gives the external diameter of a hollow shaft in terms of the diameter of a solid shaft of the same strength, when the fraction of the diameter removed at the centre is fixed. Suppose a 10-inch shaft has a hollow 4 ins. diameter. Its weight will be 16 per cent. less than that of a solid 10-inch shaft, but its strength is only 2·56 per cent. less. A hollow shaft with a hole $\frac{4}{10}$ths of the diameter, and equal in strength to a 10-inch solid shaft, would have a diameter of

$$d_1 = \sqrt[3]{\frac{10^3}{1 - 0{\cdot}4^4}} = 10{\cdot}09 \text{ inches.}$$

142. Usually in line shafting the power is taken off at various points in the length of the shaft. Hence, the shaft may be gradually reduced in diameter, and thus material is economised and the friction diminished. A long shaft consists of lengths of shafting, each of uniform diameter, the reduction of diameter being made in passing from one length to the next. When a shaft is calculated very closely in size, its ends must have bosses, to receive the keyways for fixing the couplings. The plan of graduating the size of the shaft to the work transmitted has some serious disadvantages. The bearings which support the shaft and the couplings, wheels and pulleys fixed to it, are not interchangeable in position if the diameter of the shaft is variable. This gives rise to much trouble and expense, if the machinery requires to be rearranged. Shafts of uniform diameter are now often adopted, and for such shafts the forging of bosses on the ends is unnecessary, because at most points the shaft has surplus strength.

Small shafts often give trouble from insufficient stiffness, although they have ample strength. For such shafts, $\frac{3}{8}$ in. to $\frac{5}{8}$ in. may be added to the diameter, which is sufficient for strength, in order to secure stiffness and freedom from vibration. For long shafts, and when $\frac{\text{H. P.}}{\text{N}}$ is less than 1, the diameter may be calculated by Redtenbacher's rule,

which makes the angle of torsion a fixed proportion of the length of the shaft. Then,

$$d = \beta \sqrt[4]{\frac{\text{H. P.}}{\text{N}}} \quad . \quad . \quad . \quad (40)$$

where β has the same value as before (p. 214), and the diameter is to be multiplied by the same values of n, to allow for bending action.

The span between the bearings of shafting should be so arranged as to limit the deflection of the shaft to a fixed proportion of its length. Let L be the span between the bearings, in inches. Then

$$\text{L} = \gamma \sqrt[3]{d^2} \quad . \quad . \quad . \quad . \quad (41)$$

where $\gamma = 60$ to 75, for shafting supporting its own weight only, and $\gamma = 54$ to 60, for shafting carrying the ordinary proportion of pulleys or gearing. Very commonly the bearings of mill shafting are 10 to 11 feet apart.

143. *Planished wrought-iron shafting.*—The round bars which come from the rolling-mill are rough and slightly crooked. Shafting made from such bars must be turned from end to end in the lathe, to obtain uniformity of diameter and smoothness of surface. A process has, however, lately been introduced which promises to supersede turning in many cases. By passing the bar while still hot between rapidly revolving bevelled rollers, the scale is cleaned off and the bar rendered so straight and regular that it may be used for shafting, after having been merely polished with a file and emery stick, either in the lathe or in place.

144. *Expansion of shafts.*—If the shaft is of great length, especially if it carries bevel wheels, at distances of more than 40 feet from the journal with collars, which prevent end motion, its alteration of length from changes of temperature becomes troublesome. In such cases other collar bearings are provided near the wheels, and an expansion coupling is introduced at some intermediate point. Ordinary claw couplings are sometimes used as expansion couplings, but

more commonly a box coupling is used. One shaft end is fixed in the coupling box by a taper key. The other is secured from rotating relatively to the box by two parallel keys on opposite sides fitted accurately but easily, and allowing free expansion.[1]

The expansion of iron is about 0·0012 of its length for a rise of temperature of 180° Fahr. Hence, if t_1, t_2, are the highest and lowest temperatures to which a shaft is exposed, a point l inches from the bearing with collars will move a distance

$$0{\cdot}0012\,\frac{t_1 - t_2}{180}\,l \text{ inches},$$

in consequence of variation of temperature.

145. *Centrifugal whirling of shafts.*—Prof. Rankine showed that the centrifugal force of a slightly bent shaft and the elastic stress tending to straighten it would become equal at any given speed for a certain distance between the bearings. With a greater distance such a shaft is liable to continue revolving in a bent form, if any small deflection is given to it. For a shorter distance it restores itself to straightness. The limit of length between the bearings which should not be exceeded, for a wrought-iron or steel shaft d inches in diameter, making N revolutions per minute, is,—

$$175\sqrt[2]{\frac{d}{N}} \text{ feet.}$$

With a less distance between the bearings centrifugal whirling does not occur. This does not, however, take into account the effect of rotating masses fixed on the shaft, and not near the bearings.[2]

Fencing of shafting.—To protect workmen from accident, shafting and gearing not out of reach should be fenced by

[1] Sutcliffe, 'Machinery for the Production and Transmission of Motion' ('Proc. Inst. C. E.' vol. lviii.).

[2] Rankine, 'Millwork,' p. 549.

covers of tin plate. Shafting between detached buildings should be at a considerable height above the ground, or in a covered trench below it.

Crank Shafts

146. *Forged cranked shafts.*—Forged cranked shafts are very extensively used, for inside cylinder locomotives and

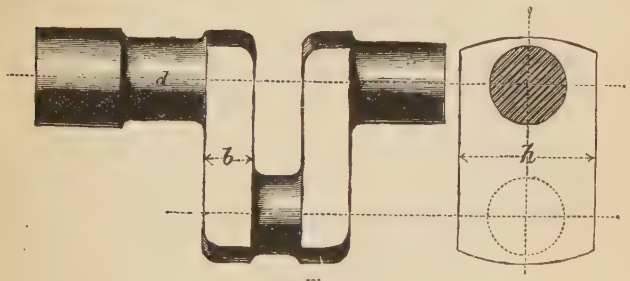


Fig. 142

for marine engines of all kinds. Fig. 142 shows the ordinary form of such a cranked shaft. In marine-engine practice the diameter may be obtained thus :—

H.P. = indicated horse power.
N = number of revolutions per minute.
d = diameter of shaft.

$$d = 4{\cdot}55 \sqrt[3]{\left(\frac{\text{H.P.}}{\text{N}}\right)} \quad . \quad . \quad (42)$$

which allows for bending as well as twisting actions. The arms of the crank may be so proportioned that

$$b h^2 = c d^3$$

where $c = 0{\cdot}9$ to $1{\cdot}0$.

The objection to the use of cranked shafts is their liability to fracture, and it is one of the principal reasons for the preference of outside to inside cylinder locomotives, that in the former cranked shafts are not required. On this

point Mr. Milton makes the following remark:[1] 'Most of the flaws for which cranked shafts are condemned occur at the angle between the web of the crank and the journal or crank-pin, at the place where the forging is most likely to be defective, and they are evidently produced more by the bending than by the twisting strains. At these places the change of form of the shaft throws great local stresses on the material, and there can be no doubt that, if these parts are made with a large radius, the strength of the shaft is materially increased. Among the causes which tend to throw great strains on the shafting may be mentioned, the presence of water in the cylinders, slackness of the brasses, and the bearings being out of line.'

147. *Graphic method of determining the straining actions in crank shafts.*—The crank arm and crank shaft are subjected to bending and twisting actions which are very conveniently dealt with graphically. Suppose the crank and shaft provisionally designed so as to fix the positions of the bearings. The moment equivalent to the bending and twisting moments at each section can then be determined and the necessary dimensions calculated.

Case I.—Taking a simple overhung crank, shown by its centre lines in fig. 143. Let A be the crank-pin centre and B and C the centres of the bearings. In general the total action will be greatest when the thrust or pull, P, of the connecting rod is perpendicular to the plane of the figure, that is when the connecting rod is at right angles to the crank arm. P will be supposed to be acting so, although drawn in the plane of the figure.

Suppose P given, then the reactions Q and R at B and C can be determined by taking moments, or more simply by drawing the diagram of forces. Take 1 2 on P's direction equal to P, and a polar distance 2 *o* at right angles of any convenient number of inches. Join 1 *o* cutting Q's direction in *a*. Join *a b* and draw *o* 3 parallel to it. Then 2 3=R

[1] 'Strains on Crank Shafts' ('Proc. Inst. of Naval Architects,' 1880).

and 1 3=Q. Also 1 *a b* is the diagram of bending moments on the crank shaft. So that the ordinate *e f* on the scale of loads multiplied by *o* 2 on the scale of lengths is the bending moment at *e*, or *e f* is proportional to the bending moment.

Take 1 *c*=1 A. Draw *c d* at right angles to the shaft, cutting 1 *o* or 1 *o* produced in *d*. Then since *c d* : *c* 1 :: 1 2 : *o* 2, ∴ *c d* × *o* 2 = *c* 1 × 1 2 = P × 1 A. But P × 1 A is the

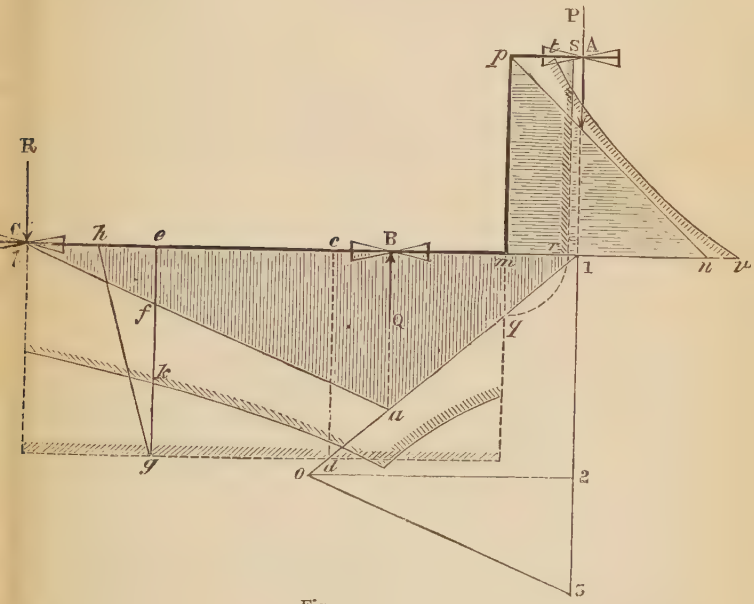


Fig. 143

twisting moment on the shaft. Consequently *c d* represents the twisting moment on the same scale as the diagram of bending moments. Drawing a line through *d* parallel to the shaft we get the diagram of twisting moments.

To combine the bending and twisting moments at any point into an equivalent bending or twisting moment, let *e f*=M be the bending and *e g*=T the twisting moment at *e*.

Take $e\ h = e\ f$ and join $h\ g$; then $h\ g = \sqrt{(e\ \overline{h^2} + e\ \overline{g^2})} = \sqrt{(M^2 + T^2)}$. Consequently $e\ f + h\ g$ is $M + \sqrt{(M^2 + T^2)}$, the equivalent twisting moment, and half this is the equivalent bending moment. The shaded curve is the curve of equivalent bending moments obtained by repeating this process at several points, and drawing a curve through the points so found. From the ordinates of this curve the dimensions of the shaft at any point necessary for strength can be calculated.

When P acts perpendicular to the plane of the figure, $P \times 1$ A is the bending moment at the point m of the crank arm. But this has already been found to be represented by $c\ d$. Take $m\ n = c\ d$ and join $n\ p$. Then $p\ n\ m$ is the diagram of bending moments on the crank arm. The twisting moment on the crank arm is $P \times A\ p$ or equal to the bending moment on the crank shaft at m. Take $m\ r = m\ q$ and complete the rectangle $s\ m$. This is the diagram of twisting moments on the crank arm. Combining the bending and twisting moments by the same process as before we get the curve of equivalent bending moments on the crank arm $t\ v$.

148. *Case II.*—An engine crank shaft is subjected at the crank pin to a pressure P of 3 tons, and carries a spur flywheel at Q weighing 5 tons. Hence only the part of the shaft between P and Q is subject to torsion. The forces are supposed normal to the plane of the drawing, fig. 144, although shown for convenience in the plane of the drawing. The comparatively small pressure at the spur-wheel teeth is neglected. This case is conveniently treated partly by calculation and partly graphically.

The moment of P about R_2 is -213; that of Q is -135 inch tons. Hence the moment of R_1 is $213 + 135 = 348$ inch tons. Hence $R_1 = 348/55 = 6{\cdot}327$ tons and $R_2 = 8 - 6{\cdot}327 = 1{\cdot}673$ tons. Now take any line $a\ b$, a being on P's direction and b being on R_2's direction. Set off $b\ c$ equal to the moment of P and join $a\ c$; then $a\ b\ c$ is P's moment area, so that any vertical intercept between $a\ b$ and $a\ c$ is the moment

of P at that point. Let d be on R_1's direction, set off $ce = R_1$'s moment and join de, then dec is R_1's moment area. Let f be on Q's direction, set off eb = Q's moment and join fb, then feb is Q's moment area. The negative moments are measured downwards and the positive moments upwards. It will now be seen that, in the areas feb and $dgbc$, a posi-

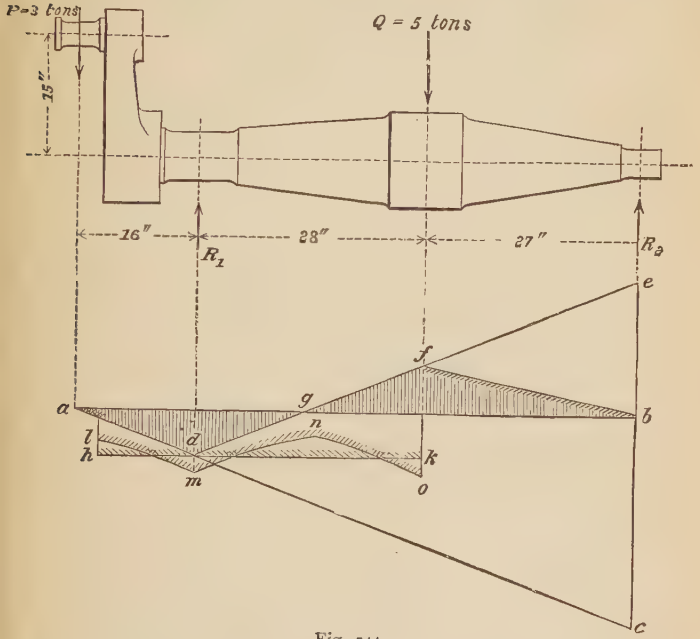


Fig. 144

tive and negative moment balance. There remains the shaded area $adgfb$, which is the bending moment diagram for the shaft.

The twisting moment of P is $3 \times 15 = 45$ inch tons. Set this off downwards and draw hk parallel to ab. This is the torsion moment area. Combining the torsion moment with

the bending moment at each point, as in the previous case, we get the curve *l m n o* as the curve of equivalent bending moments. The bending moment curve previously drawn gives the bending moments for the remainder of the shaft.

Couplings

149. Lines of shafting are made of lengths of 20 to 30 feet coupled together. The couplings should be placed near bearings and on the side farthest from the driving-point. Then if a length of shaft is disconnected the running part is supported. The shaft lengths should be exact multiples of the distance between bearings.

We may distinguish (*a*) couplings for shafts having a common axis of rotation. (*b*) Couplings for parallel shafts. (*c*) Couplings for shafts the axes of which intersect. Next as to the description of coupling we may distinguish: (1) fixed or permanent couplings, which can only be disconnected by unscrewing bolts or slacking keys. (2) Loose couplings or clutches, provided with arrangements for throwing part of the shafting out of gear as often as necessary. (3) Friction couplings, which are loose couplings having the special peculiarity that they put the driven shaft into gear gradually and slip if the resistance becomes excessive.

Fast or Permanent Couplings

150. *Box couplings.*—Figs. 145, 146, and 147 show three forms of coupling, known as 'box' or 'muff' couplings. In these a cast-iron box, or hollow cylinder, is fitted over the ends of the shafts. In figs. 145 and 147 the coupling is termed a butt coupling, and relative movement of the shafts is prevented by a wrought-iron key, which lies in a key-way, cut half into the box and half into the shaft ends. Fig. 146 is a half-lap coupling, the shaft ends overlapping, so as to prevent relative motion independently of the key,

whose chief function is to fix the box rigidly in place. The keys are proportioned according to the rules in § 98. In fig. 146 the key is a saddle key. The other dimensions

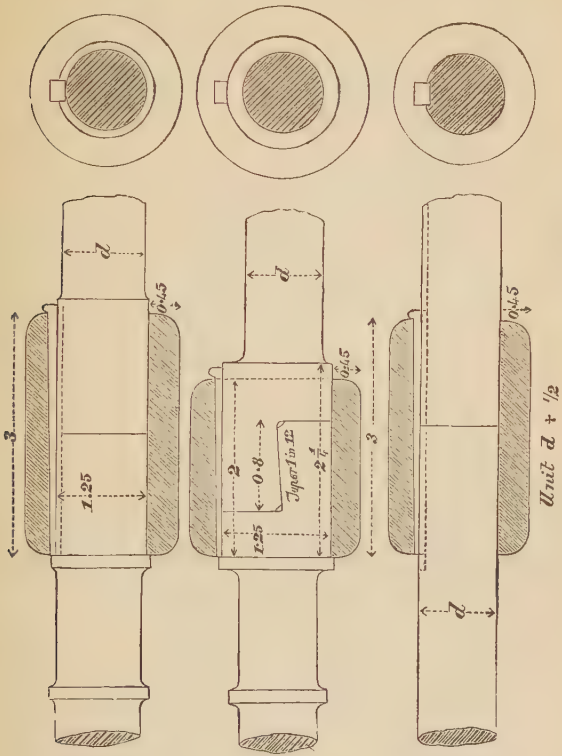


Fig. 145 Fig. 146 Fig. 147

may be obtained from the proportional numbers. The half-lap coupling is an excellent coupling for shafts not exceeding 5 ins. in diameter, but is somewhat expensive. The butt coupling is cheaper, but less secure. Both forms

are free from projections likely to catch the clothes of a workman.

151. Fig. 148 shows a flange, or face-plate coupling. It consists of two parts of cast iron, firmly fixed by keys to the two shaft ends. The face of each coupling is turned after it has been keyed on the shaft, so that it is accurately perpendicular to the axis of the shaft. The couplings being brought together are fixed by bolts, which prevent relative movement by their resistance to shearing. In the coupling shown the bolt-heads are sunk in the substance of the flange for safety. The bolt-holes must be drilled and the bolts carefully fitted. The number of bolts may be

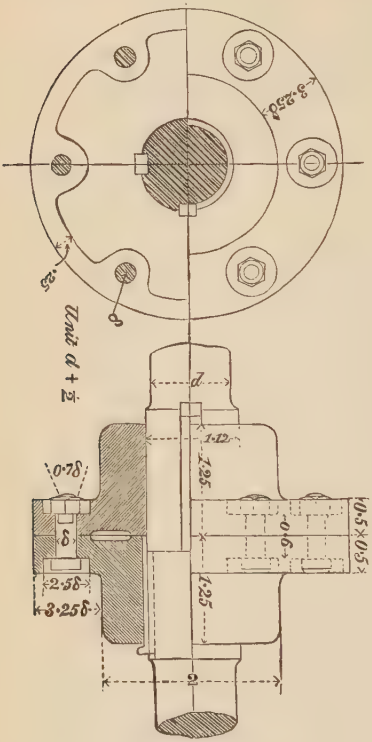


Fig. 148

$$n = 3 + \frac{d}{2} \quad . \quad (43)$$

the nearest even number being usually taken.

Let R be the radius of the bolt circle. Then, when the shaft is strained to its limit of elasticity, eq. 32, p. 214, gives, for the shearing force on the bolts,

$$P = \frac{f d^3}{5{\cdot}1\ R}$$

The resistance of the bolts to shearing is $\frac{\pi}{4} n \delta^2 f_s$. Equating this to the shearing force, we get, for the diameter of the bolts,

$$\delta = 0{\cdot}577 \sqrt{\left(\frac{d^3}{nR}\right)} \quad . \quad . \quad . \quad . \quad (44)$$

In practice the bolts are often a little larger, and may be

$$\delta = \frac{d}{n} + \tfrac{1}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

To keep the shafts in line, the end of one shaft may enter into the coupling on the other $\frac{1}{4}$ to $\frac{1}{2}$ inch.

When shafting is not closely calculated for strength, it is better to fix the flange coupling, as shown in fig. 149.

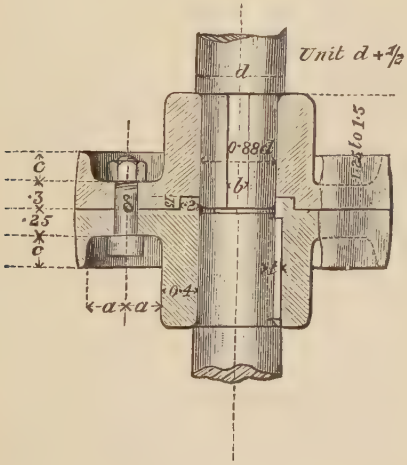


Fig. 149

Putting d for the diameter of shaft, and δ for the diameter of the bolts, the following proportions are good :—

Number of bolts $n = 3 + 0{\cdot}5\,d$

Diameter „ $\delta = \dfrac{0{\cdot}62\,d}{\sqrt{n}}$

$a = 1{\cdot}5\,\delta$

$c = 1{\cdot}25\,\delta$

$b = 0{\cdot}25\,d + 0{\cdot}125$

$t = \frac{1}{2}\,b$

152. *Propeller shaft coupling.*—Fig. 150 shows a coupling formed by flanging the shaft itself. In this case the shaft is a hollow compressed steel shaft. The unit for the pro-

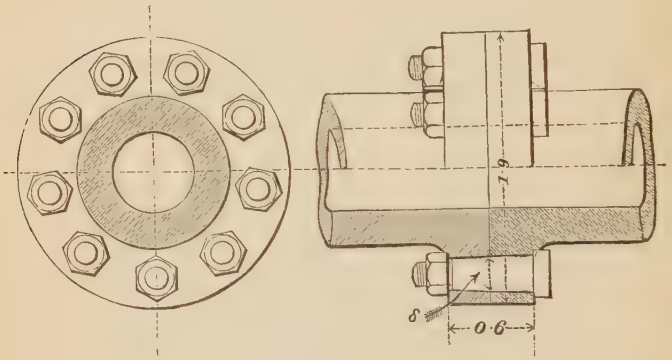


Fig. 150

portions may be the diameter of an equivalent solid shaft. Let d_1, d_2 be the internal and external diameters of the shaft; then

$$\text{Unit} = d = \sqrt[3]{\left(\frac{{d_2}^4 - {d_1}^4}{d_2}\right)}.$$

The bolt diameter is given by the equation

$$\delta = 0{\cdot}75\sqrt{\frac{d^3}{n\,\text{R}}} \quad . \quad . \quad (46)$$

where n = number of bolts; R = radius of bolt circle.

153. *Sellers's double cone vice coupling.*—With box

couplings it is generally necessary to forge bosses on the shaft ends, to receive the couplings. This prevents pulleys and wheels being put on the shafts from the ends. The face-plate coupling depends for its solidity on a taper key, and cannot be often loosened without danger of impairing its accurate adjustment relatively to the axis of the shaft. Mr. Sellers has introduced a coupling which obviates these difficulties, and which does not require such perfect fitting. Fig. 151 shows this coupling in longitudinal section, and end elevation and cross section. It consists of an outer cylindrical muff, or barrel, enclosing the ends of the shafts. The inside of this is turned to a double conical form. Between the barrel and the shaft are two sleeves, the outsides of which are conical, and fit the box, and the insides are cylindrical, and fit the shaft. These sleeves are pressed together by three screw-bolts, parallel to the shaft. The bolts are square in section, and rest in slots cut into the

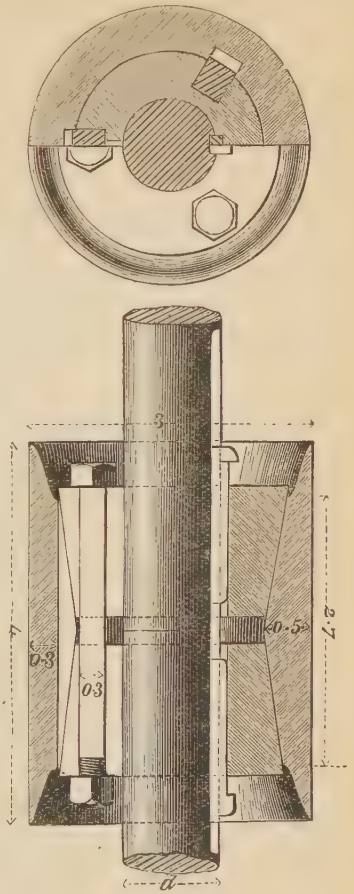


Fig. 151

sleeves and the barrel. To give elasticity to the sleeves, they are completely cut through on one side, at the bottom of one of the bolt slots. Each sleeve is drawn inwards with equal force and grasps the shaft with equal tightness. A key is driven into each shaft end, as an additional precaution, but these keys should fit sideways only, and not at top and bottom. They do not then exercise any bursting force on the coupling. Absolute equality of size of the shafts is unnecessary. When two shafts of unequal size are connected, the larger is turned down, at the end, to the size of the smaller. These couplings are sometimes difficult to disconnect. To obviate this, it has been proposed to tap a set screw through the barrel, having a conical end pressing against the two inner sleeves. When this set screw is turned it separates the sleeves. If the parts are well oiled before they are put together, there is no great difficulty in disconnecting. The bolts are taken out, and the coupling struck with a wooden mallet, or a wedge is driven into the split through the sleeves.

154. A simple and effective friction coupling, used in France and Germany, is shown in fig. 152. The coupling

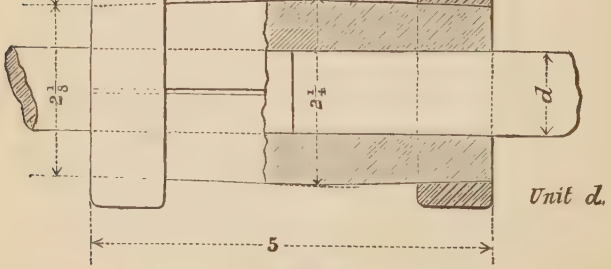


Fig. 152

consists of a pair of semicircular cast-iron clips. These are placed together with a thin plate or sheet of paper inter-

posed, and bored out to the diameter of the shaft. When placed on the shaft the clips are held by two wrought-iron rings shrunk on. The ends of the shafts are turned, but not polished. The usual proportions are given on the figure, the outside of the clips being turned slightly taper. Sometimes bolts are used to connect the two clips instead of rings, and then the clips have a somewhat different form.

Supposing the shaft transmitting a twisting moment only (eq. 32, p. 214),

$$d = a\sqrt[3]{P R};$$

or for wrought-iron shafts, P estimated at the circumference of the shaft is

$$P = 3530\, d^2.$$

Let p be the pressure of the clip on each shaft end per unit of circumference; μ = the coefficient of friction. Then

$$\mu \pi d p \overline{=} 3530 d^2.$$

Taking $\mu = 0{\cdot}2$,

$$p \overline{=} 5616 d.$$

Such a bursting pressure produces in the ring a tension of $\frac{1}{2}\, p\, d$ lb. (eq. 2, § 26). Taking the safe stress in the wrought-iron rings at 9,000 lbs. per sq. inch, each ring must have a section

$$\frac{p d}{2 \times 9000} = \frac{5616 d^2}{18000} = 0{\cdot}312 d^2;$$

or if the width of the ring is $0{\cdot}9\, d$, its thickness must be at least $0{\cdot}35 d$. The thickness given above is a little less than this; but, on account of allowance for bending, the torsion transmitted is rarely as great as that assumed above. With a stress of 9,000 lbs. the ring will extend

$$\frac{l}{L} = \frac{f}{E} = \frac{1}{3200}$$

of its circumferential length. Consequently its internal

diameter before shrinking on must be $\frac{1}{32\,00}$th less than the external diameter of the coupling.

155. *Oldham coupling.*—For two shafts which are parallel the coupling shown in fig. 153 may be used. A disc is keyed on each shaft end, and between these lies a third disc, which has a diametral feather on each side fitting

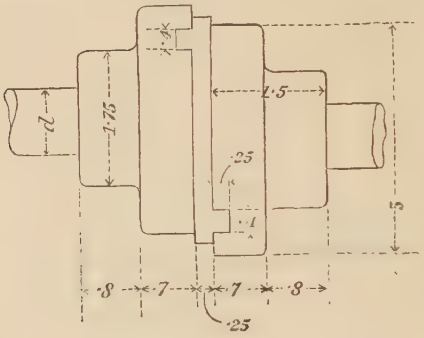


Fig. 153

in a slot in the corresponding shaft disc. The two feathers are at right angles. The middle disc revolves round an axis parallel to the shafts and midway between them. The shafts and the middle disc have all equal velocities at every period of the rotation.

Loose or Disengaging Couplings

156. *Claw coupling or clutch.*—For very large slowly rotating shafts it is desirable that the coupling should have a slight amount of play, so that, when the shafts are a little out of line, the coupling accommodates itself to the obliquity without straining the shafts. The claw coupling may then be used either as a fast coupling or so arranged that one half can be slid back and the shafts thrown out

of gear. Fig. 154 shows this coupling arranged for disengaging. It consists of two parts, like the face-plate coupling ; but each part has projections, which fit in recesses in the opposite coupling. In the coupling shown, the left-hand part is prolonged, and has a groove cut round

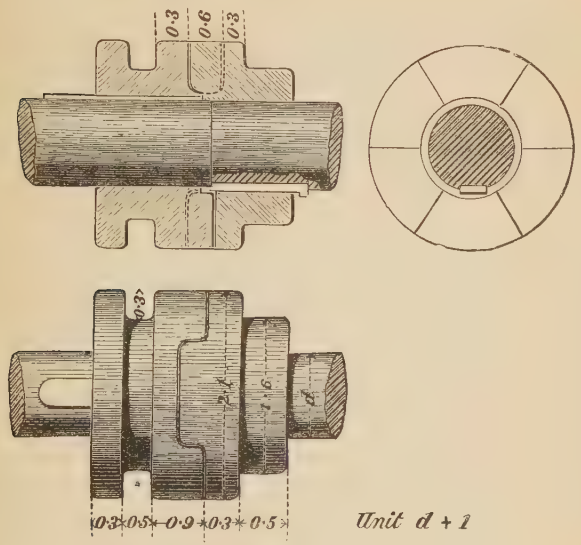


Fig. 154

it. In this fit the jaws of a lever, for sliding it back. The right-hand part is firmly keyed on its shaft. The left-hand part slides on a fixed key, or feather, which is not tapered. The claws of one coupling fit a little loosely in the recesses of the other, so as to permit a small amount of play.

157. *Friction couplings.*—The simplest form of friction coupling consists of a cone, keyed rigidly on one shaft, against which a movable cone sliding on a feather on the other shaft can be pressed (fig. 155). If the axial pressure is sufficient, the friction on the surface of the cones is greater than

the resistance of the driven shaft. The movable part should be on the driven shaft so as to be at rest when the coupling is out of gear. The mean cone radius r may be two or three times the shaft diameter, or more if the resistance of the driven shaft is considerable.

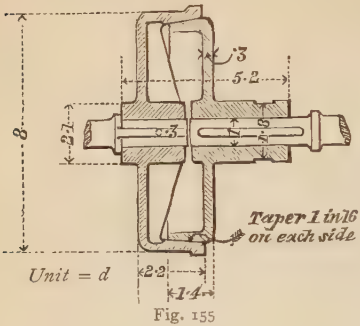


Fig. 155

If M is the moment of the effort required to drive the shaft and 2 N is the total normal pressure between the cones of mean radius r, the coefficient of friction being μ, then

$$2\,N\,\mu\,r = M$$

$$2\,N = \frac{M}{r\mu}.$$

But if P is the axial force pressing the cones together and α the angle of the cones,

$$P = 2\,N\left(\sin\frac{\alpha}{2} + \mu\cos\frac{\alpha}{2}\right)$$

Consequently

$$P = \frac{M}{r\mu}\left(\sin\frac{\alpha}{2} + \mu\cos\frac{\alpha}{2}\right).$$

When once the coupling is in gear it will retain its position with a less axial pressure. Apart from vibration, which probably has the same effect as diminishing μ, the coupling would continue to drive until the axial pressure fell to

$$P' = \frac{M}{r\mu}\left(\sin\frac{\alpha}{2} - \mu\cos\frac{\alpha}{2}\right).$$

And if the angle of the cone is small enough it may require a reversed axial force to disengage it. For iron on

iron μ may be taken 0·15, and for iron on leather 0·25. The angle at the vertex of the conical surface is usually 10° to 20°.

Weston friction coupling.—A simple form of friction coupling is shown in fig. 156, which can be used either as a shaft coupling or, as here shown, to couple at will a spur wheel to a shaft. The wheel *a* has a long boss with two feathers on which are strung wrought-iron rings. Between these are wood rings, *b*, carried by six feathers inside the coupling-box D, which slides on a feather on the shaft, *c*. If the coupling-box is pressed to the left there is friction at

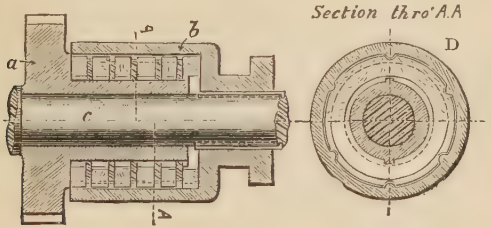


Fig. 156

each face between the wood rings and the iron rings. With n iron rings there is friction at $2n + 1$ faces. There is therefore $2n + 1$ times the friction there would be on a single face. The figure is taken from Towne on Cranes. He states that clutches of this kind have been used to transmit the power of a 1,000 H.P. steam engine to a train of rolls. For most purposes both sets of rings may be of iron or steel.

158. *Universal coupling.*—When the axes of two shafts which are not in line intersect, they may be connected by a Hooke's joint, or universal coupling, shown in fig. 157. The velocity ratio of the shafts is then variable, but if their directions make a small angle, the variation is not great, and is generally unimportant. The proportional unit for the dimensions is $d + \frac{1}{2}$ or $d + 1$.

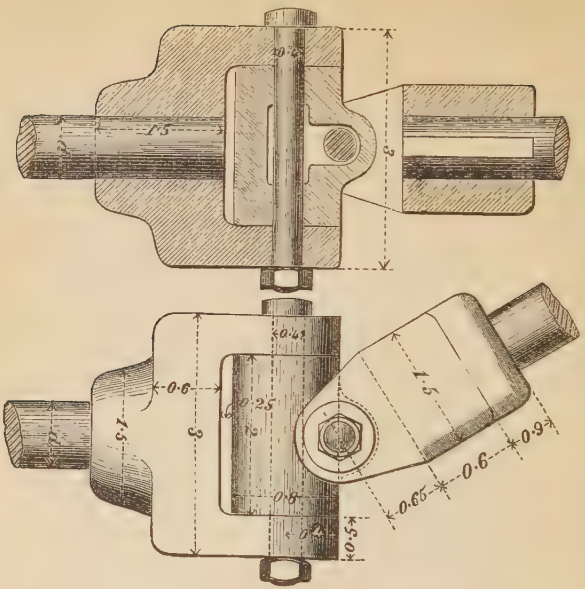


Fig. 157

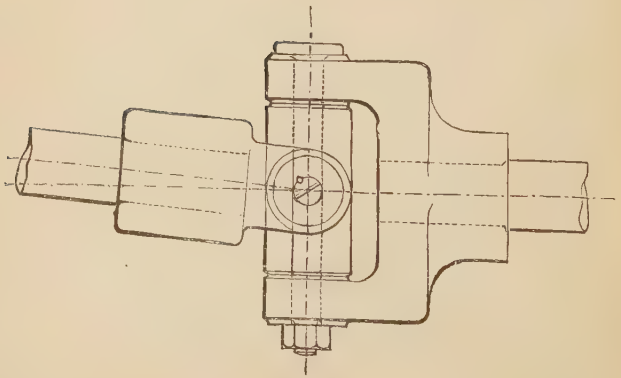


Fig. 158

The irregularity of motion is less with the form of Hooke's joint, shown in fig. 158.[1]

Let PR be the twisting moment transmitted by the shaft, and let R be measured to the middle of one of the journals in the forks. Then the maximum pressure on the journal is

$$\tfrac{1}{2}\,(P \,/\, \cos i)$$

where i is the angle between the axes of the shafts. For this pressure its diameter is calculated.

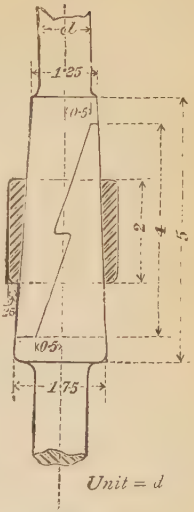


Fig. 159

159. *Disengaging coupling for transmitting an axial thrust or pull.*—Fig. 159 shows a form of coupling often used where two links have to be connected so as to be easily disengaged. The ring shown in section holds the parts together when in gear and is kept in place by gravity.

[1] The author has to thank a correspondent for this drawing.

CHAPTER VIII

BEARINGS FOR ROTATING PIECES

Pedestals

160. The simplest form of journal bearing is a cylindrical hole in the frame supporting the rotating piece. Such a hole wears oval in the course of time, and does not admit of readjustment. The hole may be lined with a brass sleeve or bush, or with soft metal; it can then be restored to its original condition by a new brass bush, or a new lining of soft metal. A better, though still a very simple, form of bearing is shown in fig. 160. In this the bearing is still in part formed in the frame of the machine, but it is in two parts, which are so arranged that the upper part can be tightened down on the journal by bolts, as it wears. The projections on the cap prevent any horizontal movement. The wear, being most often due to the weight of the pieces supported, takes place vertically. When this is not the case, the division of the bearing should be at right angles to the direction of the resultant pressure on the journal.

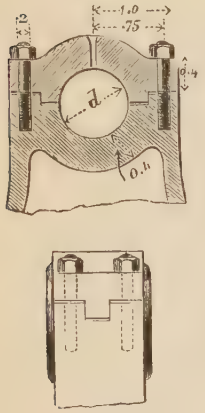


Fig. 160

Unit $d + \frac{1}{2}$

Fig. 161

By making the bearing separate from the framing of the machine, a means of initially adjusting its position is secured. Further, by lining the bearing with brasses or steps it is made practically independent of wear. The steps are divided so as to permit adjustment from time to time, and they can be removed and replaced by new ones when so much worn that their adjustment is no longer sufficient to keep the journal steady. When such a bearing instead of being fixed on the framing of a machine has to be fixed on masonry or brickwork, a wall plate or foundation plate is commonly used. This is adjusted in position tolerably accurately, so that the final placing of the bearing can be effected with little trouble. The foundation plate serves also to spread the pressure of the pedestal over a sufficiently large area of the masonry.

Fig. 161 shows a typical pedestal, with the foundation, or wall plate, on which it is fixed. This wall plate spreads the pressure of the pedestal over a larger area, and affords a levelled surface, on which the pedestal can be adjusted with less trouble than on the rough masonry of a wall. The steps are shown externally of octagonal form, the shape most convenient for hand fitting. They are often cylindrical, and are then turned in the lathe, and the pedestal is bored out to receive them. The steps have flanges, to prevent lateral movement. The under surface of the pedestal, and the upper surface of the wall plate, have narrow chipping strips, to facilitate the adjustment of level. The bolt holes in the wall plate and pedestal base are oblong, so that the pedestal can be shifted laterally in either direction. When adjusted to its true position, it is fixed by hard wood or iron wedges, driven between the ends of the pedestal and jaws cast on the wall plate.

The diameter and length of the steps are the same as those of the journal. The other dimensions may be obtained from the proportional figures, the unit for which is $d + \frac{1}{2}$.

161. *Brasses or steps.*—Fig. 162 shows some ordinary forms of steps for journal bearings. In many cases the pressure of a journal on its support acts always in one sense. In such cases a single step is sufficient, as at *a*, which represents the arrangement adopted for railway axles. The part of the journal not in contact with the step is then protected from dust by a shell cap. More commonly the pressure of a journal acts alternately in opposite senses, and then two steps are required as shown at *b*, *c*, *d*, *e*. The dividing plane of the steps is placed in the direction in

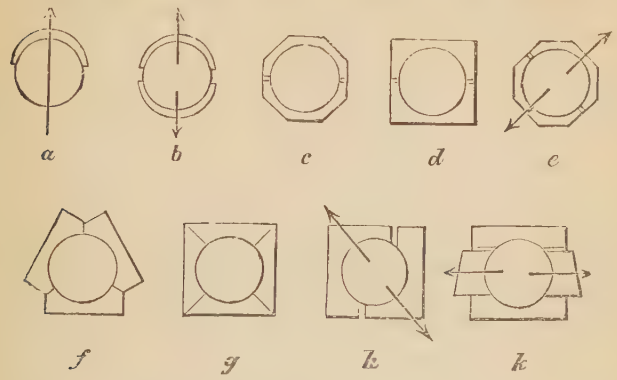


Fig. 162

which the wear is least, or at right angles to the direction of the resultant pressures. When the direction of the pressure varies, more complex adjustment is necessary, and three or four separately adjustable steps may be used, as shown at *f* and *g*; *h* is an arrangement adopted where the pressure of the journal is inclined, but where the adjustment can be most conveniently made horizontally; *k* is an arrangement permitting horizontal adjustment.

162. Fig. 163 shows sections of three ordinary forms of brass or gun-metal steps, and a half-plan, half-longitudinal, section of a step. The step is fitted to the pedestal for a

portion of its width only, at each end, the intermediate part being recessed, and left rough. When the step is turned, instead of being fitted by hand, it sometimes bears on the pedestal over its whole width. The hexagonal step cannot turn in its seat. The cylindrical step requires a snug, to

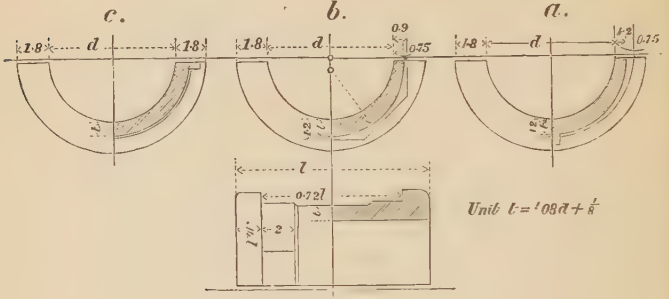


Fig. 163

prevent turning. The composition of the gun-metal, white brass and phosphor bronze, used for steps, is given in Chapter I. When antifriction metal is used, it is usually applied as a lining to a gun-metal step, and is cast in shallow recesses formed to receive it.

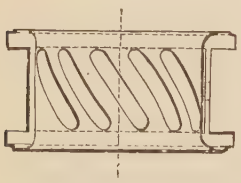


Fig. 164

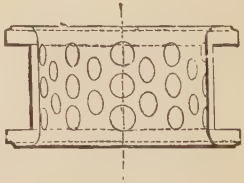


Fig. 165

Figs. 164, 165 show two good methods of applying antifriction metal or white brass in pedestal steps. The shallow recesses in which the white metal is placed are either helical shallow slots or circular shallow depressions. It appears that from some difference of expansion of the

white metal and brass, channels for distributing the lubricant are formed when the white metal is applied in this way, and the bearing wears better than when a complete white metal lining is applied.

The thickness of the steps at the bottom, where the wear is greatest, may be

$$t = 0{\cdot}07d + \tfrac{1}{8} \text{ to } 0{\cdot}1d + \tfrac{1}{8}.$$

At the sides the thickness may be $\frac{3}{4}t$. The proportional unit for the dimensions of the steps is t.

Table of Pedestal Proportions.

Diameter of journal, in ins.	Length of bearing, in ins.	Height to centre	Diameter of bolts	Size of bolt holes	Length of base	Centres of cap bolts	Centres of base bolts	Thickness of step at bottom.
$1\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{8}$	$\frac{1}{2}$	$\frac{5}{8} \times 1$	$8\frac{7}{8}$	$3\frac{1}{2}$	$7\frac{1}{4}$	$\frac{1}{4}$ to $\frac{5}{16}$
2	3	$2\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{4} \times 1\frac{1}{4}$	11	$4\frac{3}{4}$	9	$\frac{5}{16}$ to $\frac{3}{8}$
$2\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$\frac{3}{4}$	$\frac{7}{8} \times 1\frac{1}{2}$	$13\frac{1}{4}$	$5\frac{1}{4}$	$10\frac{7}{8}$	$\frac{5}{16}$ to $\frac{7}{16}$
3	4	$3\frac{3}{4}$	$\frac{7}{8}$	$1 \times 1\frac{5}{8}$	$15\frac{1}{2}$	$6\frac{1}{8}$	$12\frac{5}{8}$	$\frac{3}{8}$ to $\frac{1}{2}$
$3\frac{1}{2}$	$4\frac{1}{2}$	$4\frac{5}{16}$	1	$1\frac{1}{8} \times 1\frac{3}{4}$	$17\frac{1}{2}$	7	$14\frac{5}{8}$	$\frac{3}{8}$ to $\frac{1}{2}$
4	5	$4\frac{7}{8}$	$1\frac{1}{8}$	$1\frac{1}{4} \times 2$	20	$7\frac{7}{8}$	$16\frac{1}{4}$	$\frac{7}{16}$ to $\frac{9}{16}$
5	6	6	$1\frac{3}{8}$	$1\frac{3}{8} \times 2\frac{1}{4}$	24	$9\frac{5}{8}$	$19\frac{1}{4}$	$\frac{1}{2}$ to $\frac{3}{4}$
6	7	7	$1\frac{5}{8}$	$1\frac{7}{8} \times 2\frac{1}{2}$	$28\frac{1}{2}$	$11\frac{3}{8}$	$23\frac{3}{8}$	$\frac{9}{16}$ to $\frac{13}{16}$
7	8	$8\frac{1}{8}$	Two $1\frac{1}{4}$	$1\frac{3}{8} \times 2\frac{1}{4}$	...	$12\frac{1}{4}$	...	$\frac{5}{8}$ to $\frac{7}{8}$
8	9	$9\frac{1}{8}$	,, $1\frac{1}{2}$	$1\frac{3}{4} \times 2\frac{1}{2}$	...	14	...	$\frac{11}{16}$ to 1
9	10	$10\frac{1}{4}$	,, $1\frac{5}{8}$	$1\frac{7}{8} \times 2\frac{1}{2}$	...	$15\frac{3}{4}$	...	$\frac{3}{4}$ to 1
10	11	$11\frac{1}{2}$	,, $1\frac{3}{4}$	$2 \times 2\frac{3}{4}$	...	$17\frac{1}{2}$	...	$\frac{7}{8}$ to $1\frac{1}{8}$
12	13	$13\frac{1}{2}$	,, 2	$2\frac{3}{8} \times 3\frac{1}{8}$	...	21	...	1 to $1\frac{1}{8}$

From 7″ upwards, the pedestals have two bolts on each side, both in cap and base plate.

163. Large steps when heated by the friction of the journal tend to grip the journal at the sides. They should therefore be eased so as to fit the journal a little loosely at the sides. At one or more bearings of a shaft, collars are used to prevent longitudinal motion, and the steps at these bearings sometimes give trouble from heating. Mr. Sutcliffe points out that the steps heat more and expand more rapidly than the shaft. Hence some clearance should be

provided between the ends of the steps and the collars. Generally a very small amount of clearance is sufficient. But in shafts driven by belting, where a small amount of longitudinal motion is unobjectionable, the clearance may be one-tenth of the length of the journal. The longitudinal play tends to make the journal wear uniformly.

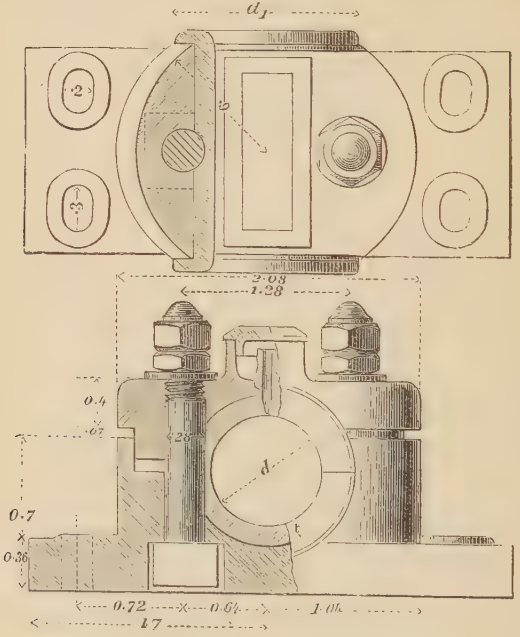


Fig. 166

164. *Weight of pedestals.*—The approximate weight of the cast iron in pedestals is given approximately by the following equation :—

$$W = 1{\cdot}1d^3 + 18 \text{ lbs.}$$

and the weight of a pair of steps is

$$W = 0{\cdot}23d^3 + 6 \text{ lbs.}$$

165. At times a pedestal requires to be contracted in dimensions. Fig. 166 shows a very neat and compact pedestal, designed by Mr. Arthur Rigg, C.E. The cap fits in a cylindrical recess in the body, which can be turned out in the lathe. The bolt-holes are bored out, and the recess for the steps also. The pedestal may be still further contracted, by making the cap bolts double-ended, and using them both for attaching the cap to the body and the body to its support. The base is then absent.

166. *Long bearings for high-speed shafts.*—When a shaft runs at a high speed, the bearings must be long, to secure durability. The steps are then often of cast iron, which answers well as a support for wrought iron if sufficient bearing surface is given. But the longer the bearings are the more important it becomes that they should be exactly concentric and in line. For long shafts it is then desirable to give the steps a spherical seat, so that they may, to some extent, adjust themselves to the position of the shaft. In America, fast-running shafts, supported on cast-iron bearings four diameters long, have been extensively used ; and for carrying these shafts Mr. Sellers has introduced the pedestal shown in fig. 167. The steps are supported on the spherical parts, and can rotate slightly, either horizontally or vertically adjusting themselves to the shaft. The unit for the proportional

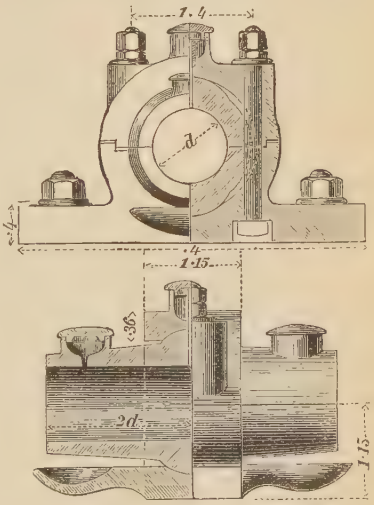


Fig. 167

figures is $d + \frac{1}{2}$. Drawings of these pedestals are given in a paper by Mr. Sellers in the 'Journal of the Franklin Institute,' 1872. The lubrication of these pedestals is peculiar. The ordinary lubrication is at the centre of the pedestal; in addition to this, two cup-shaped hollows are formed near the ends of the top step. These are filled with a mixture of tallow and oil, which is solid at ordinary temperatures, and melts at about 100° F. If the step heats from failure of the ordinary lubrication the tallow melts, and prevents injury to the shaft. A drip cup is provided under each end of the pedestal.

167. *Self-lubricating pedestals.*—Many pedestals have been designed with oil reservoirs, which enable the pedestal to run six months without additional lubrication. Fig. 168 shows Möhler's pedestal. This has a lower brass only

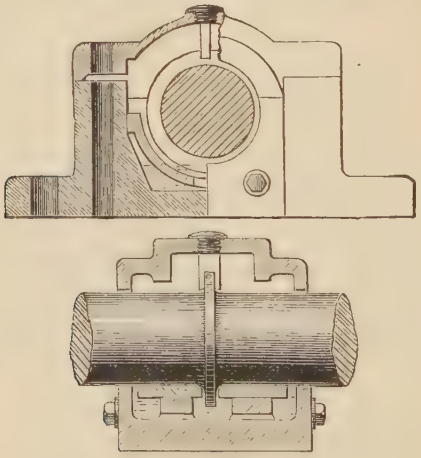


Fig. 168

divided into two portions by a collar on the shaft. The lower part of the pedestal is hollow, and forms a reservoir, into which the collar dips. As the shaft revolves, the collar lifts the oil and distributes it to the shaft on either side.

The surplus oil flows back into the reservoir. The objection to these pedestals is that they require a large supply of oil at first, which gradually becomes viscid by absorption of oxygen, and is then useless.

168. *Lignum vitæ bearings.*—In a few cases where ordinary oil lubrication is difficult, lignum vitæ bearings are used. Thus the stern tube bearing of propeller shafts consists of strips of lignum vitæ, 1 to 4 inches wide, placed parallel to the shaft. The strips are fixed in a cast-iron tube, into which they are dovetailed. They project $\frac{1}{4}$ to $\frac{3}{8}$ inch, and have spaces of about $\frac{1}{2}$ to 1 inch wide between them. Through these spaces the sea water finds free access to the rubbing surfaces, acting as a lubricant and keeping the surfaces cool. The shaft is cased with gun metal in order that there may be no corrosion of the rubbing surface. The bearing varies in length from 2 feet for engines of 100 I.H.P. to 6 feet for engines of 1,000 I.H.P.

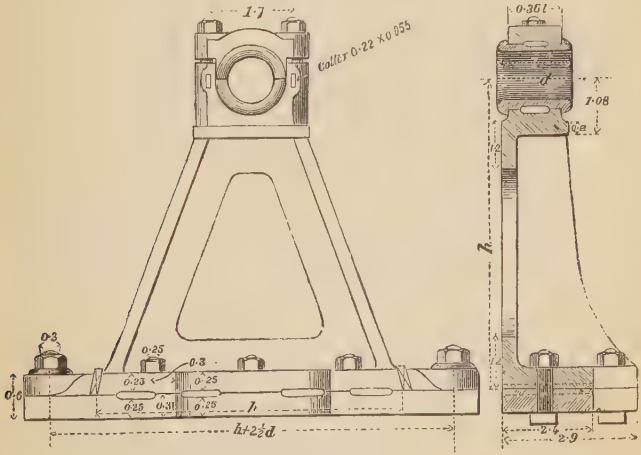


Fig. 169

169. When a pedestal requires to be elevated above its support, the form shown in fig. 169 is used. The propor-

tions of the steps, cap and cap bolts, are the same as for an ordinary pedestal. The other dimensions are given on the figure, the proportional unit being, as before, $d + \frac{1}{2}$.

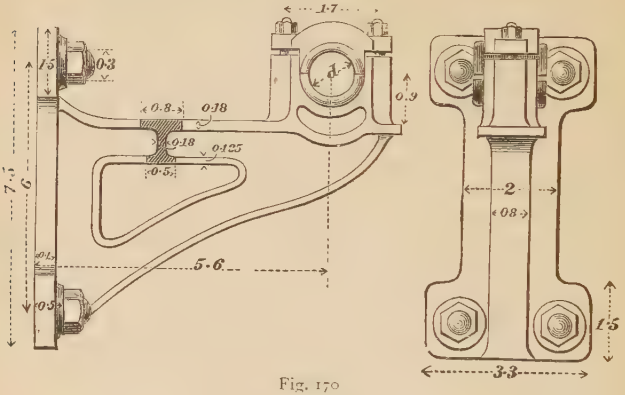


Fig. 170

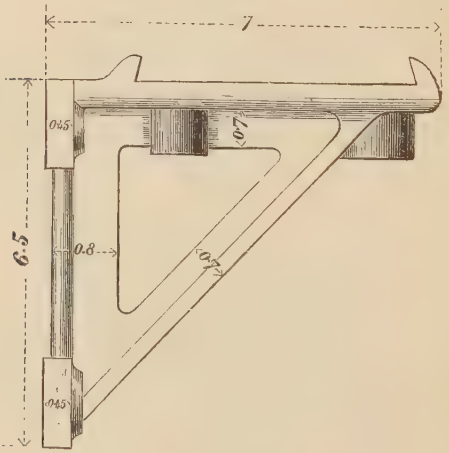


Fig. 171

Sometimes a pedestal has to be fixed to a wall. Then the bracket pedestal shown in fig. 170 is used. The unit

for the proportions is $d+\frac{1}{2}$. An ordinary pedestal may be

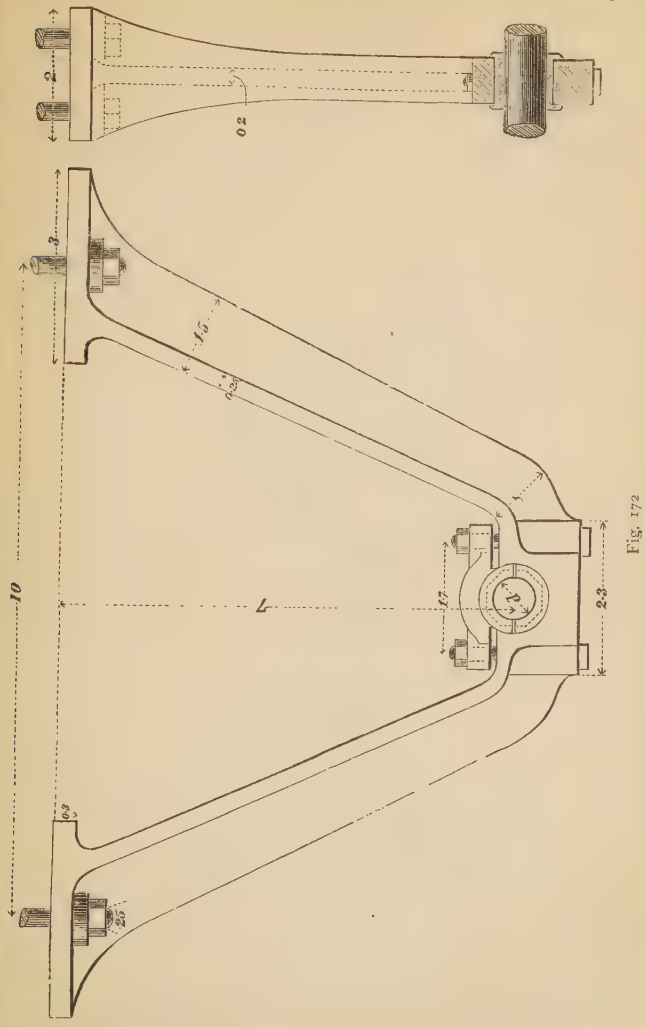


Fig. 172

used, fixed on a bracket such as that shown in fig. 171. The

recess under the bearing in fig. 170 serves to receive a tin dish, which catches the oil drippings.

170. *Hangers.*—When a shaft is supported from the ceiling girders, the pedestal is modified in form, and is

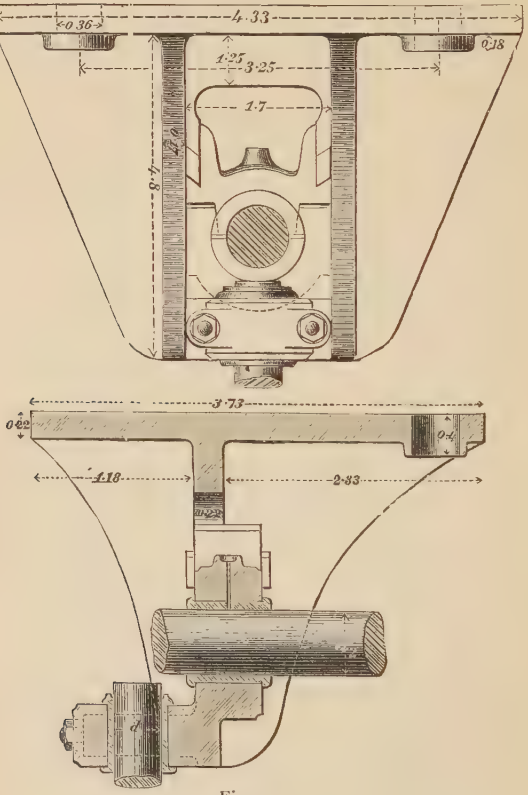


Fig. 173

termed a hanger. Two forms are used; in fig. 172, the pedestal base is bent round and upwards, and attached to the ceiling on both sides; in the other, the pedestal is supported on one side only. The latter arrangement facilitates

the dismounting of the shaft, but requires more metal in the hanger.

171. Fig. 173 shows a hanger for two shafts, whose directions intersect. This happens when one shaft drives another by bevil gearing. The cap of the upper pedestal is kept in place by keys. The steps, bolts, and caps of this hanger may be designed as for ordinary pedestals. The proportional unit for the remainder of the pedestal is

$$d_2 + 0{\cdot}4d_1 + \tfrac{1}{2},$$

where d_2 is the diameter of the greater, and d_1 that of the smaller, of the two shafts.

172. *Wall Fixings.*—When a pedestal is fixed in a wall, a wall box is used. These wall boxes (fig. 174) not only

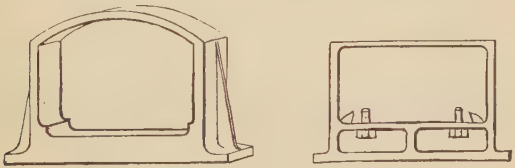


Fig. 174

give a firm and level support to the pedestal, but they carry the wall over the opening, and give a regular form to the opening.

When wall boxes are provided and built into the wall during its erection, they may have broad outside flanges on each face of the wall to prevent movement. They are also often conveniently built of four separate plates, instead of being in a single casting (fig. 175). All wall boxes giving a clear opening through a wall should be provided with internal flanges, to which wrought-iron plates can be bolted to form a fireproof barrier. (Sutcliffe, *Proc. Inst. C.E.*, vol. lviii.)

Figs. 175, 176, taken from Mr. Sutcliffe's paper, are examples of the most modern type of wall fixings, for carrying shafts of considerable size in well-arranged factories. In fig. 176 a small wall box is shown, which forms

a support for a bracket fixing, on which a pedestal is to be placed. The wall box is built into the wall between two ashlar stones, which carry the pressure of the ironwork

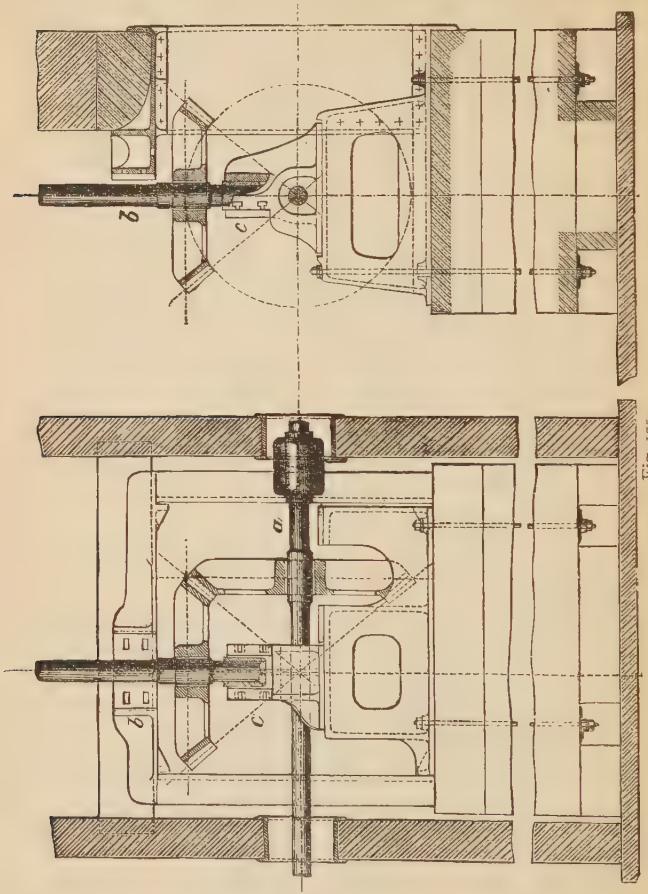


Fig. 175

better than the brickwork of the wall. The face of the iron wall box is planed to receive the base plate of the bracket, and is provided with two vertical grooves which receive the

T-shaped heads of the bolts which fix the bracket in place. Thus a very accurate vertical adjustment of the bracket can be effected. When this is done, the bracket is additionally secured by wedges or floats driven between the ends of the bracket and snugs on the wall box. The horizontal adjust-

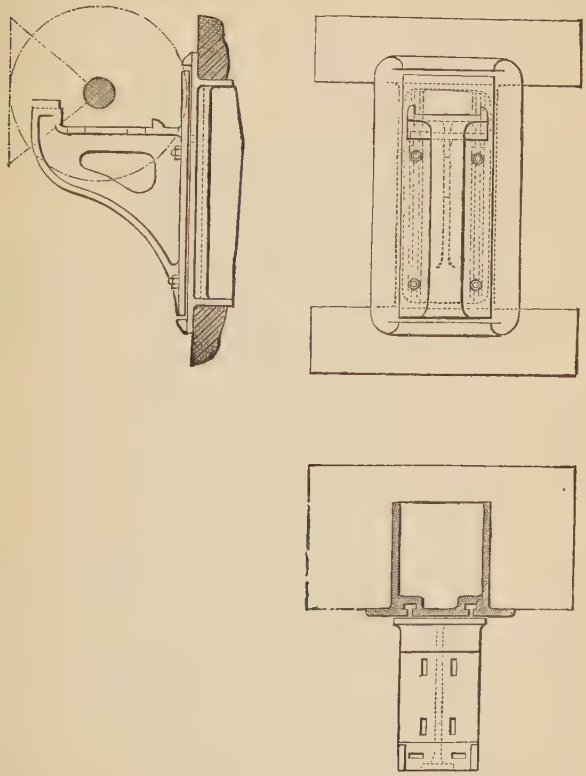


Fig. 176

ment of the pedestal can be similarly effected, and thus the adjustment of the position of the shaft is completely provided for.

In most mills the engines drive a horizontal shaft, which gives motion to a vertical shaft passing up through all the

floors of the mill, and in turn driving the shafting on each floor. The vertical shaft, therefore, carries the whole power of the engines, and any disarrangement of its action affects the whole factory. Great care, therefore, is taken in the fixing of this shaft. The shaft is now often placed in a separate walled tower with stages or floors at each pair of wheels. The footstep is carried on a massive plate, which rests on a masonry and brick pier carried up from a solid foundation. Openings to the tower are closed by iron doors to prevent the communication of fire from floor to floor. Fig. 175 shows the arrangement of the fixing at the foot of such an upright shaft on a scale of $\frac{1}{6}$th of an inch to the foot. The drawing shows the horizontal shaft, with its coupling, and the vertical shaft with its pivot. At *a* is the position of the pedestal for the horizontal shaft; at *b*, that of the pedestal for the upright shaft. The bevil wheels work in the aperture formed by the wall box. The casting on which the footstep is fixed is partly built into the wall, partly fixed by long foundation bolts. The front plate of the footstep *c* is fixed by T-headed bolts. When this is removed the footstep can be taken out.

Axle Boxes

173. Axle-boxes are peculiarly formed journal-bearings, by which the weight of locomotive engines and railway carriages is transmitted to the axles. The axle-boxes of carriages are of very various forms, and will not be treated here. The axle-boxes of locomotive engines are more simple, and may be briefly considered, as illustrations of modified pedestals. In a good axle-box the lubrication should be constant, and not wasteful; the journal should be protected from grit; and should fit easily in the step, with a moderate amount of end play. Axle-boxes consist of an outer casing, a step of gun-metal or other alloy, and a hollow shell, closing the under side of the box, and receiving the surplus oil. The outer casing is accurately faced on

both sides, to fit the space prepared for it in the horn plates, between which it slides vertically, and it is provided with flanges, which permit a small amount of lateral play. The upper part is formed into an oil box, from which copper tubes conduct the oil to the journal. It has also usually a socket, to receive the end of the spindle, through which the

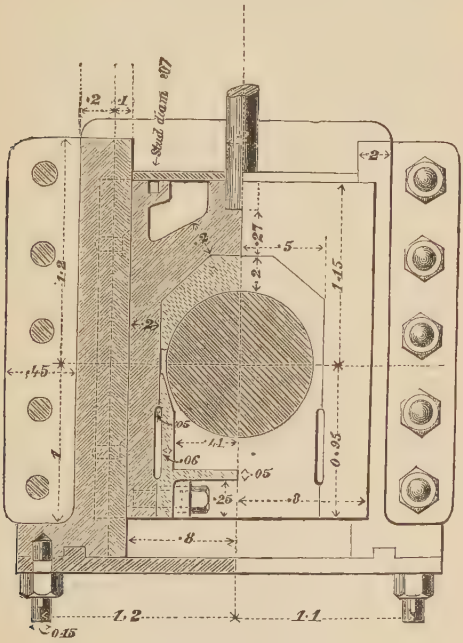


Fig. 177

weight of the engine is transmitted to the box. This rod bears on a plate spring at its upper end. External axle-boxes are closed in front, and being of rather complex form, are of cast iron. Axle-boxes for internal framings are simpler, and are of forged wrought iron, or of cast steel. In the older axle-boxes, wedges were provided on each side, to com-

pensate for wear, but these are now generally omitted. The step is like that of an ordinary pedestal, and is sometimes lined with soft metal.

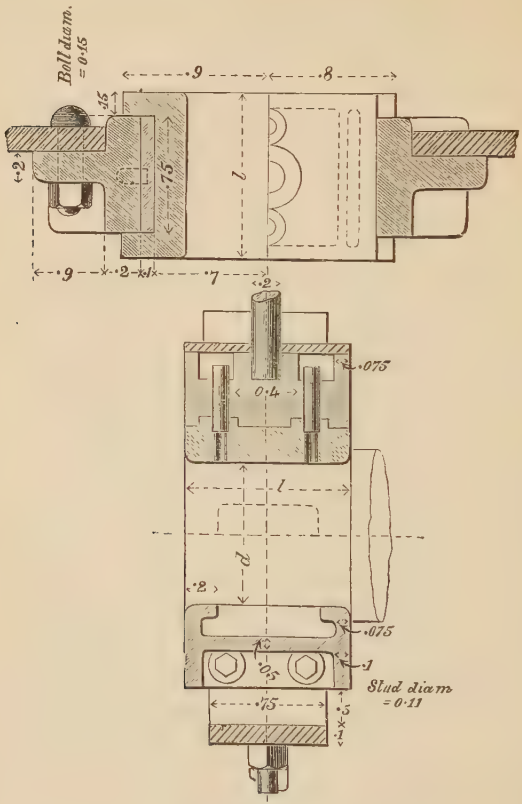


Fig. 178

Let d be the diameter, l the length of a journal. Then the product $d\,l$ is called the bearing surface of the journal. Let P be the load on the journal, then $P \div d\,l$ is the intensity of the pressure on the bearing surface.

The following table gives the dimensions of some actual axle journals, the ordinary speed and the pressure allowed per unit of bearing surface.

Material of axle		Size of journal	Revolutions per minute	Load per unit of bearing surface in lbs.
Steel	Passenger carriage .	$6'' \times 3''$	240 to 360	340
,,	Goods waggon . .	$6'' \times 3\frac{1}{2}''$	180	380
,,	Tender axle . . .	$7'' \times 3\frac{1}{2}''$	240 to 360	330
,,	Locomotive axle .	$6'' \times 4\frac{1}{2}''$	240	210
Wrought iron	Goods engine axle .	$7'' \times 5\frac{1}{4}''$	180	250
,,	Goods waggon . .	$6'' \times 3\frac{1}{2}''$	180	380

The axle-box shown in figs. 177, 178, is a trailing axle box of cast iron. Let D be the diameter of the cylinder. The bearing surface on each side may be $0{\cdot}4\ D^2$. The thickness of the step is $\frac{d}{5}$ to $\frac{d}{6}$, where d is the journal diameter. Lengthways, the step may be $\frac{1}{16}$th inch shorter than the journal, to allow a little end play. The unit for the proportional figures is $d+\frac{1}{2}$.

Footstep Bearings

174. When a shaft is vertical, its lower end rests in a kind of pedestal, termed a footstep. The ordinary arrange-

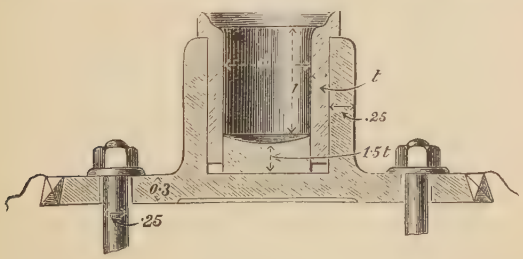


Fig. 179

ment is shown in fig. 179. The end of the shaft is steeled or has a steel end welded to it. Lateral motion is prevented

by a brass bush, fitting in a cast-iron fixing, and end movement, by a tough brass or steel, slightly cup-shaped, disc, on

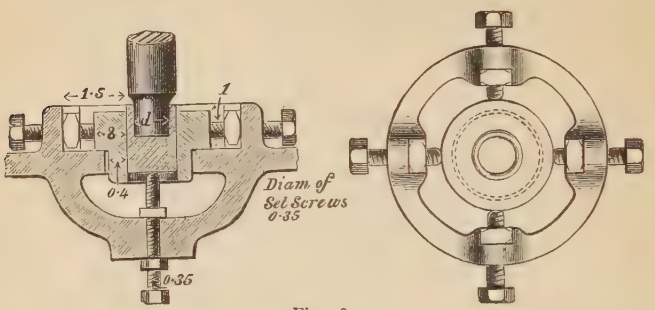


Fig. 180

which the shaft pivot revolves. The thickness of the brass may be $t = 0{\cdot}07d + \frac{1}{8}$. The unit for the other dimensions is $d + \frac{1}{2}$.

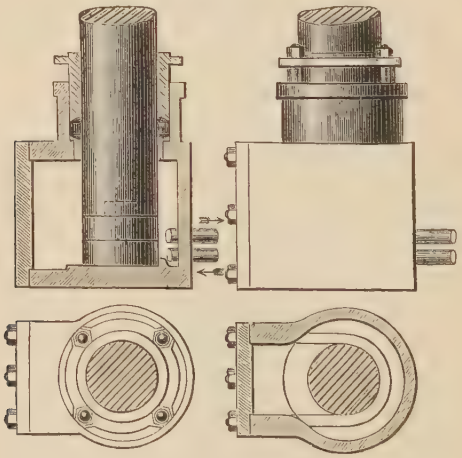


Fig. 181

In the case of important upright shafts of factories, the weight of the shaft is so great that the brass disc is liable to split. To prevent this, it may be hooped with a wrought-

iron ring shrunk on. Fig. 175 shows a pivot footstep in position, and has already been described.

When exact vertical and lateral adjustment of the footstep is necessary, the arrangement in fig. 180 is adopted. The lateral adjustment is effected by four set screws, and the vertical adjustment by a single set screw. The horizontal screws are tapped into the casting, and fixed by lock nuts. The vertical screw has two nuts. Unit $d + \frac{1}{2}$.

When a footstep works under water, there is difficulty in insuring proper lubrication of the pivot. Fig. 181 shows a turbine pivot, enclosed in an oil casing, through which a flow of oil can be insured. Two small copper pipes are connected with the casing, and these are conducted to points above the water level. The shaft passes into the casing through a stuffing box. The end of the shaft is provided with a steel disc, working on a similar disc fitted in the casing.

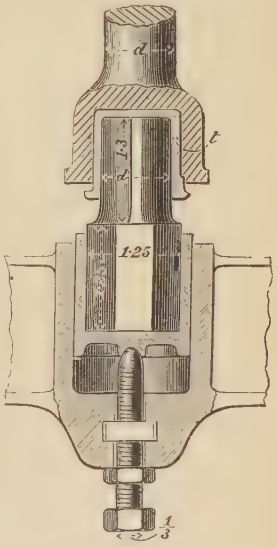


Fig. 182

Another arrangement, which is very effective, is to dispense with the ordinary metal pivot, and replace it with a pivot of lignum vitæ. For metal on wood, water is an excellent lubricant, and such bearings work with very little wear under great pressures. The pivot, fig. 182, is inverted, so that grit is less likely to enter. The pivot is adjusted vertically by a set screw. The end of the shaft is enlarged, bored out, and fitted with a brass step. A groove is cut round the pivot, which, being always filled with water, insures proper lubrication.

175. *Pivot bearing for suspended shaft.*—In the construc-

tion of turbines which have a vertical shaft, the difficulty of lubricating the pivot under water has led most Continental constructors to adopt the arrangement shown in fig. 183. A is a fixed wrought-iron pillar carrying at its top the footstep C, which is in any position above the water in which the turbine works convenient for access. D is the wrought-iron shaft carrying the power of the turbine which rests on the pivot, and which is coupled to a hollow cast-iron shaft B B surrounding the fixed pillar A, and carrying at its lower end the turbine wheel. The lower hollow shaft is, therefore, suspended on the pivot. The diameter of the upper wrought-iron shaft d is calculated by the rules for wrought-iron shafting. The pillar has a diameter d_3 which is often the same as d; but it must be considered if this diameter is sufficient to carry the weight of the turbine and gearing when the pillar is treated as a long column virtually rounded at the ends (Rule II. Table VIII. p. 80). The inside diameter, d_1, of the

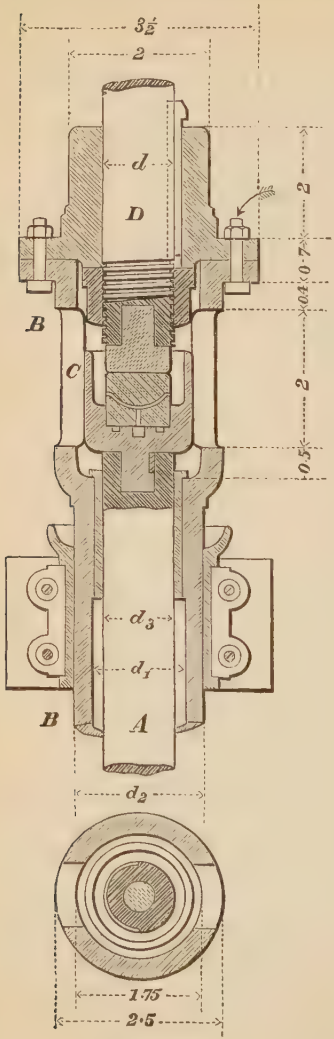


Fig. 183

hollow shaft must be sufficient to allow room for the brass sleeve which steadies the top of the pillar A. Usually $d_1 = 1{\cdot}35d_3$. Lastly, d_2 is calculated so that the section of the hollow shaft is equivalent in torsional resistance to the solid shaft of diameter d. If the two shafts were of the same material we should have

$$\frac{d_2^4 - d_1^4}{d_2} = d^3.$$

Taking the shearing resistance of wrought iron to be three times as great as that of cast iron, we must have

$$3\,\frac{d_2^4 - d_1^4}{d_2} = d^3.$$

Generally d_1 is known. Then

$$d_2^4 - \tfrac{1}{3}d_2 d^3 - d_1^4 = 0,$$

an equation difficult to solve directly. Let δ_2 be any approximation to the value of d_2. Then

$$\delta_2 - \frac{\delta_2^4 - \tfrac{1}{3}\delta_2 d^3 - d_1^4}{4\delta_2^3 - \tfrac{1}{3}d^3}$$

is a nearer approximation. By repeating the calculation with the new value a still nearer approximation is obtained.

Fig. 184

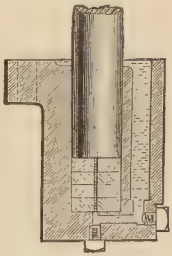


Fig. 185

176. *Pivots with loose discs.*—Fig. 184 shows a form

of collar-bearing for drilling-machine spindles. The three loose washers distribute the pressure and wear, and if the lubrication of any one surface becomes temporarily imperfect the rubbing occurs only at the other surfaces. Fig. 185 shows another arrangement with loose washers. Here provision is made to introduce oil at the centre of the discs, whence it distributes itself over the rubbing surfaces by centrifugal action.

CHAPTER IX

ROLLING CONTACT

Friction and Wedge Gearing

177. Gearing is a general term for the means of transmitting motion, but it is especially employed to denote the wheels by which motion is transmitted from one shaft to another. The wheels employed for transmitting motion are almost always toothed wheels, but it is convenient to study first the action of toothless rollers, because each kind of toothed wheel is equivalent cinematically to a toothless roller.

In the following chapters the notation and units employed will be as follows :—

r=radius, d=diameter of wheel in inches.
N=number of rotations per minute.
p=pitch in inches. h=height of tooth in inches.
b=width of face of wheel in inches.
t=thickness of tooth in inches.
T=number of teeth.
P=pressure of one wheel on another, measured in direction of motion or along tangent to pitch line, in lbs.
nP=load on one tooth in lbs.
f=safe stress in lbs. per sq. in.
H=horses power transmitted.
V=velocity of pitch line in ft. per sec.

Communication of Motion by Rolling Contact
Constant Velocity Ratio

178. *Parallel shafts.*—Let two accurately turned cylindrical rollers (fig. 186) be keyed on the shafts, of such a size that they are in contact. Then, if one shaft revolves the other must revolve also, unless the resistance to motion,

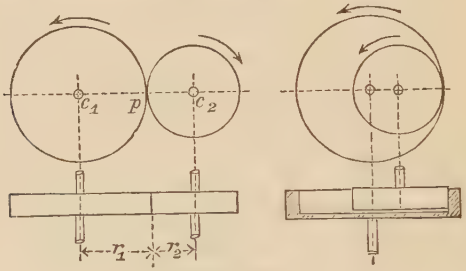


Fig. 186

reckoned at the point of contact, is greater than the frictional resistance to slipping. The contact may be external contact, *a*, or internal contact, *b*. External contact is more common in ordinary cases. If there is no slipping, the two rollers must have the same velocities at the point of contact. Let N_1, N_2, be the speed of the shafts in revs. per min., r_1, r_2, the radii, d_1, d_2, the diameters of the rollers. The velocities at the point of contact are $2\pi r_1 N_1$ and $2\pi r_2 N_2$. Since these are equal

$$\frac{N_1}{N_2} = \frac{r_2}{r_1} = \frac{d_2}{d_1} \quad . \quad . \quad . \quad . \quad (1)$$

and the velocity ratio is constant. Toothed wheels corresponding to rollers of this kind are called spur wheels. The surfaces of the rollers are termed *pitch surfaces*. Planes normal to the shafts intersect these surfaces in circles termed *pitch lines*. The point of contact, p, is the *pitch point*.

Shafts the directions of which intersect.—If two conical rollers are placed on the shafts (fig. 187), then one shaft will drive the other with constant velocity ratio, as in the last case, the contact being external or internal. Toothed

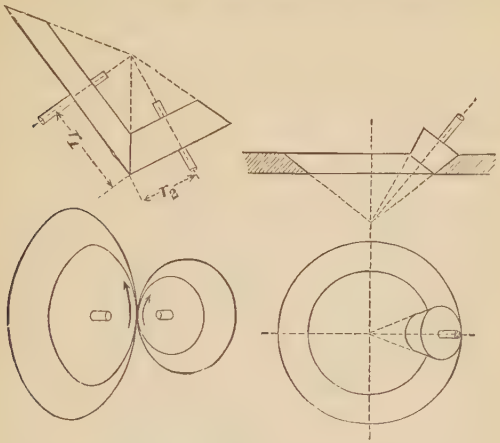


Fig. 187

wheels corresponding to these rollers are termed bevil wheels. In practice the shafts are in most cases at right angles. The radius or diameter of bevil wheels is conventionally measured at the larger end of the cones.

179. *Shafts the directions of which are not parallel and do not intersect.*—Let *o o* (fig. 188) be a vertical axis. Let *a b* (in plan *a'b'*) be a line inclined to the axis and not in the same plane. If *a b* revolves round the axis *o o*, it will describe the hyperboloid of revolution A B C D, the edges of which in elevation are hyperbolas A E C and B F D having vertices at E and F. *c d* is a projection in elevation of a second line also represented in plan by *a'b'*, and equally inclined to the axis which by rotation would describe the same hyperboloid Since the point *f'* of *a'b'* describes the circle *f'g'* in plan, the corresponding point in the outline of the solid may be found

by projecting f' to f on $a\,b$ and g' to g on an horizontal through f. If $o'e'$ is the perpendicular from the axis on $a'b'$, the circle through e' is called the gorge circle and corresponds to the section of the solid through the vertices E, F of the hyperbolas. $a\,b$ and $c\,d$ are asymptotes to the hyperbolas A E C and B F D. If through f' a tangent $h'k'$ is drawn to the gorge circle, this is the plan of a second generator passing through f', which lies wholly in the surface of the hyperboloid. Its projection in elevation is $h\,k$.

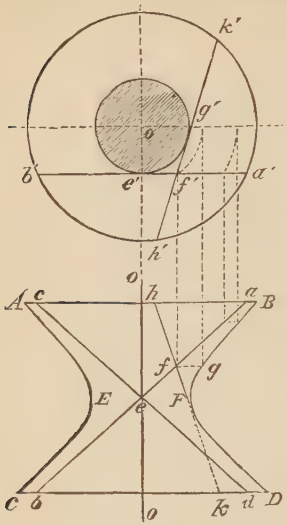


Fig. 188

Two hyperboloids (fig. 189) can be constructed by the rotation successively of the same line round two axes which are not parallel and do not intersect. These two hyperboloids touch always along the straight line, which is the common position of the generator in its two successive rotations. A frustum of one of these hyperboloids will communicate motion to a frustum of the other, but not by pure rolling. There is always sliding in the direction of the common generator $d\,d$. Toothed wheels corresponding to these rollers are called skew bevil wheels.

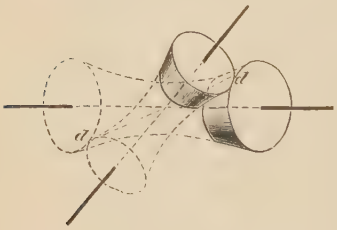


Fig. 189

Let the upper part of fig. 190 be supposed drawn on a plane parallel to the two axes $o\,o$ and $p\,p$, the lower part on

a plane normal to $o\,o$, the left-hand part on a plane normal to $p\,p$. Let $a\,b$ be the common line of contact of the hyperboloids, and $o_1p_1 = o_2p_2$ their common normal at b. Then p_1p_1 and o_2o_2 will be the projections of the axes. Through any point a draw $a\,c\,d$ perpendicular to $a\,b$. Then $c\,d$ is the projection of the common normal to the surfaces at the point of contact a. Projecting d to d_1 and c to c_2, then d_1o_1 and c_2p_2 will be projections of $c\,d$. Projecting a to these

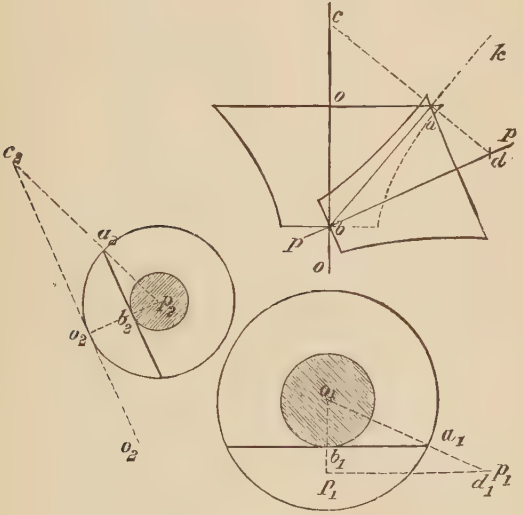


Fig. 190

lines at a_1a_2, these points will be on the circles of rotation passing through a, and p_2a_2, o_1a_1 are the radii of the bases of the hyperboloids. Drawing a_1b_1, a_2b_2 parallel to p_1p_1 and o_2o_2, these will be projections of the line of contact $a\,b$. The gorge circles are drawn touching these lines.

At the common point of contact b, fig. 191, let $b\,v$ be the velocity and direction of motion of the upper hyperboloid and $b\,w$ that of the under hyperboloid. The angular velocity

round $b\,d$ is $b\,v/b\,e$ and that round $b\,c$ is $b\,w/b\,f$. Hence the velocity ratio of the wheels is

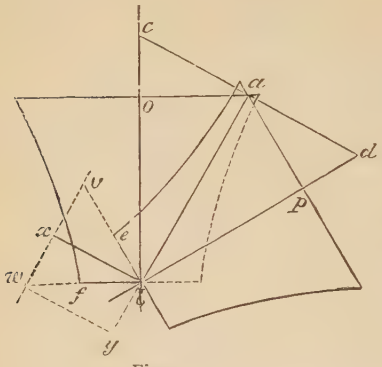


Fig. 191

$$\frac{b\,v}{b\,e}\times\frac{b\,f}{b\,w}=\frac{b\,v}{b\,w}\cdot\frac{b\,f}{b\,e}.$$

If these hyperboloids are to have teeth parallel to the line of contact $a\,b$, their velocities must be so arranged that all the slipping at the point of contact is in the direction of $b\,a$ parallel to the teeth. At right angles to $a\,b$, or normal to the teeth, the velocities must be the same. Hence $b\,v$ and $b\,w$ must have such a relation that they have a common component $b\,x$ at right angles to $b\,a$. But the triangle $b\,v\,w$ is similar to $b\,d\,c$. Hence $b\,v/b\,w=b\,d/b\,c$. And comparing the previous figure with this, $b\,f/b\,e=o_1b_1/b_1p_1=o_1a_1/a_1d_1=c\,a/a\,d$. Substituting these values, the angular velocity ratio

$$\frac{b\,v}{b\,w}\cdot\frac{b\,f}{b\,e}=\frac{b\,d}{b\,c}\cdot\frac{c\,a}{a\,d}=\frac{b\,d}{a\,d}\cdot\frac{c\,a}{b\,c},$$

but $b\,d/a\,d=b\,a/a\,p$, and $c\,a/b\,c=o\,a/a\,b$. Hence the velocity ratio

$$=\frac{b\,d}{a\,d}\cdot\frac{c\,a}{b\,c}=\frac{b\,a}{a\,p}\cdot\frac{o\,a}{a\,b}=\frac{o\,a}{a\,p},$$

consequently the velocity ratio of skew bevil wheels is inversely proportional to the projections of their radii at the point of contact on a plane parallel to both axes.

Suppose the directions of two axes given, their perpendicular distance apart, and their required velocity ratio. Take any two distances in that ratio, and setting them off

from the axes draw lines parallel to the axes intersecting in k, fig. 190. A line from b through k is then the line of contact, and the construction of the hyperboloids by the methods given above is easy.

When hyperboloidal or skew bevil wheels have a comparatively narrow width of face b, fig. 192, a conical surface, tangent to the hyperboloidal surface, may be substituted as a pitch surface for the curved surface. Draw $a'e'$ tangent to the gorge circle. Then $a'e'$ is a plan of one position of the generator whose elevation is found by projecting e' to e and joining A e. Produce A e to m. Then m is the apex of a conical surface B m A tangent to the hyperboloidal surface in the circle A B. If A n is drawn perpendicular to A m, n is the apex of the normal cone or cone which cuts the surface B m A normally in the circle A B.

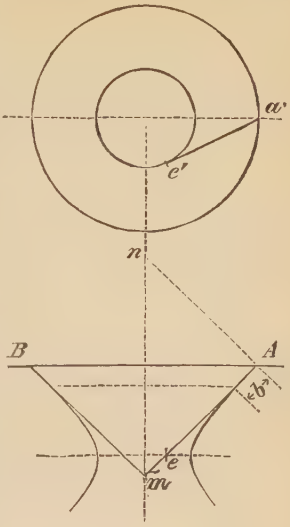


Fig. 192

Velocity Ratio Variable

180. Non-circular rollers may be used to transmit motion with a variable velocity ratio.

Cylindrical wheels with non-circular pitch-lines

Let A A and B B, fig. 193, be two portions of the pitch-lines, $c_1 c_2$ the axes of rotation. Then $c_1 p + c_2 p = d$, the distance between the axes, which is constant. If in a short element of time dt the wheels turn through angles $\psi_1 \psi_2$ and

the points a and b come into contact on $c_1 c_2$, then $c_1 a + c_2 b = d$ as before.

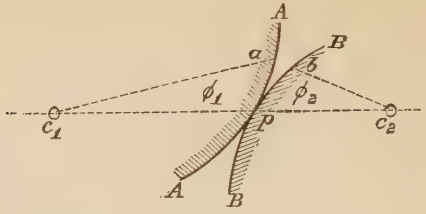


Fig. 193

If the pitch-lines are to roll in contact, then also $p\,a = p\,b$. Let $p\,a_1\,a_2\,a_3$, fig. 194, be a given curve, and let $a_1\,a_2\,a_3$

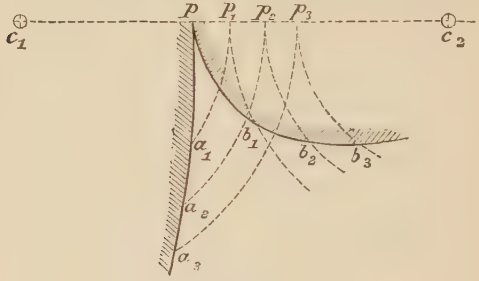


Fig. 194

be points which come successively to $p_1\,p_2\,p_3$. Draw the concentric circles with centre c_2 and passing through $p_1\,p_2\,p_3$;

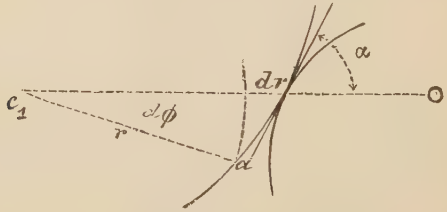


Fig. 195

take arc $p\,b_1$ = arc $p\,a_1$; arc $b_1\,b_2$ = arc $a_1\,a_2$; arc $b_2\,b_3$ = arc $a_2\,a_3$. Then $p\,b_1\,b_2\,b_3$ is a curve which will roll in con-

tact with $p\, a_1\, a_2\, a_3$. Now because the curves always touch at the point of contact, they must have a common tangent at that point. Consequently the tangents to the curve $a_1\, a_2\, a_3$ must make the same angle with the radius to c_1 at any point that the tangent of the corresponding point of $b_1\, b_2\, b_3$ makes with the radius to c_2. If the tangent makes a constant angle with the radius for one of the curves, it must also make a constant angle with the radius for the other curve. From fig. 195 we have

$$\tan \alpha = \frac{r\, d\phi}{dr}$$

$$d\phi = \frac{\tan \alpha}{r}\, dr$$

$$\phi = \tan \alpha \log_e \frac{r}{r_0}$$

$$\log_e r = \frac{\phi}{\tan \alpha} + \log_e r_0$$

where r_0 is the value of the radius for $\phi = 0$. The curve is a logarithmic spiral. Hence two logarithmic spirals of equal obliquity will roll together.

Let fig. 196 represent two similar and opposite logarithmic spirals centred at their poles and in contact on the line of centres. These will roll in contact through a limited angle. Usually the greatest and least required velocity ratios will be given, the distance of the centres, and the angles through which the wheels are to turn.

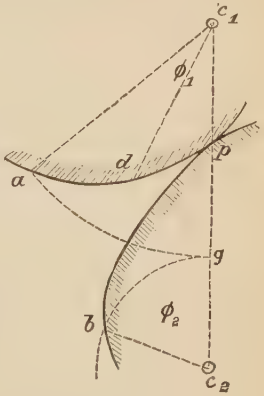


Fig. 196

Divide the line of centres $c_1\, c_2$ in p and g in the required extreme velocity ratios. Let ϕ_1 be the angle through which the upper wheel is to turn while the velocity ratio changes from $c_1\, p / p\, c_2$ to $c_1\, g / g\, c_2$. A circle

through g determines the point a on the curve of the upper wheel at which rolling ends. The angle of obliquity is given by the equation

$$\tan \alpha = \frac{\phi_1}{\log_e \frac{c_1 a}{c_1 p}},$$

and this being known the second spiral can be drawn. A circle through g cuts this in the point b where rolling ends and determines the angle ϕ_2, through which the lower wheel turns. Intermediate points of the curve are easily found thus. A vector which bisects the angle between two other vectors is a mean proportional to those vectors. Thus, if $c_1 d$ bisects $a\, c_1\, p$, then

$$c_1 d = \sqrt{(c_1 a \times c_1 p)}.$$

181. *Lobed wheels.*—By taking alternately reversed logarithmic spirals, lobed wheels are constructed capable of con-

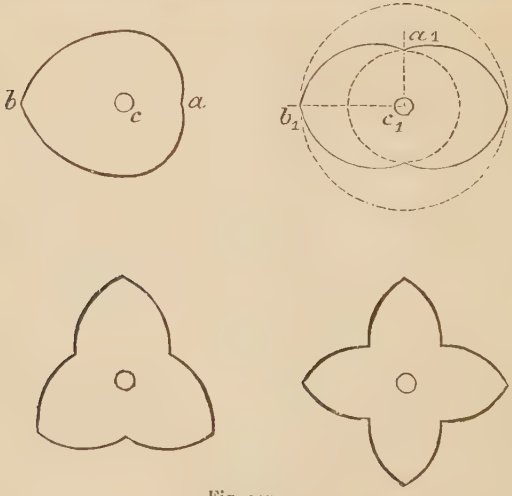


Fig. 197

tinuous rotation and having a series of maximum and minimum velocity ratios.

Equal unilobes or equal multilobes will work together. But also a unilobe will work with a bilobe or trilobe. The condition in this case is this. Suppose the unilobe above is to work with the bilobe. Then the distances $c\,a + c_1\,b_1$, $c\,b + c_1\,a_1$ are at two different times the distances between the centres, and these must be the same. Consequently $c\,b - c\,a = c_1\,b_1 - c_1\,a_1$. Hence, having constructed the spiral and chosen a part of the curve for the unilobe, we have next to find by trial and error a part of the curve such that the lifference of two radii at right angles is equal to $c\,b - c\,a$. This will be the part of the curve to be taken for the bilobe. Similarly for the trilobe the difference of two radii at 60° must be equal to $c\,b - c\,a$.

182. *Elliptical wheels.*—Two equal ellipses centred about two foci and placed in contact so that the distance between the centres is equal to the sum of the focal distances will roll in contact.

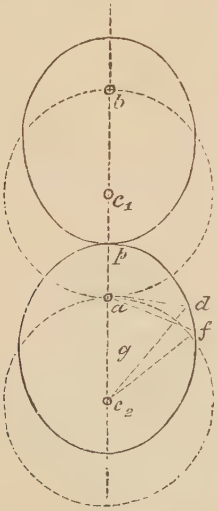


Fig. 198

Let $c_1\,c_2$, fig. 198, be the given distance of centres.

Divide $c_1 c_2$ in p and g so that $c_1 p / p c_2$ and $c_1 g / g c_2$ are the extreme velocity ratios. Obviously it is necessary that $c_1 p / p c_2 = g c_2 / g c_1$. Bisect $c_1 c_2$ in a. Then a is the other focus of the lower ellipse. Take $c_1 b = c_2 a$. Then b is the other focus of the upper ellipse. Any point d of the ellipse is found by taking $a d + d c_2 = c_1 c_2$. Thus, if $a f = f c_2 = \frac{1}{2} c_1 c_2$, then f is the extremity of the minor axis. The two free foci $a b$ remain at a constant distance apart during rotation, so that they are sometimes connected by a link.

If we put $e = c_1 c_2$ and a and b for the major and minor semi-axes of the ellipse, then the radii from the centres of rotation to the point of contact vary from

$$r_{\max} = a + \frac{e}{2}$$

to

$$r_{\min} = a - \frac{e}{2}.$$

Hence the velocity ratio varies from

$$\left(a + \frac{e}{2}\right) \Big/ \left(a - \frac{e}{2}\right) \text{ to } \left(a - \frac{e}{2}\right) \Big/ \left(a + \frac{e}{2}\right).$$

The ratio of the greatest and least velocity ratios is

$$\rho = \left(\frac{a + \frac{e}{2}}{a - \frac{e}{2}} \right)^2.$$

and $\frac{e}{2} = \sqrt{(a^2 - b^2)}$.

Consequently

$$\frac{b}{a} = \frac{2 \sqrt[4]{\rho}}{1 + \sqrt{\rho}}.$$

An equation which gives the ratio of the semi-major and minor axes for a given range of velocity ratio.

Friction Gearing

183. If two wheels of any of the forms just described are pressed together by a force acting normally to the surfaces at the line of contact, there is a frictional resistance to the slipping of one wheel on the other.

Hence, if one wheel is rotated, the other will rotate also, provided the resistance to motion, measured at the pitch surfaces, is less than the frictional resistance to slipping.

With toothless rollers it is difficult, however, to transmit much force without causing a slipping of the rollers, and in consequence noise and wear. The rollers are sometimes covered with leather or india-rubber, which compress a little and so neutralise inaccuracy of form, and have also a greater coefficient of friction. But such arrangements are little used. Cutting wedge-shaped grooves, as will be described presently, also reduces the liability to slip.

Fig. 199 shows simple friction gearing of this kind. The wheels at A are for parallel shafts, those at B for shafts at right angles. In both these cases, if the arrangements are perfect there is simple rolling contact of the wheels. Such wheels may be used (*a*) when the power to be transmitted is not very great ; (*b*) when the speed is so high that toothed wheels would be noisy ; (*c*) when the shafts require to be frequently put into or out of gear. At C, fig. 199, is shown another form of friction gearing, which has been used with a different object. Here the wheels take the form of discs, and the shafts are at right angles. By moving the

smaller wheel towards or away from the axis of the larger wheel the velocity ratio is varied, and that while the gear is in motion and without any abrupt change. As, however, the smaller wheel must have a sensible thickness, its edge is in contact with parts of the larger disc having different velocities. Consequently there must be sliding and wear of the surfaces in contact. To avoid this, the arrangement shown at D has been invented by Prof. James Thomson. *a* is a disc, *b* a heavy metal sphere, and *c* a cylinder. The

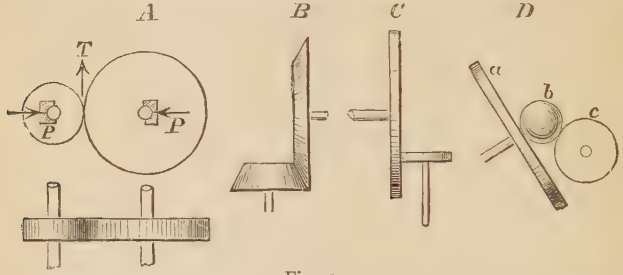


Fig. 199

disc rotates the sphere by friction at the point of contact, and this in turn communicates motion to the cylinder. The contact is simple rolling contact. By moving the sphere across the face of the disc, the velocity ratio is altered.

Let P be the pressure acting between two friction wheels normally to the surfaces at the line of contact; μ the coefficient of friction; T the tangential resistance to motion of the driven wheel, at the pitch surface. Then

$$\mu P = T$$

$$P = \frac{T}{\mu}$$

Let V be the velocity of the pitch surfaces in feet per second, H the horses power transmitted. Then

$$T = \frac{550\ H}{V}$$

$$P = \frac{550\ H}{\mu V}.$$

This gives the magnitude of the external force which must be applied to prevent slipping. We may take for the coefficient of friction, the surfaces being dry,

For metal on metal . .	0·15 to 0·20
„ wood on metal . .	0·25 to 0·30
„ millboard on metal . .	0·20.

In the case of bevil wheels, B, fig. 199, the pressures acting on each wheel may be resolved into a force Q parallel and a force N normal to the shaft. Putting N_1 Q_1 for the forces acting on one wheel, and N_2 Q_2 for those acting on the other ; P for the normal pressure at the pitch surfaces of the wheels ; r_1, r_2, for the radii of the wheels ; and δ for the angle the tangent of which is $\frac{r_1}{r_2}$; we get

$$Q_1 = P \sin \delta = N_2$$
$$Q_2 = P \cos \delta = N_1$$

in which equations P has the value given above, and the friction of the supports of the shafts is neglected. Q_1, Q_2, are the external forces which must be applied along the shafts to prevent the slipping of the wheels.

Friction wheels may be of metal, but generally one of the pair has a surface of wood, of millboard, or of leather, to secure greater resistance to slipping.

Fig. 200 shows the construction of bevil and spur wheels with wood faces ; and millboard may be also used in the same way. It is best to make the wheel with wood or millboard face the driver, as it is then less liable to wear irregularly if slipping occurs. It appears that friction gearing of this kind is frequently used in America, especially for driving sawing machinery. From details given by Mr. Wicklin [1] it appears that it is the practice to make the width of face of the wheels equal to the width of a single leather belt to transmit the same power at the same speed, or that

[1] Cooper, 'Use of Belting,' p. 288.

the tangential force t per inch width of face is about 30 lbs. for maple wood, and about 15 to 20 lbs. for pine or other soft wood. Hence if β is the width of face,

$$T = t\beta = \frac{550\ H}{V}$$

where t has the values given above. The pressure between the wheels must be 3T to 4T. In the case of a wheel with millboard face, the force transmitted was observed by the author to be much greater, amounting to about 80 lbs. per inch width of face.

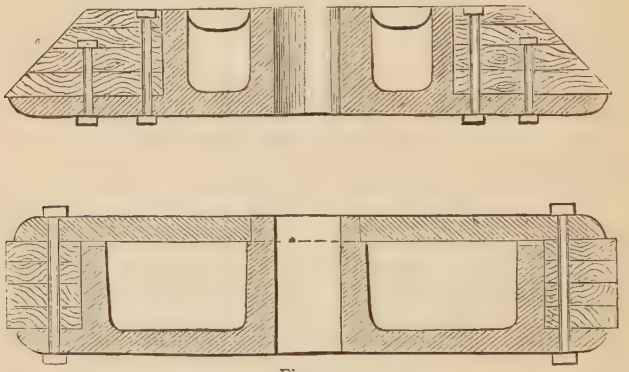


Fig. 200

Since a very slight movement puts friction wheels out of gear, they are convenient when rapid disconnecting is necessary. Fig. 201 shows an arrangement for disconnecting. The end of the shaft of the driven wheel is carried in an eccentric disc, a slight rotation of which puts the wheels into or out of gear.

A lever and weight is shown for producing the pressure necessary to prevent slipping. This is so arranged as to move both ends of the shaft simultaneously.[1]

[1] Fig. 201 is a diagrammatic sketch of the gearing of a crab invented by Mr. H. Lewis of the firm of Lewis and Lewis, and erected at the Great Eastern Railway Goods Station.

184. *Wedge gearing* (fig. 202) is a modification of friction gearing intended to secure a given resistance to slipping with less pressure at the pitch surfaces, and therefore with less wear of the supports of the wheels than ordinary friction gear. The circumferences of the wheels are cut into wedge-shaped projections by circumferential grooves. Let N be the normal pressure on all the wedge surfaces in contact,

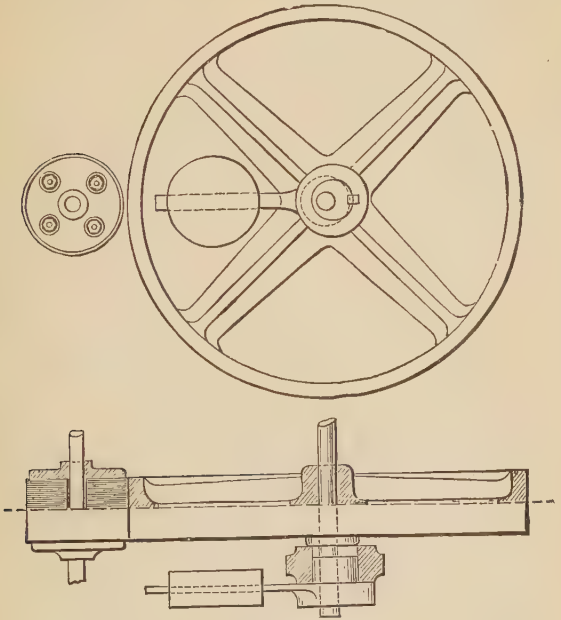


Fig. 201

P the force pressing the wheels together, T the tangential force transmitted—

$$P = N (\sin \alpha + \mu \cos \alpha)$$

where 2α is the inclination of the sides of the wedges

$$\mu N \gtrless T$$

$$P \gtrless \frac{T}{\mu} (\sin \alpha + \mu \cos \alpha)$$

The objection to these wheels is that the contact is sliding contact, and therefore the wheels grind, and in some cases are noisy. To diminish this evil, the depth of the surface in contact should be made small. The inclination of the sides of the wedge projections is usually 30° to 40°. The number of projections on each wheel is usually 1 to 6, but sometimes a greater number are used. The number of projections has no influence on the power transmitted, but only on the durability. The depth t of the acting surface may be taken about

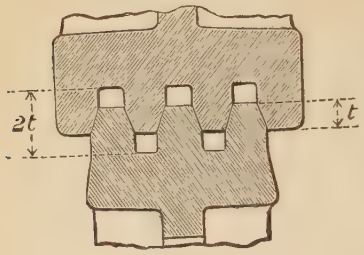


Fig. 202

$$t = 0{\cdot}025\sqrt{T}.$$

The rim and arms may be of the same strength as those of a spur wheel transmitting the same power.

An interesting record of experience with large wedge friction wheels transmitting a circumferential effort of nearly 4,000 lbs. will be found in 'Proc. Inst. Mechanical Engineers,' 1888, p. 363. The most serious evil was found to be a tendency to wear at the soft spots, leading to slipping.

Some extremely ingenious arrangements of friction gear were invented by the late Prof. Fleeming Jenkin and termed *nest gearing*. (See 'Report of the British Association,' 1883, p. 387.) Practical experience, however, seems to have shown that in all these forms of gear, where motion is communicated by frictional contact at a line or point of contact, slipping occurs and the wear and disintegration of the surfaces prove to be a very serious evil.

CHAPTER X

SLIDING CONTACT. TOOTHED GEARING

185. A pair of well-formed toothed wheels have a relative motion identical with that of a pair of friction wheels, rolling in contact. The surfaces of the friction wheels correspond to surfaces in the toothed wheels, which are termed *pitch surfaces*. Hence, if r_1 r_2 are the radii, d_1 d_2 the diameters measured to the pitch surfaces, N_1 N_2 the rotations per minute,

$$\frac{N_1}{N_2} = \frac{r_2}{r_1} = \frac{d_2}{d_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In designing the wheels the pitch surfaces are first drawn and the teeth are constructed with reference to the pitch surfaces, so that the teeth on one wheel fall into the spaces on the other, a small clearance being allowed to provide for inaccuracy in construction.

The importance of forming the teeth so that the velocity ratio is constant is very great. It is not only that an irregular motion is injurious to the machinery driven, but since the inertia of the heavy moving parts resists alteration of velocity, the driven wheel will alternately fall back and overtake the driving wheel, the gearing working with noise and vibration. This action is called 'back lash.'

The greater the clearance and the less the care taken in forming the teeth, and the greater the speed of the pitch line, the more violent is the back lash. The limiting speeds of pitch line for different classes of gearing are about as follows :—

	Speed of pitch line in feet per second.
Ordinary cast-iron gearing . . .	30
Helical or mortice wheels	40
Machine-cut wheels	50

Large moulded spur-wheels and pinions with iron teeth have sometimes a pitch-line speed of 42 feet per second, and when they are exceptionally good and well fitted run smoothly and noiselessly.

186. *Material of gearing.*—Ordinary gearing is made of cast iron. Where great strength is required cast-steel wheels are used. Gearing subjected to shock is often of gun-metal, phosphor bronze, or Delta metal, materials which possess greater toughness than cast iron. When the pitch-line speed is great, one wheel of the pair has wood cogs to diminish noise.

Pattern and Machine-moulded Wheels

Formerly all toothed wheels were moulded from a complete wood pattern, and as this was expensive to make, it was stored and used over and over again. Such patterns almost always become distorted from shrinkage, and then the wheels cast from them work badly. About 1860 Messrs. Jackson, of Manchester, introduced a wheel-moulding machine. A pattern of two or three teeth only is constructed and fixed to a radial arm, which can be rotated accurately through any required fraction of the circumference of the wheel. The whole toothed rim is thus moulded in successive portions. The spaces between the arms are then cored out by dry sand cores moulded in core boxes. Machine-moulded wheels are superior to pattern-moulded wheels if the wheel-moulding machine is kept in good order and not allowed to become inaccurate by wear. In pattern-moulded spur wheels a slight taper or draught is given to the teeth to facilitate withdrawing the pattern from the sand. The wheels should be placed in gear with the draught of the

teeth in opposite directions, but this is not always attended to. Then the teeth bear chiefly on one side. Machine-moulded wheels do not require this taper. Machine moulding has facilitated the production of accurate helical and screw gearing.

Machine-cut wheels.—For a long time small wheels have been cast with a blank rim and the tooth spaces cut out by a milling tool or cutting tool in a wheel-cutting machine. Very much greater accuracy of pitch is secured than when the teeth are moulded, and if care is taken a better tooth form also. Recently the production of accurate machine-cut wheels of all sizes, up to the largest, has been greatly extended in the United States. Sometimes milling cutters of the form of the tooth space are used. At other times automatic machines are employed to cut accurate cycloidal or involute templets of the tooth form, and these are then employed as guides of a milling tool or cutting tool in a wheel-cutting machine. Such wheels are altogether superior to ordinary wheels, and by adopting a system, the number of cutters and templets required is diminished. For particulars reference may be made to the valuable treatise of Prof. MacCord on 'The Kinematics of Mechanical Movements.'

In the case of heavy spur flywheels with rims of large section, it is not a good plan to cast the teeth on the rim. The teeth warp in cooling, owing to the contraction of the large mass of metal in the rim, and are often spongy and weak at the roots. The toothed rim should be moulded by machine and cast as a solid ring or with parting plates. The ends of the segments so formed may then be planed and fitted with distance pieces to insure accuracy of pitch. Both the outer circumference of the flywheel and the inner circumference of the ring should be turned to fit. (See Longridge, 'Report of Boiler Insurance Association,' 1883, p. 18.)

187. *Mortice wheels.*—When wheels are run at high

velocities, the teeth of one of each pair are of wood, and are termed 'cogs.' These cogs are morticed into an iron rim, and shaped by hand. The iron wheel, which works with a mortice wheel, is usually 'pitched and trimmed;' that is, the rough surface of the teeth is chipped away, and the teeth are filed perfectly smooth. This insures greater accuracy in the form of the teeth, and prevents the destruction of the wood cogs by the rough surface of the casting. With machine-moulded wheels, it is only necessary to clear off the sand from the surfaces of the teeth and to file them smooth. No chipping is necessary.

As the wood cogs are of a weaker material than iron, they are usually of greater thickness on the pitch line than the iron teeth working with them.

188. *Relation between the number of teeth and the diameter of the wheel. Circular and diametral pitch.*—The distance measured *along the pitch line* from the centre of one tooth to the centre of the next tooth is the *pitch* or circular pitch. A length which is the same fraction of the diameter as the pitch is of the circumference is the *diametral pitch.*

Let p be the circular pitch, s the diametral pitch, d the diameter of pitch line, T the number of teeth. Then, since p T must be the circumference of the pitch line,

$$\left.\begin{aligned} d &= \frac{p}{\pi}\,\mathrm{T} = 0{\cdot}3183\,p\,\mathrm{T} \\ \mathrm{T} &= \frac{\pi}{p}\,d = 3{\cdot}1416\,d/p \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (2)$$

$$s = p/\pi = 0{\cdot}3183\,p$$

$$\left.\begin{aligned} d &= \mathrm{T}\,s \\ \mathrm{T} &= d/s \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

Values of p/π and π/p are given in the following table to facilitate calculation. It will be seen that the relations (2*a*) in terms of the diametral pitch are simpler than the relations (2) in terms of the circular pitch. Further, the diameters of wheels will be rational numbers if a series of simple

Table giving Diameter or Number of Teeth of Wheels in Terms of the Pitch

Pitches in inches	$\frac{\pi}{p}$	$\frac{p}{\pi}$
$\frac{1}{2}$	6·2832	0·1592
$\frac{5}{8}$	5·0266	0·1989
$\frac{3}{4}$	4·1888	0·2387
$\frac{7}{8}$	3·5904	0·2786
1	3·1416	0·3183
$1\frac{1}{8}$	2·7926	0·3581
$1\frac{1}{4}$	2·5132	0·3979
$1\frac{3}{8}$	2·2848	0·4377
$1\frac{1}{2}$	2·0944	0·4775
$1\frac{3}{4}$	1·7952	0·5570
2	1·5708	0·6366
$2\frac{1}{4}$	1·3963	0·7162
$2\frac{1}{2}$	1·2566	0·7958
$2\frac{3}{4}$	1·1424	0·8754
3	1·0472	0·9549
$3\frac{1}{4}$	0·9668	1·0345
$3\frac{1}{2}$	0·8976	1·1141
$3\frac{3}{4}$	0·8377	1·1936
4	0·7854	1·2732
$4\frac{1}{4}$	0·7392	1·3528
$4\frac{1}{2}$	0·6981	1·4324
$4\frac{3}{4}$	0·6615	1·5120
5	0·6283	1·5916
$5\frac{1}{2}$	0·5711	1·7507
6	0·5236	1·9099
$6\frac{1}{2}$	0·4833	2·0691
7	0·4488	2·2283
$7\frac{1}{2}$	0·4188	2·3875
8	0·3927	2·5465
9	0·3491	2·8647
10	0·3142	3·1829
11	0·2856	3·5014
12	0·2618	3·8200

values are chosen for the diametral pitches instead of for the circular pitches. Hence the diametral pitch is more convenient than the circular pitch in forming an arbitrary system of sizes of wheels. The number of teeth per inch diameter of wheel is $1/s$. Messrs. Browne and Sharpe, of Providence, term s the diameter pitch and its reciprocal $1/s$ the diametral pitch. This is confusing. The quantity $1/s$ will here be termed the 'pitch number.'

189. *To lay off the pitch on the pitch line.*—The following construction, due to Rankine, is convenient, when the wheel is so large that it is impossible to find the exact pitch, by

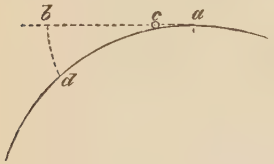


Fig. 203

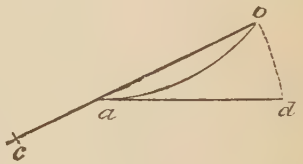


Fig. 204

stepping round the pitch line. Let the circle, fig. 203, be the pitch line. At any point, a, draw the tangent $a\,b$. Make $a\,b =$ the pitch. Take $a\,c = \frac{1}{4}\,a\,b$. With centre c, and radius $c\,b$, draw the arc $b\,d$. Then the arc $a\,d$ is $= a\,b$, and is the pitch laid off on the pitch line. When the wheel has many teeth the arc $a\,d$ sensibly coincides with its chord, but, if it has few teeth, there is an appreciable error in taking the chord $a\,d$ equal to the pitch.

The corresponding approximate construction for rectifying a circular arc is shown in fig. 204. Let $a\,b$ be a circular arc, $a\,d$ its tangent. Draw the chord $b\,a$ and produce it. Take $a\,c = \frac{1}{3}\,a\,b$. With centre c describe the arc $b\,d$. Then $a\,d =$ arc $a\,b$ very nearly.

190. *Parts and proportions of teeth.*—Fig. 205 shows the general form of wheel teeth drawn for convenience on a straight pitch line. $f\,e$ is the pitch (circular pitch) divided

into $f b$, the tooth thickness, and $b e$ the tooth space. $d e$ is the side clearance, which may be very small in machine-cut wheels, and must be larger the less the accuracy of the wheels. $b a$ is the face, and $b c$ the flank of the tooth. $g h$ is the bottom clearance.

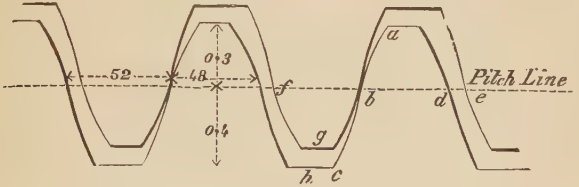


Fig. 205

In working, the faces of the teeth of one wheel come into contact with the flanks of the teeth of the other wheel. Also, while the point of contact is approaching the line of centres, the flank of the driving acts on the face of the driven tooth; while the point of contact is receding from the line of centres, the face of the driving acts on the flank of the driven tooth. The arc of the pitch circle, through which the wheel turns during contact, is called the 'arc of contact,' or 'arc of action,' and the portions into which it is divided by the pitch point are the arcs of approach and recess. The length of the arc of approach depends on the length of face of the driven tooth, because contact begins at the point of the driven tooth and travels to the pitch point; the arc of recess on the length of the face of the driving tooth, because contact travels from the pitch point to the point of the driving tooth.

Fig. 205 shows ordinary proportions of teeth expressed in the simplest numbers. The following proportions, however, are better:—

PATTERN-MOULDED WHEELS

Thickness of tooth . .	$0{\cdot}47\,p - 0{\cdot}02$	to $0{\cdot}48\,p - 0{\cdot}03$
Width of space . . .	$0{\cdot}53\,p + 0{\cdot}02$	to $0{\cdot}52\,p + 0{\cdot}03$
Clearance (side) . . .	$0{\cdot}06\,p + 0{\cdot}04$	to $0{\cdot}04\,p + 0{\cdot}06$
Height above pitch line .	. . $0{\cdot}3\,p$	to $0{\cdot}35\,p$
Depth below ,, ,, .	$0{\cdot}35\,p + 0{\cdot}08$	to $0{\cdot}4\,p + 0{\cdot}08$
Total height of tooth . .	$0{\cdot}65\,p + 0{\cdot}08$	to $0{\cdot}75\,p + 0{\cdot}08$

MACHINE-MOULDED WHEELS

Thickness of tooth . .	$0{\cdot}485\,p - 0{\cdot}03$	
Width of space . . .	$0{\cdot}515\,p + 0{\cdot}03$	
Height above pitch line .	. . $0{\cdot}3\,p$	to $0{\cdot}35\,p$
Depth below ,, ,, .	$0{\cdot}35\,p + 0{\cdot}08$	to $0{\cdot}4\,p + 0{\cdot}08$
Total height of tooth .	$0{\cdot}65\,p + 0{\cdot}08$	to $0{\cdot}75\,p + 0{\cdot}08$

MACHINE-CUT WHEELS

(*Messrs. Browne and Sharpe.*)

Thickness of tooth and width of space .	$0{\cdot}5\,p$ or $1{\cdot}571\,s$
Height above pitch line	$0{\cdot}318\,p$ or s
Bottom clearance	$p/20$ or $0{\cdot}157\,s$
Total height of tooth	$0{\cdot}686\,p$ or $2{\cdot}157\,s$

Mr. Longridge has lately expressed the opinion that wheel teeth are commonly made of too great height. Perhaps this is so for wheels with many teeth. It will be seen presently that the height of tooth determines the length of the arc of contact, and the essential condition is that the arc of contact should not be less than the pitch. For safety it should probably never be less than $1\frac{1}{2}$ time the pitch.

For mortice wheels the wood cog may be thicker than the iron tooth working with it. Then the following proportions are good:—

Thickness of iron tooth and space of mortice teeth .	$= 0{\cdot}4\,p$
,, wood cog and space of iron teeth . .	$= 0{\cdot}6\,p$

The following table gives Browne and Sharpe's proportions for machine-cut wheels, and is inserted chiefly to show how a convenient series of diametral pitches may be chosen:—

Pitch Number	Diametral pitch	Circular pitch	Thickness of tooth	Height above pitch line	Depth below pitch line	Total height of tooth
$\frac{1}{s}$	s	p	$0{\cdot}5p$	s	$s+f$	$2s+f$
$\frac{1}{2}$	2	6·283	3·142	2·000	2·314	4·314
$\frac{3}{4}$	1·333	4·189	2·094	1·333	1·543	2·876
1	1	3·142	1·571	1·000	1·157	2·157
$1\frac{1}{4}$	0·8	2·513	1·257	·800	·926	1·726
$1\frac{1}{2}$	0·667	2·094	1·047	·667	·771	1·438
$1\frac{3}{4}$	0·571	1·795	·898	·571	·661	1·233
2	0·5	1·571	·785	·500	·578	1·078
$2\frac{1}{4}$	0·444	1·396	·698	·444	·514	·959
$2\frac{1}{2}$	0·4	1·257	·628	·400	·463	·863
$2\frac{3}{4}$	0·364	1·142	·571	·364	·421	·784
3	0·333	1·047	·524	·333	·386	·719
$3\frac{1}{2}$	0·286	·898	·449	·286	·331	·616
4	0·25	·785	·393	·250	·289	·539
5	0·2	·628	·314	·200	·231	·431
6	0·167	·524	·262	·167	·193	·360
7	0·143	·449	·224	·143	·165	·308
8	0·125	·393	·196	·125	·145	·270

The diameter of the wheel is the pitch number multiplied by the number of teeth. As the pitch numbers chosen are simple numbers, the diameter will also generally be a simple number.

Conditions determining the Form of Teeth

191. *Condition of continuous contact of a pair of teeth.*— Let A and B be two spur wheels rotating at any moment with angular velocities $+\omega_1$ and $-\omega_2$. Nothing will be changed in the relative motion of the two wheels, if a rotation $-\omega_1$ is impressed on each. Then A's angular velocity of rotation will be $\omega_1 - \omega_1 = 0$, that is, it will be at rest. The centre of B will rotate about A with the velocity $-\omega_1$, and B will rotate about its own centre with the velocity $-(\omega_1 + \omega_2)$. Hence, the motion of B will be the same as if it were at the moment rotating about an axis placed at a point dividing the line of centres in the ratio ω_1/ω_2, or it will roll on the pitch

line of A. Now let a tooth be fixed to B. In order that that tooth may remain continuously in contact with a tooth on A, the form of the latter must be the envelope of the successive positions of the tooth on B, as it moves round A. The form of the tooth on B is not arbitrary. Only certain forms give to the envelope shapes which are practically realisable as wheel teeth.

If two solids, such as two wheel teeth, move in contact, they must have equal velocities in the direction of their common normal at the point of contact. For if the velocities were not equal, one tooth would be penetrating into the space occupied by the other. But, in the case above, one tooth is at rest. Therefore the other can have no velocity in the direction of the normal to its surface at the point of contact. Hence, the point of contact must always fall in such a position that the normal to the tooth at that point passes through the axis of rotation at the moment, that is, the point which divides the line of centres in the ratio $\omega_1 : \omega_2$, which coincides with what is commonly termed the 'pitch point.'

In ordinary wheels with circular pitch lines any tooth of one wheel may have to gear with any tooth of the other wheel. Hence all the teeth must be formed so as to come into gear at the same point and to remain in action while the wheel turns through the same angle. All the teeth of each wheel should therefore be of the same form.[1]

Condition of constancy of velocity ratio. – To a certain extent this is secured by having the teeth numerous and small. But, in addition, the single and sufficient condition which insures the constancy of the velocity ratio, during the action of each pair of teeth, is this:—The common normal to two teeth at the point of contact must always pass through the pitch point,[2] a condition which is fulfilled if

[1] This is, of course, not true of wheels with non-circular pitch lines, when the same pairs of teeth always gear together.

[2] If the pitch line is not circular, and the common normal to the

one tooth is the envelope of the relative positions of the other.

Influence of the form of the tooth on its strength.—It will be seen presently that the teeth tend to break across at the root. The teeth are stronger the shorter they are, and the thicker they are at the root. They cannot be shortened without reducing the arc of contact, and their length should be such as to insure a sufficient, but not excessive, arc of contact. The thickness at the root depends on the form selected for the teeth. Involute teeth are generally stronger than cycloidal teeth. With cycloidal teeth, the teeth are stronger the smaller the diameter of the describing circle used for the flanks. Seldom should the flanks be described with a rolling circle the diameter of which is greater than half the diameter of the pitch line, inside which it is rolled.

Conditions of durability.—The rolling of the teeth over each other so as to spread the contact over a considerable area of tooth surface is advantageous. But the sliding of one tooth on the other is disadvantageous. The amount of sliding is the difference of the length of the faces of the teeth and the flanks which work with them. By increasing the height of the tooth the amount of sliding is increased. So that in general, to spread the wear over a larger surface, it is preferable to widen the face of the wheel instead of making the height of the teeth greater. It was an old rule to secure durability that the pressure between the teeth should not exceed 400 lbs. per inch of width.

192. *Arcs of approach and recess. Arc of action.*—Let fig. 206 show the teeth of two wheels in the positions at which contact begins and ends. During the first part of the action of two teeth, the flank of the driver acts on the face of the driven tooth, and the point of contact moves towards the pitch point p. During the remainder of the action the

teeth always passes through the pitch point, the wheels have the same varying velocity ratio as smooth rollers, coinciding with the pitch lines.

face of the driving tooth is acting on the flank of the driven tooth, and the point of contact is travelling away from p. Contact begins, therefore, at the point a_1 of the driven tooth, and ends at the point a_2 of the driving tooth. If the tooth contours intersect the pitch lines in b_1, c_1, b_2, c_2, then b_1 and

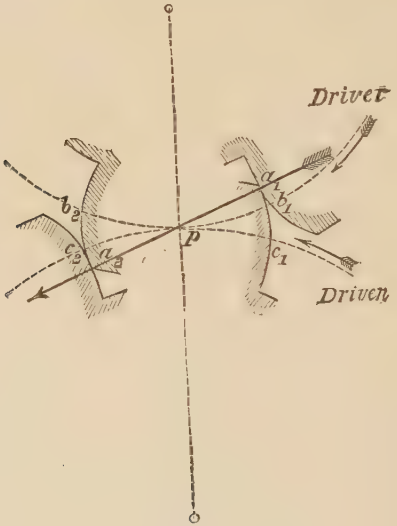


Fig. 206

c_1 come together at p and recede to b_2, c_2; the arcs $b_1 p$, $c_1 p$ are called the arcs of approach, and the arcs $p b_2$, $p c_2$ the arcs of recess. The arcs $b_1, p b_2$, $c_1 p c_2$ are the arcs of action. The arc of approach depends on the length of face of the driven tooth; the arc of recess on the length of face of the driving tooth.

In order that a pair of teeth may not go out of gear before another pair of teeth has come into gear, the pitch (circular) must not exceed either the arc $b_1 p b_2$ or $c_1 p c_2$. Usually these arcs of action are 1·5 to 2 times the pitch.

In fig. 207, which repeats the positions of the teeth shown in the previous figure, join the points of beginning and ending contact $a_1\ a_2$ to the centres of the wheels. Then,

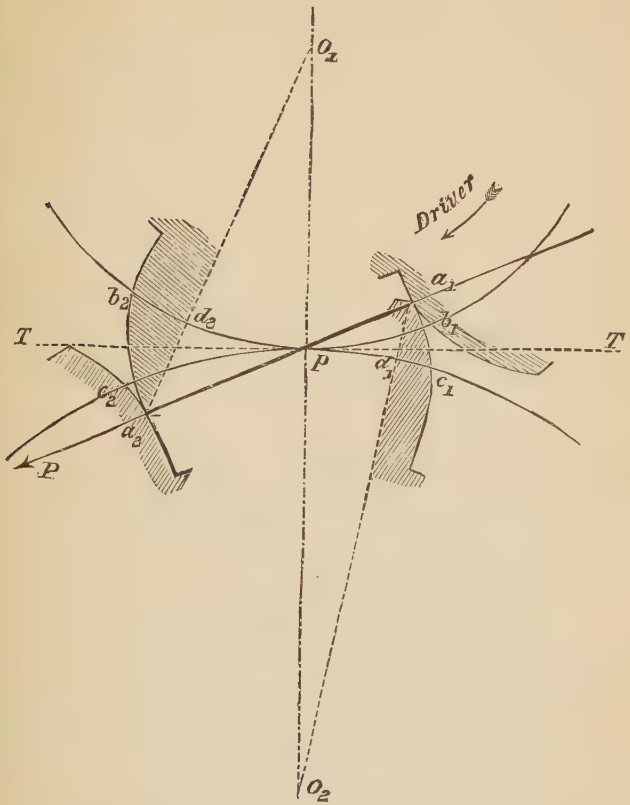


Fig. 207

if $c_1\ d_1$ is one-fourth $c_1\ c_2$, the tooth of the lower wheel is pointed, and any greater pitch is impossible. Similarly, if $b_2\ d_2$ is greater than one-fourth $b_2\ b_1$ the tooth of the upper

wheel is impossible. Hence possible values of the pitch lie between these limits :

$$\text{pitch} < b_1 b_2 \text{ and } c_1 c_2$$
$$> 4\,(c_1 d_2) \text{ and } 4\,(b_2 d_2).$$

193. *Obliquity of action.*—Neglecting the friction of the teeth, the pressure P between them must be in the direction of the normal at the point of contact, which in properly formed teeth always passes through p. The angle which this line makes with the tangent T T to the pitch circles is called the *angle of obliquity of action*. In some forms of teeth this angle has a constant value during the contact of a pair of teeth. Its value is then commonly $15\frac{1}{2}°$. With other forms of teeth this angle varies, its greatest values $a_1 p$ T, and $a_2 p$ T, occurring at the moment of beginning or ending contact (fig. 207). These values are then usually limited to 30°. Professor Kennedy[1] has pointed out that, if friction is taken into account, P is more inclined to the tangent during approach and less during recess, and that this is probably the reason for the statement that the action of the wheels is less smooth and the friction greater during approach than during recess. For ordinary gearing, however, this difference may be neglected. Its importance has probably been overrated.

In wheels for clockwork, where the friction is specially injurious, it is usual in certain cases to design the wheels so that the driving-teeth have no flanks and the driven teeth no faces. Then contact is entirely confined to the period of recess. The arcs of recess must then be at least equal to the pitch.

194. *Given the form of tooth of one wheel, to find the proper form of the tooth of another wheel to gear with it.*—Let $a\,b\,c$, fig. 208, be the given tooth, b B, b' B the pitch lines of the wheels, B the pitch point or point of contact of the

[1] 'Mechanics of Machinery,' p. 605.

pitch lines. From any points, $a\,c$, draw normals, $a\,\alpha$, $c\,\gamma$, to the curve of the given tooth, cutting the pitch line in α, γ. Then the points $a\,b\,c$ should be points of contact, when α, b, γ, are at the pitch point. From B set off B A $= a\,\alpha$, B C $= c\,\gamma$. Then A is the point where a is in contact; B the point where b is in contact, and C the point where c is in contact, and some line, passing through A B C, is the path of contact. Through A, C, draw circles A a', C c'. Set off arc B a' = arc B α; also $a'\,a' = \alpha\,a$. Then a' is a point in the tooth of the second wheel, which will come in contact with a at A, and will have a common normal, passing through the pitch point. Set off arc B b' = arc B b; then b' will come in contact with b at B. Also, set off arc B γ' = arc B γ, and $\gamma'\,c' = \gamma\,c$; then c' will come in contact with c at C. A curve $a'\,b'\,c'$ through the points so found will be the required tooth. In certain cases the construction becomes impossible, and the given tooth is of unsuitable form. Forms should be avoided which make a tooth entirely concave.

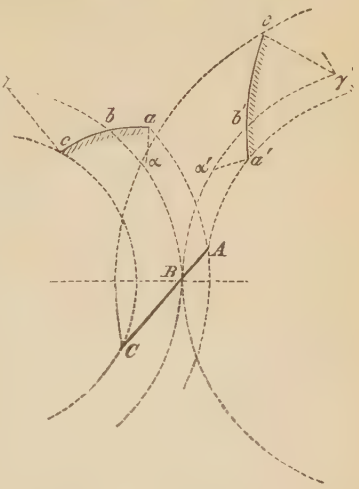


Fig. 208

Mr. Last's method.—A very ingenious method of solving the problem of finding the form of a tooth to work with a given tooth is due to Mr. Last.[1] Let fig. 209 represent a rubbing taken from an existing wheel. Choose on this the most probable pitch circle, B C. On a piece of tracing-

[1] 'Proc. Inst. Civil Engineers,' vol. lxxxix. p. 341.

paper is described the pitch circle X Y of the wheel which is to gear with the given wheel. Placing the tracing-paper so that the pitch lines touch at p, the needle point of a pair of bow compasses is placed at p and the bows opened till they will describe a circle touching the outline of the given adjacent tooth. A small arc is struck on the tracing-paper. Without removing the bow compass point the tracing-paper is rotated a little till the circle X Y cuts the circle B C at a

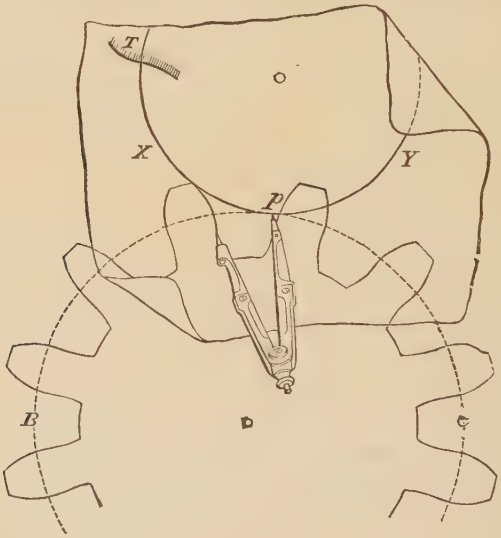


Fig. 209

point near p. The compass point is removed to this point and the tracing-paper is then a little further rotated till the two circles apparently touch. The whole operation does not sensibly differ from rolling the circle X Y on B C. From this new centre a new small arc is struck touching the outline of the tooth. The operation is repeated as often as necessary. Thus the whole outline, shown at T, of the new tooth is obtained, the flank by arcs touching the face of the

old tooth, the face by arcs touching the flank of the old tooth.

Cycloidal Teeth

195. If a circle rolls on a straight line, a tracing point on its circumference describes a *cycloid*. If the circle rolls outside another circle the tracing point describes an *epicycloid*. If the circle rolls inside another circle, the tracing point describes a *hypocycloid*. If for any of these curves a line is drawn, from the tracing point at any moment, to the touching point of the rolling circle and base line (or circle) at that moment, that line is the normal to the curve.

Cycloidal teeth with external contact.—Consider the two pitch lines and the upper rolling circle, shown touching at the pitch point p, in fig. 210. Suppose these three circles on three different planes turning about the centres, c_1, o, c_2, of the circles so as to roll in contact at p. Then a tracing point attached to the rolling circle would describe the arc $p\,a$, in space. But at the same time, it would mark out on the plane of the lower pitch line an epicycloid $b\,a$ and on the plane of the upper pitch line a hypocycloid $d\,a$. These two curves, generated simultaneously, by the tracing point attached to the rolling circle at a, must be in contact at a and have always a common normal $a\,p$, which passes through the pitch point p. Consequently they are suitable curves for the face of the tooth of the lower wheel and the flank of the tooth of the upper wheel, which act together as the point of contact moves from p to a. The point of contact of the two curves is always on the circumference of the rolling circle. Consequently $p\,a$ is the path of contact. But b and d are the points of the curves which were in contact at p. Hence the arcs, $p\,a$, $p\,d$, $p\,b$, are corresponding distances rolled ; these arcs are equal. The difference of length of $a\,d$ and $a\,b$ is the whole amount of sliding of the teeth in the same time. To complete the teeth, the faces of the teeth on the upper pitch line

and the flanks on the lower must be described by another rolling circle.

As in § 192, contact begins at the point a' of the driven tooth, and during approach the point of contact moves over the arc $a'\ p$ of the rolling circle. Contact ends at the point a of the driving tooth, and during recess the point of contact

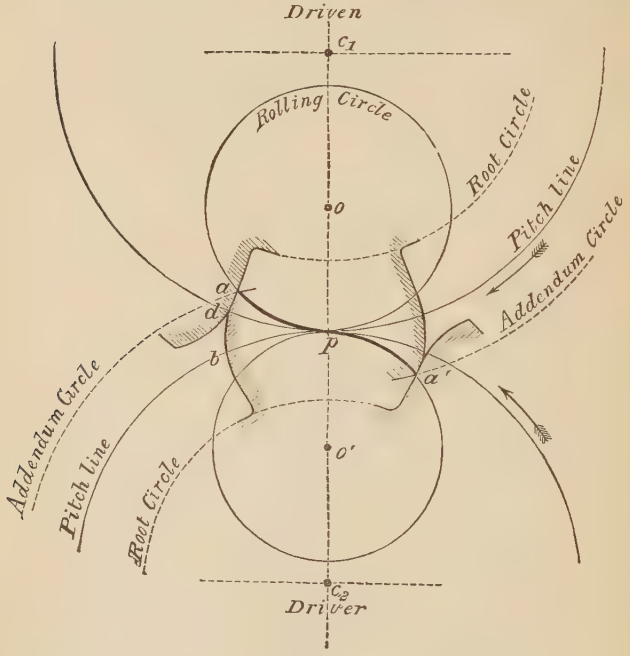


Fig. 210

moves over the arc $p\ a$ of the rolling circle. The whole path of contact is the double arc $a'\ p\ a$. The arcs of recess are the arcs $p\ d$, $p\ b$ of the pitch lines, and each of these is equal to the arc $p\ a$ of the rolling circle. Similarly the arcs of approach on the pitch lines are each equal to $p\ a'$.

The obliquity of action of a pair of cycloidal teeth is

the angle which $a' p$ makes with the tangent at p to the pitch lines when contact begins. It decreases as the point of contact moves along the arc $a' p$ and becomes zero when the teeth are in contact at p. It then increases again to the angle which $a p$ makes with the tangent. The maximum obliquity in cycloidal gearing is about 30° and its mean value is about half its maximum for any one pair of teeth.

196. *To determine the lengths of cycloidal teeth for given arcs of approach and recess.*— Let A_1, A_2 be the given pitch lines, R_1, R_2 the given rolling circles. Set off $b_1 p$, $p b_2$, the

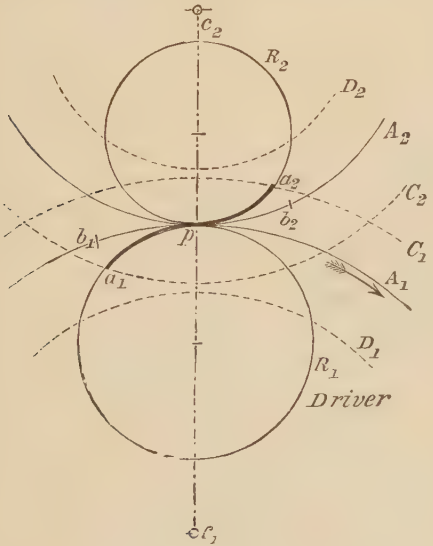


Fig. 211

given arcs of approach and recess. Step off arc $p a_1 = p b_1$, and arc $p a_2 = p b_2$. Then, $a_1 p a_2$ is the required path of contact. Contact begins at a_1 and ends at a_2. Circles c_1 c_2, through a_2 a_1 are the required addendum circles. The root circles D_1 D_2 must be taken so that the points of the

teeth of each wheel clear the other wheel, or allow the given amount of bottom clearance (see § 190).

197. *Internal cycloidal teeth.*—Let A_1, A_2 (fig. 212) be the pitch lines, and p the pitch point; $a_1 c_1$ the tooth belonging to A_1, and $a_2 c_2$ the tooth belonging to A_2. The flank $p c_1$ works on the face $p a_2$; both are epicycloids described by the rolling circle R, rolling outside A_1 and A_2. The face $p a_1$ and the flank $p c_2$ work together, and are hypocycloids described by R′ rolling inside A_1 and A_2. Through a_1 draw an arc $a_1 x$ concentric with A_1, and through a_2 an arc $a_2 y$ concentric with A_2, cutting the rolling circles in x and y, then $x p y$ is the path of contact. As before, lengths equal to the arcs $x p$, $p y$ set off along the pitch lines will be the arcs of approach and recess.

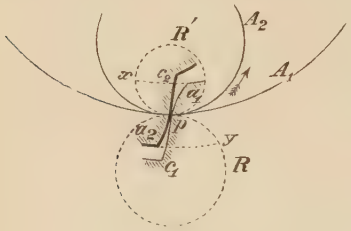


Fig. 212

198. *Choice of the diameter of the rolling circle.*—The diameter of the rolling circle is not usually greater than the radius of the pitch circle, inside which it is rolled. When it is equal to the radius, the flanks of the teeth become radial straight lines. Teeth with radial flanks were at one time much used. Two rolling circles were then taken for each pair of wheels, the diameters of which were the radii of the wheels. When a pair of wheels only are required, the rolling circle, for both faces and flanks of both wheels, may be made with convenience equal in *diameter* to the *radius* of the smaller wheel.

If the rolling circle is taken too large, the teeth are weak at the roots, but no great inconvenience arises if the diameter of the rolling circle does not exceed $1\frac{1}{4}$ times the radius of the pitch line inside which it is rolled.

When a set of wheels have to be constructed any two of

which are to work together the same rolling circle must be taken for both faces and flanks of all the wheels of the set. The diameter of the rolling circle may then be the radius of the smallest wheel of the set. Frequently for sets of wheels, the diameter of the rolling circle is equal to the radius of a wheel of fifteen teeth. Let p be the pitch, T the number of teeth in a wheel the *radius* of which is d, so that d may be taken as the *diameter* of the largest suitable rolling circle. Then:

$$d = \frac{T\,p}{2\,\pi} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

No. of teeth T=	Diam. of rolling circle d=
11	1·751 p
12	1·910 p
13	2·068 p
14	2·228 p
15	2·387 p
16	2·546 p
20	3·183 p
25	3·981 p

Since the introduction of machine-moulding, it is less an object than it used to be to make wheels in sets.

Fig. 213 shows the influence of the size of the rolling circle on the form of the teeth. Let R be the radius of the wheel, r the radius of the describing circle. The tooth curve 1 p 1 is described with rolling circles of radius $r = \frac{1}{4}$ R, and $x'\,y'$ is the corresponding path of contact. The tooth curve 2 p 2 is described with $r = \frac{1}{2}$ R, the flank of the tooth is radial, and $x''\,p\,y''$ is the path of contact. The tooth 3 p 3 is described with $r = \frac{1}{8}$ R, and $x'''\,p\,y'''$ is the path of contact. It will be seen that the smaller the rolling circle, the stronger the tooth is at the root. On the other hand, the smaller the rolling circle, the shorter is the path of contact and the greater the obliquity of action for a given length of tooth.

Taking the pitch at 2 ins., which corresponds to the height of tooth shown, the arcs of contact are 1·22, 1·5, and

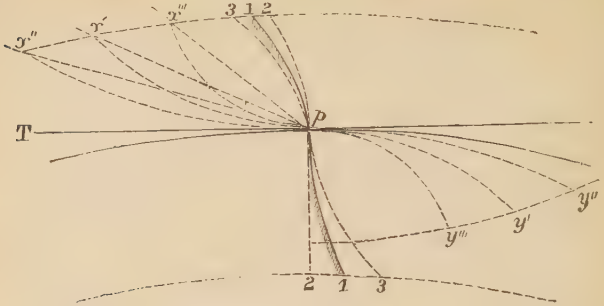


Fig. 213

0·97 times the pitch. The corresponding maximum angles of obliquity are $x'\ p\ T = 24°$; $x'\ p\ T = 15°$ and $x'''\ p\ T = 38°$.

199. *Gee's Patent Gearing.*[1]—Messrs. Jackson, of Manchester, have introduced a peculiar form of tooth, which is

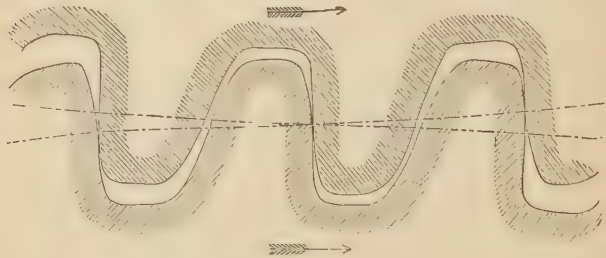


Fig. 214

35 per cent. stronger than the usual form. In this gearing the driving faces of the teeth (fig. 214) are of the usual form. The other faces have much more obliquity than ordinary teeth. Gearing of this kind should therefore be used to

[1] These teeth were described originally in Willis's 'Mechanism,' second edition, p. 142. Willis proposed that the backs of the teeth should be involutes of considerable obliquity.

drive in one direction only. The back faces may be cycloidal curves described with very small describing circles, or a form may be selected for the back of the tooth of one wheel, and the suitable curve for the tooth of the other may be obtained by the process given in § 194.

200. *Methods of drawing cycloidal teeth.*—The curves of the teeth may be found by rolling a templet of the size of the rolling circle inside and outside templets of the size of the pitch circle. A pencil held in contact with the rolling templet describes the required curve. The curves may also be obtained by the ordinary rules for describing cycloidal curves. When they have been drawn, it is usually convenient to replace the cycloidal curves by circular arcs, sensibly coinciding with them, and which can be used by the pattern-maker more conveniently than the true curves. In proceeding thus, two sources of error are introduced. It is not easy to draw small cycloidal arcs very exactly, and in fitting circular arcs to them a new source of error is introduced. To obviate these objections, it was proposed by Professor Willis to find directly the centres of circular arcs which would approximate to the cycloidal arcs. The method of Professor Willis, however, does not give a very good approximation, the teeth being too thin at the points and too thick at the roots.

The following method, founded on Rankine's rules for rectifying circular arcs, gives a much nearer approximation to the true curves, in fact, with teeth of ordinary size, there is no appreciable difference between the cycloidal and circular arcs. The method is also much easier in practice than that of drawing first the true curves. The method is based on this principle. For each cycloidal arc a circular curve is found, which coincides with it at the pitch line, and at $\frac{3}{5}$ its length from the pitch line, and which has at the latter point a common normal with it.

In fig. 215, the strongly marked circle $b\ p\ b'$ is the pitch circle, and p the pitch point. The complete dotted circles

are the rolling circles. The height of the tooth outside the pitch line is $p\,r$, and its depth within it is $p\,s$, so that the circles through r and s are the addendum and root circles.

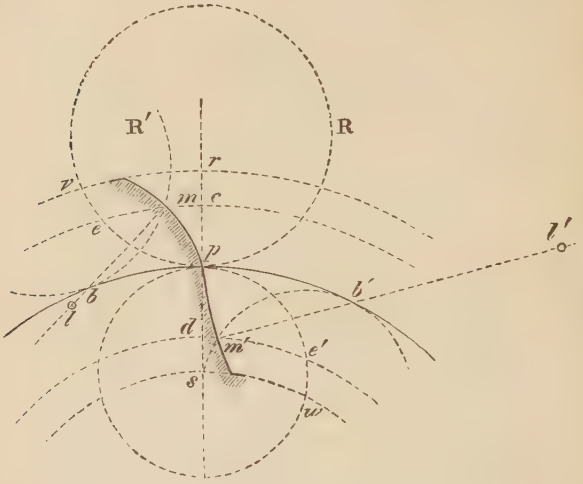


Fig. 215

The arcs $v\,p\,w$ mark the path of contact. As the rolling circle R rolls to the position R′, the tracing-point moves from p to m, marking out the epicycloid $p\,m$, which forms the face of the tooth.

Method 1.—Take $p\,c = \frac{2}{3}\,p\,r$, and draw the arc $c\,e$ concentric with the pitch line. Step off arc $p\,b$ = arc $p\,e$. Take the chord $p\,e$ in the compasses, and with centre b mark off $b\,m = p\,e$; then m is a point of the true epicycloid, and $m\,b$ is the normal to the curve at m. It is then easy to find in $m\,b$, by trial, a centre l for a circular arc, which will pass through m and p. That circular arc will be the required approximation to the epicycloid.

For the flank of the tooth, make $p\,d = \frac{2}{3}\,p\,s$, and draw the arc $d\,e'$. Step off with the compasses the arc $p\,b'$ = arc $p\,e'$.

With centre b' and radius = chord $p\,e'$, cut $d\,e'$ in m'. Then m' is a point in the hypocycloid, and $m'\,b'$ is the normal to the curve at m'. Find, by trial, a centre l' on $m'\,b'$, for a circular arc, passing through m' and p. That arc is the required approximation to the hypocycloid.

Method 2.—The following method (fig. 216) is the same as the last, except that all the points are found by construc-

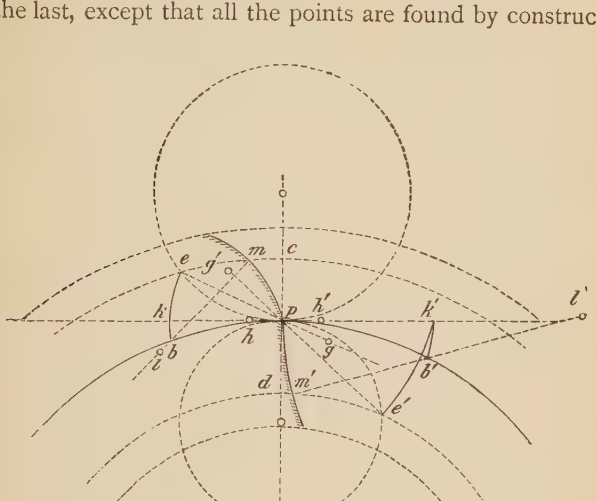


Fig. 216

tion instead of by trial. Take as in the last method, $p\,c = \frac{2}{3}$ of the height of the tooth, outside the pitch line, and $p\,d = \frac{2}{3}$ the depth within the pitch line. Draw the arcs $c\,e$, $d\,e'$ concentric with the pitch line. Through the pitch point p, draw a tangent to the pitch circle. Join $e\,p$, produce it, and make $p\,g = \frac{1}{2}\,p\,e$. With centre g and radius

$g\,e$, describe an arc $e\,\frac{1}{2}$, cutting the tangent at the pitch point in k. Then, $p\,k$=arc $p\,e$. In $p\,k$ take $p\,h=\frac{1}{4}\,p\,k$. From centre h, with radius $h\,k$, describe an arc $k\,b$, cutting the pitch line in b. Then, arc $p\,b$=arc $p.e$. With centre b and radius=chord $p\,e$ cut $c\,e$ in m. Join $m\,b$, and in $m\,b$ find a centre l of a circular arc, passing through m and p. That point l may be found by joining $m\,p$, and drawing a line, bisecting $m\,p$ at right angles. The line so drawn will intersect $m\,b$, produced in l. Then the arc $p\,m$, drawn with centre l and radius $l\,m$, is the required approximation to the epicycloid. The same construction gives the flank of the tooth, and the same description is applicable if accented letters are substituted for unaccented letters.

201. *Method* 3. *Mr. Heys's method.*—The following method, adopted by Mr. Heys of Manchester, gives very good approximations, so far as the author has tested it. Let O A be the line of centres, P the pitch line, and x the pitch point. At x, draw the tangent $x\,g$, and make $x\,g$ = 0·571 of the diameter of the rolling circle. Through g, parallel to O A, draw B C. Make g B = $g\,x$, and g C = diameter of rolling circle. Join O B, O C, and produce O C. Draw $a\,y$, $b\,z$ parallel to $x\,g$, and at a distance from it=$\frac{1}{8}\,x\,g$. Take $b\,z'$ =$b\,z$. Then y is the centre for a circular arc, approximating to the hypocloidal flank of the teeth, and $y\,x$ is its radius. Also, z' is the centre for a circular arc, approxi-

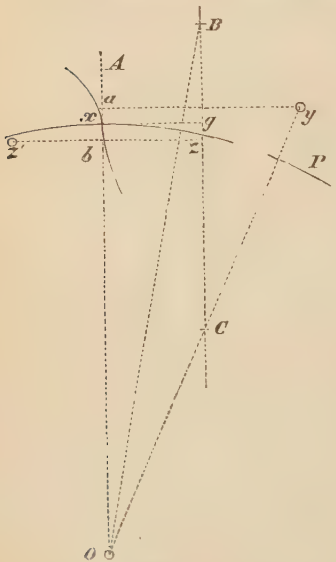


Fig. 217

mating to the face of the tooth, and $z'x$ is its radius. In practice it is accurate enough to take, if p=pitch,

$$xg = gB = 1{\cdot}125p$$
$$BC = 3p$$
$$xa = xb = \tfrac{1}{7}p$$

202. *Method* 4. *Mr. Last's method of drawing cycloidal curves.*—The same method already described for approximating to a tooth to work with a given tooth may be applied

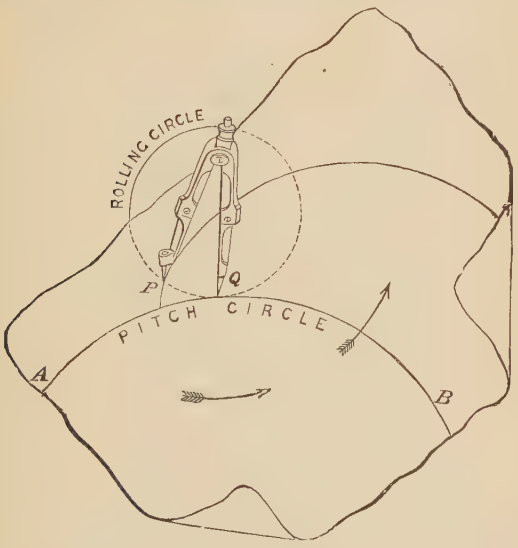


Fig. 218

to drawing cycloidal curves. Let the rolling circle be drawn on a sheet of drawing paper, and the pitch line on a sheet of tracing-paper (fig. 218). Let the circles first touch at P. Place there the compass point and shift the tracing-paper till the pitch line cuts the rolling circle in a near point Q. Remove the compass point to Q and shift the tracing-paper till the circles touch at Q. Then with Q P as radius,

describe a small arc. Next shift the tracing-paper till the pitch line again cuts the rolling circle in a near point and repeat the process. The new radius is to be taken so as to continue the arc already drawn. In the figure an epicycloid is drawn, a hypocycloid or cycloid is described with equal facility. The process with care is very accurate.

Involute Teeth

203. When the path of contact is a straight line, inclined to the line of centres, the form of the teeth is an involute of a base circle, concentric with the pitch circle, and having the path of contact for a tangent. The pressure between the teeth is in the direction of their common normal very nearly, and this normal coincides in involute teeth with the path of contact.

Involute Teeth with external Contact

Let B_1 B_2 be two circles centred at C_1 C_2, fig. 219. Suppose a string wrapped round and fastened to both circles and lying between them in the direction d_1 d_2 of their common tangent, which cuts the line of centres at p. If the upper wheel turns it will drive the lower wheel. A tracing-point attached to the string will move in space along the straight line d_1 d_2. At the same time it will mark out on the plane of the upper wheel an involute d_1 a_1, and on the plane of the lower wheel an involute d_1 b_1. At any moment these two curves will be in contact at the then position of the tracing-point which is moving along d_1 d_2 and will have a common normal, namely, the line d_1 d_2 which cuts the line of centres in the constant point p. These curves are suitable, therefore, for the teeth of wheels. Circles A_1, A_2 drawn through p will be the pitch circles. In the case of the string which acts by pulling, the upper wheel is the driver, but in the case of the teeth which act by pushing the lower wheel is the driver. Contact begins at d_1 and

ends at d_2, and d_1 d_2 is the greatest possible length of the path of contact. Circles c_1, c_2 through d_1, d_2 will be the addendum circles if the full possible length of the path of contact is utilised.

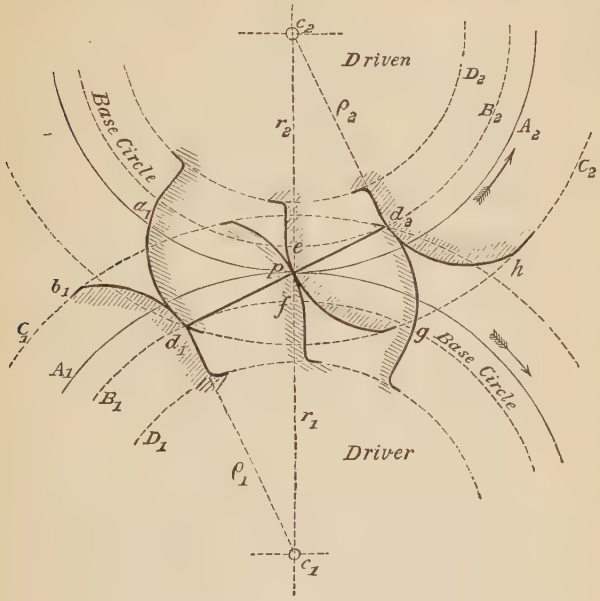


Fig. 219

The triangles, $c_2\ p\ d_2$ and $c_1\ p\ d_1$, are similar. Hence, if r_1 r_2 are the radii of the pitch lines and ρ_1 ρ_2 the radii of the base circles,

$$\frac{r_1}{r_2} = \frac{\rho_1}{\rho_2} = \frac{pd_1}{pd_2}. \quad . \quad . \quad . \quad (4)$$

Most commonly the path of contact d_1 d_2 makes an angle of $15\frac{1}{2}°$ with the tangent to the pitch lines, this being the constant angle of obliquity of action of the teeth, and makes an angle of $74\frac{1}{2}°$ with the line of centres. Then the

radii of the base circles are $\frac{63}{65}$ths (or 0·968) of the radii of the pitch lines.

If we move the centres c_1 c_2 further apart, the obliquity of action is increased, but we obtain for the teeth the same involute curves Hence in the new position the curves of the teeth are the same. With involute wheels, if the distance between the axes of the wheels alters by wear of journals or otherwise, the teeth still act correctly. But the obliquity of action and length of the path of contact is changed. Further, any pair of involute wheels of the same pitch gear correctly together.

The figure shows the tooth contours at the pitch point, and at beginning and ending contact. Hence arcs of the pitch lines, measured from p to the tooth contour of the wheel at beginning or ending contact, are the arcs of approach and recess for the length of tooth shown. Thus the arc of the pitch line of the upper wheel from p to the contour d_1 a_1 is the arc of approach, and to the contour through d_2 the arc of recess (see § 192).

With the length of path of contact shown the point d_1 of the tooth of the upper wheel enters some way within the base circle of the lower wheel and the point d_2 of the tooth of the lower wheel within the base circle of the upper wheel. Consequently the root circles D_1 D_2 must be taken inside the base circles so as to clear sufficiently the addendum circles C_1 C_2. The teeth may be continued from the base circle to the root circle by radial lines, these parts not being working parts of the teeth.

The arcs of action, the arc of the pitch line from the contour b_1 d_1 to d_2 g, or from d_1 a_1 to d_2 h, must be at least equal to the pitch. This fixes a maximum value for the pitch. Join d_1 c_2 and d_2 c_1. Then if these lines cut either of the arcs of action on the pitch lines in parts having the ratio 1 to 3, the tooth of that wheel is pointed and the pitch has a minimum value.

The obliquity of action is constant for all positions of the teeth, for d_1 d_2 is always the normal at the point of contact

204. *Involute wheels with internal contact.*—Let c_1 c_2 (fig. 220) be the centres of the wheels; A_1, A_2 the pitch lines. Draw $a_1\ p\ a_2$ inclined at the given angle of obliquity. Perpendiculars on $a_1\ p\ a_2$, from c_1 c_2, are the radii ρ_1 ρ_2 of the base circles. If the smaller wheel is the driver, contact

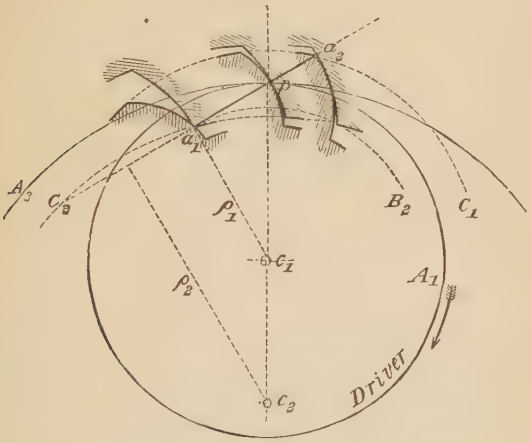


Fig. 220

begins at a_1, and C_2, through a_1, is the addendum circle of the larger wheel. Choose a length for the addendum of the teeth of the smaller wheel and draw the addendum circle C_1. Then contact ends at a_2. The arcs of contact during approach and recess are found by laying off $a_1\ p$, and $p\ a_2$, on the base circles, and finding the corresponding pitch circle arcs. If the arc of action is too large the teeth may be shortened.

205. *Approximate method of describing the involute.*—The involute is not difficult to describe, but the following method gives a very accurate circular approximation. Let $e_1\ e_2$ (fig. 221) be the working height of the teeth, or the distance between circles C_1 B_1, in fig. 219, measured along the line of centres. Take $e_1\ g = \frac{2}{3}\ e_1\ e_2$. Draw a tangent $g\ h$ to the base circle. Take $h\ k = \frac{1}{4}\ h\ g$. Then, a circle $m\ n$, struck

from k with radius $k g$, will be the required approximation to the involute. It will coincide with the involute at n and g, and will have the same normal at g. The part of the tooth

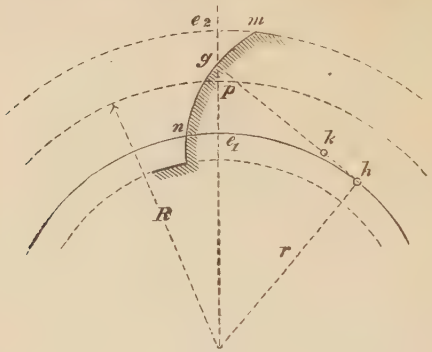


Fig. 221

below the base circle may be radial. This part does not come in contact with a tooth of the other wheel.

206. *To determine the length of involute teeth for given arcs of approach and recess.*—Let $A_1 A_2$ be the pitch lines. Draw $a_1 p a_2$ at the required obliquity. Perpendiculars $\rho_1 \rho_2$ on $a_1 p a_2$ are the radii of the base circles $B_1 B_2$. Let the line of centres $c_1 c_2$ cut the base circles in the points $n o$. Now lay off on the pitch lines the required arcs of approach and recess $b_1 p$ and $p b_2$. Draw radii from b_1, b_2 to c_1, c_2, the centres of the wheels, cutting off the corresponding arcs $n e_1$, $o e_2$ on the base circles. Now laying off on $a_1 p a_2$, $p d_1 =$ arc $n e_1$ and $p d_2 =$ arc $o e_2$, $d_1 p d_2$ is the required path of contact. Contact begins at d_1 and ends at d_2, and circles $C_1 C_2$ through d_2 and d_1 are the addendum circles. The root circles $D_1 D_2$ must be drawn for each wheel to allow room for the points of the teeth of the other wheel.

Take a radius through d_1, then $d_1 h$ is the working height of the tooth of the lower wheel. Take $d_1 k = \frac{2}{3} d_1 h$, and from k draw a tangent to the base circle. Take $m k = \frac{3}{4}$

of this tangent, then m is the centre of a circular arc $f k g$ approximating closely to the involute. The part of the tooth between g and the root circle D_1 may be radial. A

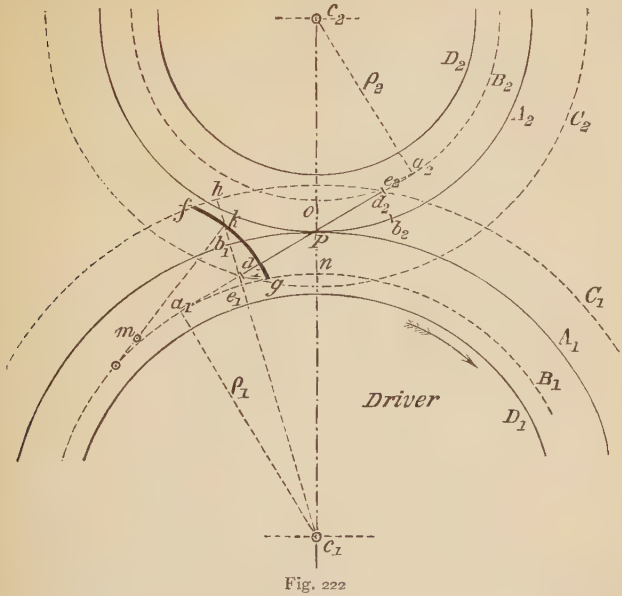


Fig. 222

corresponding construction gives the form of tooth for the other wheel.

Involute teeth have two remarkable properties. All involute wheels, whose teeth have the same pitch and the same obliquity of the line of contact, work well together. A pair of involute wheels may be drawn a little further apart without the accuracy of action of the teeth being impaired, though the arc of contact is diminished. Involute wheels cannot be made with very long teeth, because then the obliquity of the line of contact must be great. Hence, the centres cannot be moved much further apart than their normal distance, without too much reducing the arc of con-

tact. But this property of involute wheels is a valuable one, as it neutralises the injurious effect of wear of the supports of the wheels. With the angle of obliquity given above, the smallest number of teeth in an involute wheel should be twenty-five. With fewer teeth the arc of contact is too small. The obliquity of action is ordinarily alleged as a serious objection to involute wheels. Its importance has perhaps been overrated.

207. *Knuckle gearing* (fig. 223) is an imperfect form of gearing used for cranes and slow-moving machinery. The

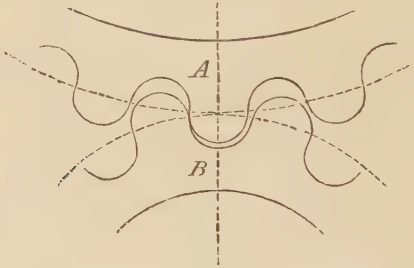


Fig. 223

action of the teeth is, however, very imperfect except for an arc of contact smaller than the pitch. There must be, therefore, a change of velocity ratio as the teeth come into and go out of contact.

Teeth of Bevil Wheels

208. The teeth of bevil wheels may be cycloidal or involute, and are described in the same way as the teeth of spur wheels, upon a development of the conical surfaces which limit their length. Let fig. 224 represent the section of a bevil wheel rim. o *a* is the intersection of the conical pitch surface with the plane of the paper, o *d* the axis of the wheel. Let *a c* be the width of face of the wheel. Draw

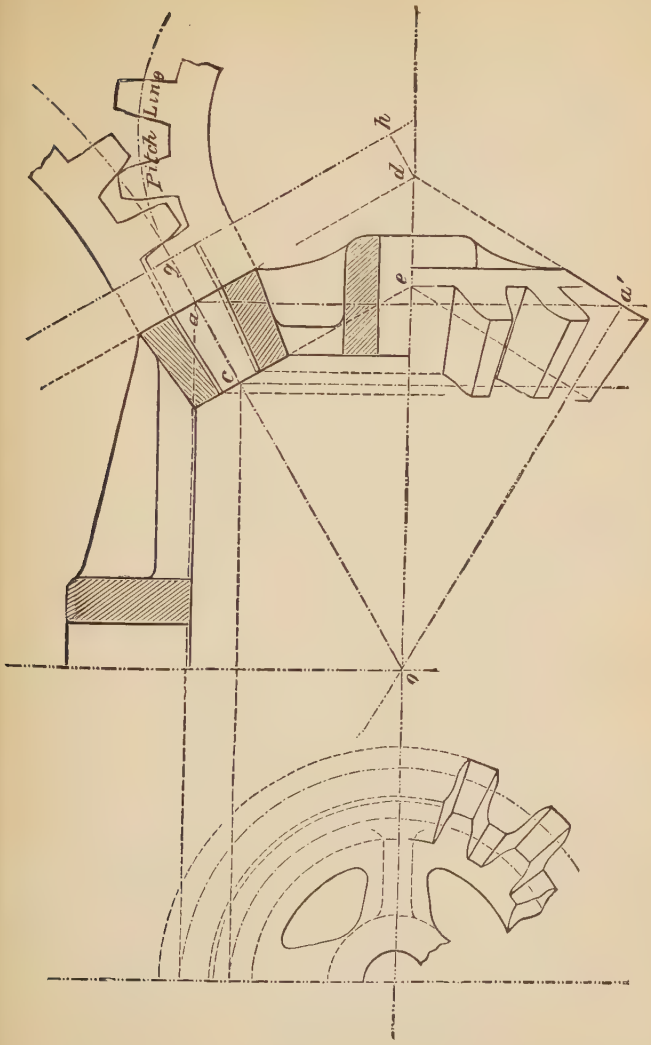


Fig. 224

a d, *c e*, perpendicular to o *a*, cutting the axis of the wheel in *e* and *d*. Then the teeth are limited in length by the conical surfaces, whose intersections with the paper are *e c*, *d a* and which have o *d* as axis.

Project *d a* to *h g* for convenience. Then a circle drawn with *h g* as radius is the virtual pitch line of the ends of the teeth, and the teeth are described on that circle as if it were the actual pitch line.

If R_1 R_2 are the radii of two bevil wheels on shafts at right angles, and r_1 r_2 the corresponding radii of the virtual pitch lines,

$$\frac{r_1}{R_1} = \frac{\sqrt{(R_1^2 + R_2^2)}}{R_2}; \quad \frac{r_2}{R_2} = \frac{\sqrt{(R_1^2 + R_2^2)}}{R_1}.$$

Summary of Curves for Toothed Wheels

209. R_1, R_2 = the radii of the wheels; r = radius of rolling circle; p the pitch of the wheels; T the number of teeth in the smallest wheel of the set; ρ_1 ρ_2 radii of base circles of involutes.

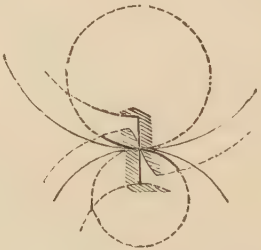


Fig. 225

External Contact

Cycloidal Curves

Case I.—*Pair of wheels* (fig. 225).

Face of R_1, epicycloid, $r = \frac{1}{2} R_2$.

Flank of R_1, radial line, $r = \frac{1}{2} R_1$.
Face of R_2, epicycloid, $r = \frac{1}{2} R_1$.
Flank of R_2, radial line, $r = \frac{1}{2} R_2$.

Case II.—*Set of wheels of which any two are to work together* (fig. 226).

Faces of R_1 and R_2, epicycloids.
Flanks of R_1 and R_2, hypocycloids.
Radius of rolling circle for all the cycloidal curves $r = \frac{1}{2} \frac{p T}{2 \pi}$.

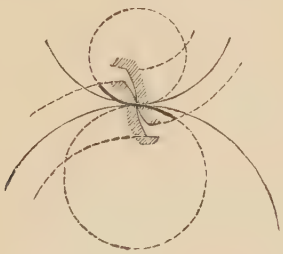


Fig. 226

Case III. — *Pair of wheels, contact during recess only* (fig. 227).

Flank of R_1, radial line.

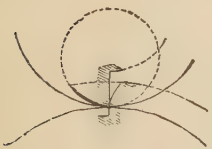


Fig. 227

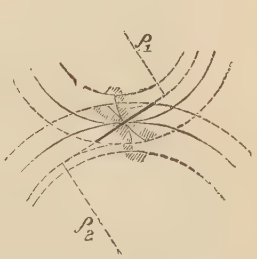


Fig. 228

Face of R_2, epicycloid, $r = \frac{1}{2} R_1$.
R_2 is the driver.

Involute Curves

Case IV.—*Set of wheels, any two of which work together.*

The curve of each tooth (fig. 228) is an involute, the base circles being chosen so that

$$\frac{\rho_1}{\rho_2} = \frac{R_1}{R_2}.$$

The parts of the teeth beyond the region of contact may be radial.

Internal Contact

Cycloidal Curves

Case V.—*Set of wheels, any two of which work together* (fig. 229).

Face of R_1 and flank of R_2, epicycloids.
Flank of R_1 and face of R_2, hypocycloids.

Radius of rolling circle for all the curves $r = \frac{1}{2} \cdot \frac{p\,T}{2\,\pi}$.

Fig. 229

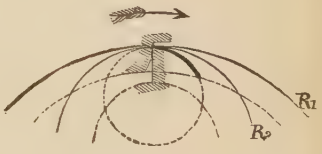


Fig. 230

Case VI.—*Two wheels having contact only during recess.*

Face of R_1, epicycloid, $r = \frac{1}{2} R_2$.
Flank of R_2, radial line.
R_1 is the driver (fig. 230).

Involute Curves

Case VII.—*Set of wheels any two of which gear together.*

The curve of each tooth is an involute (fig. 231), the base circles being chosen so that

$$\frac{\rho_1}{\rho_2} = \frac{R_1}{R_2}.$$

Beyond the region of contact the tooth of R_2 may be tangential to the curve.

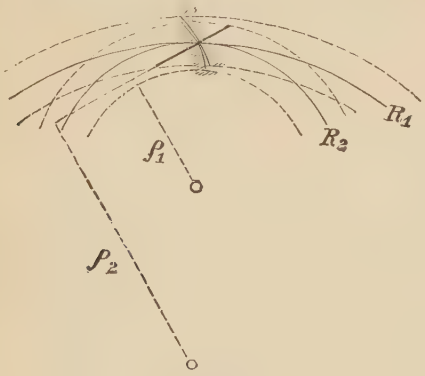


Fig. 231

PINION AND RACK

Case VIII.—*Set of wheels to work with one rack* (fig. 232).

Face of pinion tooth, epicycloid.

Flank of pinion tooth, hypocycloid.

Face and flank of rack tooth, cycloids.

Radius of rolling circle for all the curves $r = \frac{1}{2}\frac{p}{2\pi}\frac{T}{}$.

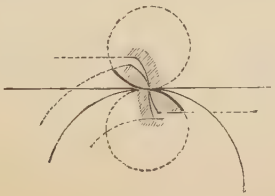


Fig. 232

Case IX.—*Single wheel to work with rack* (fig. 233).

Face of pinion tooth an epicycloid described with $r = \infty$, consequently an involute.

Flank of pinion tooth, hypocycloid, $r = \frac{1}{2} R_1$

Face of rack tooth, cycloid, $r = \frac{1}{2} R_1$.

Flank of rack tooth, hypocycloid described with $r = \infty$, and therefore a straight line perpendicular to the pitch line.

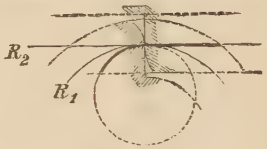


Fig. 233

Involute Curves

Pinion tooth an involute, with tangential prolongation beyond the region of contact.

Rack tooth, a straight line perpendicular to the path of contact.

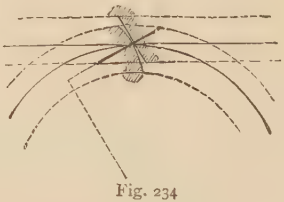


Fig. 234

In all the figures, the pitch lines are thin full lines, the rolling or base circles dotted lines, the path of contact a thick full line.

Strength of Wheel Teeth

210. In determining the strength of wheel teeth, it is not usually necessary to take into account their curved form. It is sufficiently accurate to treat the tooth as a rectangular cantilever (fig. 235), of thickness 36, uniform and equal to the thickness of the actual tooth at the pitch line. Usually two pairs of teeth are simultaneously in contact. The pressure transmitted is therefore shared by two pairs of teeth. The wheels cannot be made accurately enough to insure an equal distribution of the pressure to both pairs. Hence, if P is the whole pressure transmitted, the greatest pressure on one pair of teeth is nP, where n is a fraction lying between $\frac{1}{2}$ and 1. The teeth are in contact at a line which, in spur wheels, is parallel to the axis of rotation. The line of contact varies in position during the action of the teeth, and either at the beginning or end of contact coincides with the extreme edge of the tooth. Ordinarily, in teeth which have worn a little by mutual friction, the pressure will be distributed with approximate uniformity along the edge of the tooth, and will tend to break the tooth across at its root along its whole breadth. Another contingency less favourable to the strength of the tooth is possible. From inaccurate form in the teeth or inaccurate fixing of the wheels, the pressure may be restricted to a small portion of the edge of the tooth. In that case, to insure safety, the tooth must be strong enough to sustain the pressure nP applied at one corner, as shown in fig. 235.

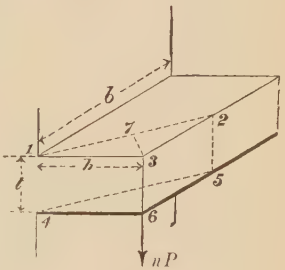


Fig. 235

211. Case I. *Strength of wheel teeth, when from inaccuracy of form or fixing the pressure may come on a corner of the tooth.*— In crane gearing and other cases where the wheels are of moderate width and where permanence of the initial adjustment cannot be relied on, the strength should be determined on the assumption that the whole pressure transmitted comes on a corner of the tooth.

Let P be the whole pressure of one wheel on the other, estimated in the direction of motion; H the number of horses' power transmitted by the wheel; N the number of revolutions of the wheel per minute; R its radius in inches; V the velocity of the pitch line in ft. per sec.

$$V = \frac{2\pi RN}{12 \times 60} = \cdot 00873\, RN$$

$$P = \frac{550\, H}{V} = 63{,}020\, \frac{H}{RN} \quad . \quad . \quad . \quad . \quad (5)$$

Let p be the pitch of the wheels in inches, T the number of teeth. Then the velocity of the pitch line in feet per second is

$$V = \frac{pTN}{12 \times 60};$$

consequently the pressure in lbs. on all the teeth in action at any moment may be expressed in the form

$$P = \frac{550 \times 12 \times 60 \times H}{pTN};$$

$$= 396{,}000\, \frac{H}{pTN} \quad . \quad . \quad . \quad (5a)$$

Let the height of the tooth 13, fig 235, $= h$; its thickness $14 = t$; the width of face $= b$. Then, if the pressure n P is applied at a corner, it tends to break off a triangular prism, bounded by a plane 1 2 5 4, which passes through the root of the tooth. Draw 37 perpendicular to that plane, and let the angle 2 1 3 $= \theta$; then,

$$\overline{37} = \overline{13} \sin\theta = h \sin\theta.$$
$$\overline{12} = \overline{13} \sec.\theta = h \sec.\theta.$$

The bending moment of n P, with respect to the section 1 2 5 4, is n P h sin θ. The moment of resistance of that section to bending is $\frac{1}{6} f h t^2 \sec\theta$. Equating the bending moment and moment of resistance, we get for the greatest stress due to bending,

$$f = \frac{3\, n\, \mathrm{P}}{t^2} \sin 2\theta,$$

which will be a maximum when $\theta = 45°$ and $\sin 2\theta = 1$. Then,

$$f = \frac{3\, n\, \mathrm{P}}{t^2}.$$

If f is the greatest safe stress,

$$\left.\begin{aligned} \mathrm{P} &= \tfrac{1}{3} \frac{f t^2}{n} \\ t &= \sqrt{\left(\frac{3\, n\, \mathrm{P}}{f}\right)} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (6)$$

212. It is convenient to express t in terms of the pitch p. For unworn teeth $t = 0{\cdot}48\, p$ for cast-iron teeth. Since, however, the teeth must be strong enough when worn, we may take $t = 0{\cdot}36\, p$. For mortice teeth of hard wood $t = 0{\cdot}6\, p$ when the teeth are new, and we may take $t = 0{\cdot}45\, p$ for worn teeth. Then, introducing these values in eq. 6,

$$\left.\begin{aligned} p &= 4{\cdot}8 \sqrt{\frac{n\, \mathrm{P}}{f}} \text{ for iron teeth} \\ &= 3{\cdot}85 \sqrt{\frac{n\, \mathrm{P}}{f}} \text{ for wood teeth} \end{aligned}\right\} \quad . \quad . \quad . \quad (7)$$

These formulæ cease to be applicable if $b < h$, but this does not occur in wheels of ordinary proportions.

In obtaining the above formulæ, some assumptions are made, and the value of n is undetermined. For different

wheels p is simply proportional to $\sqrt{P}$, and we may therefore write,

$$p = K\sqrt{P} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and determine K from existing wheels. K will be found to vary considerably in different cases. In slowly moving gearing, especially in gearing worked by hand and not subjected to much vibration or shock, K = ·04 for iron wheels. In ordinary mill-gearing, running at a greater speed and subjected to considerable vibration, K = ·05, and in wheels subjected to excessive vibration and shock, as in the gearing which drives machine tools, K = 0·06. For mortice gearing K = ·06.

Safe Pressure P *at pitch line of Wheels from Equation* 8.

Pitch in ins.	Safe pressure on teeth in lbs.			
	Iron teeth, Little shock	Iron teeth, Moderate shock	Iron teeth, Excessive shock	
1	625	400	277	The pressures in the last column are applicable to mortice teeth calculated by eq. 8.
1¼	975	624	432	
1½	1,406	900	623	
1¾	1,912	1,224	848	
2	2,500	1,600	1,108	
2¼	3,162	2,024	1,402	
2½	3,906	2,500	1,732	
2¾	4,726	3,024	2,094	
3	5,625	3,600	2,493	
3¼	6,600	4,224	2,926	
3½	7,658	4,900	3,393	
3¾	8,787	5,624	3,895	
4	10,000	6,400	4,432	
4½	12,656	8,100	5,608	
5	15,625	10,000	6,924	
5½	18,900	12,100	8,379	
6	22,500	14,400	9,972	

If, now, it is assumed that $n = \frac{2}{3}$, which cannot be very far from the truth, we get for the values of f for cast iron corresponding to the three cases above, 9,600, 6,100, and 4,300 lbs. per sq. in.; values which are quite consistent

with ordinary practice in the use of cast iron, to resist transverse straining actions. For hard wood, when $\kappa = \cdot 06$ $f = 2{,}740$.

213. Case II. *Strength of wheel teeth in carefully fitted gearing, when the pressure is assumed to be distributed along the whole width of the tooth.*—In the foregoing investigation, the pressure is assumed to be concentrated at the corner of the tooth, and consequently the strength is independent of the width of the tooth. For well-constructed and carefully erected mill-gearing, this is a very improbable condition. Let P be the total pressure and n P the pressure on one pair of teeth, as before. If the pressure is distributed along the edge of the tooth, the bending moment at its root is n P h. The moment of resistance of the section of the tooth is $\frac{1}{6} f b t^2$. Equating these,

$$\text{P} = \frac{1}{6} \frac{b t^2}{n h} f.$$

Let $t = 0{\cdot}36\, p$ for iron, and $0{\cdot}45\, p$ for wood teeth as above; $h = 0{\cdot}7\, p$ for iron, and $0{\cdot}6\, p$ for wood teeth; $n = \frac{2}{3}$.

$$\left.\begin{array}{l} \text{P} = \cdot 046\, b\, p\, f \text{ for iron teeth} \\ \quad = \cdot 084\, b\, p\, f \text{ for wood teeth} \end{array}\right\} \quad . \quad . \quad . \quad (9)$$

It must be remembered, however, that the wider the tooth the greater is the care necessary in adjusting the wheels so that the pressure may be equally distributed along the face of the teeth.

The following is the most convenient form of these equations:—

$$p = \kappa_1 \sqrt{\frac{p}{b}} \sqrt{\text{P}} \quad . \quad . \quad . \quad . \quad (10)$$

Where κ_1 in practice ranges from 0·0707 to 0·0884 for iron wheels and 0·085 to 0·106 for wood teeth. These values give stresses of 4,350 and 2,780 lbs. per sq. in. for iron teeth and 1,650 and 1,060 for wood teeth.

The following table facilitates the use of this equation :—

$\frac{b}{p}$ =	2	$2\frac{1}{4}$	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
Iron teeth. $K_1\sqrt{\frac{p}{b}}$ =	·0500	·0475	·0447	·0408	·0378	·0354
to	·0625	·0594	·0559	·0510	·0472	·0442
Wood teeth. $K_1\sqrt{\frac{p}{b}}$ =	·0600	·0565	·0536	·0490	·0453	·0424
to	·075	·0706	·0670	·0612	·0566	·0530

Safe pressure P *at pitch line of wheels in pounds from Equation* 10.—Pressure assumed to be distributed along the edge of the teeth.

K_1 taken at 0·0707.

Pitch in ins.	Pressure on teeth in lbs. when $\frac{b}{p}$ = 2	$2\frac{1}{4}$	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
1	400	450	500	606	700	800
$1\frac{1}{4}$	624	702	780	930	1,092	1,248
$1\frac{1}{2}$	900	1,012	1,150	1,400	1,550	1,800
$1\frac{3}{4}$	1,224	1,377	1,530	1,836	2,142	2,448
2	1,600	1,800	2,000	2,400	2,800	3,200
$2\frac{1}{4}$	2,024	2,277	2,530	3,036	3,542	4,048
$2\frac{1}{2}$	2,500	2,812	3,125	3,750	4,375	5,000
$2\frac{3}{4}$	3,024	3,402	3,780	4,536	5,292	6,048
3	3,600	4,050	4,500	5,400	6,300	7,200
$3\frac{1}{4}$	4,224	4,752	5,280	6,336	7,392	8,448
$3\frac{1}{2}$	4,900	5,512	6,125	7,350	8,575	9,800
$3\frac{3}{4}$	5,624	6,327	7,030	8,436	9,842	11,248
4	6,400	7,200	8,000	9,600	11,200	12,800
$4\frac{1}{2}$	8,100	9,112	10,125	12,150	14,175	16,200
5	10,000	11,250	12,500	15,000	17,500	20,000
$5\frac{1}{2}$	12,100	13,612	15,125	18,150	21,175	24,200
6	14,400	16,200	18,000	21,600	25,200	28,800

K_1 taken at 0·0884.

Pitch in ins.	Pressure on teeth in lbs. when $\frac{b}{p}$ =					
	2	2¼	2½	3	3½	4
1	256	288	320	384	448	51
1¼	400	450	500	600	700	800
1½	576	736	736	900	1,000	1,150
1¾	783	880	980	1,175	1,370	1,560
2	1,024	1,152	1,280	1,536	1,792	2,048
2¼	1,295	1,457	1,620	1,943	2,267	2,590
2½	1,600	1,800	2,000	2,400	2,800	3,200
2¾	1,935	2,177	2,420	2,903	3,386	3,870
3	2,304	2,592	2,880	3,456	4,030	4,608
3¼	2,703	3,040	3,380	4,055	4,730	5,407
3½	3,136	3,528	3,920	4,704	5,488	6,270
3¾	3,600	4,050	4,500	5,400	6,300	7,200
4	4,095	4,608	5,120	6,145	7,168	8,192
4½	5,184	5,830	6,480	7,770	9,070	10,370
5	6,400	7,200	8,000	9,600	11,200	12,800
5½	7,740	8,700	9,660	11,600	13,550	15,500
6	9,210	10,370	11,520	13,800	16,130	18,430

The latter table may be taken to apply to cases where there is some vibration and shock.

For mortice wheels multiply the pitch for iron teeth by 1·25. For wrought iron or steel wheels multiply by 0·75. For gun-metal wheels multiply by 1·4. Mr. Longridge has pointed out (Report 1884) that the very large contraction of steel in cooling makes it difficult to obtain accurate diameter or pitch, and he expresses an opinion unfavourable to steel wheels.

No specific rule can be given to decide between the cases in which eq. 8 and eq. 10 should be used. It is really a question of the degree of security against accident which is desired.

214. The equation above may be put in another convenient form. For the most usual proportion, $b = 2\frac{1}{2}p$, and iron teeth

$$p = 0{\cdot}0447\sqrt{P}$$

but

$$P = \frac{550\,H}{V} = \frac{550\,H \times 12 \times 60}{p\,T\,N}$$

Inserting this value,

$$p = 28{\cdot}13\sqrt{\left(\frac{H}{p\,T\,N}\right)} \quad . \quad . \quad . \quad (10a)$$

or inverting, we get the number of teeth of a given pitch necessary for strength.

$$T = 791\,\frac{H}{p^3\,N} \quad . \quad . \quad . \quad . \quad (10b)$$

From this equation the following table is calculated. It gives the least number of teeth suitable for strength, when $\frac{H}{N}$ is known and p assumed.

For mortice wheels, when $\frac{b}{p} = 3$, we have

$$p = 0{\cdot}049\sqrt{P};$$

and this gives

$$T = 951\,\frac{H}{p^3\,N}. \quad . \quad . \quad (10c)$$

Consequently, if the numbers in the table are multiplied by 1·2 (or, what is the same thing, if the numbers are increased one-fifth), they will be the proper numbers for mortice wheels.

215. *Limiting velocity of toothed wheels.*—If the wheels are run at a sufficiently high velocity, the wheel rim bursts in consequence of the centrifugal tension. Toothed wheels

Table giving least Numbers of Teeth for a Wheel of given Pitch and Speed, from equation 10b.

Pitch in inches p	p^3	Least number of teeth for $\frac{HP}{N}=$																																
		0·012	·025	·05	·075	·1	·15	·2	·25	·3	·35	·4	·45	·5	·6	·7	·8	·9	1·0	1·25	1·5	1·75	2·0	2·25	2·5	2·75	3·0	3·5	4·0	4·5	5·0	6·0	7·0	8·0
1	1·00	10	20	39	60	79	119	158	200	236	276	316	356	394																				
1½	3·37			12	17	24	35	47	59	70	82	94	106	114	141	164	187	211	234	293	351													
1¾	5·36				11	15	22	30	37	44	52	59	67	74	89	103	118	133	148	184	221	258	295												
2	8·00	. .	. .	.	. .	10	15	20	25	30	35	40	45	50	59	69	79	89	99	124	148	173	198	223										
2¼	11·4					11	14	18	21	25	28	31	35	42	49	56	63	70	87	104	123	138	156	173	191	208	243							
2½	15·6						10	13	16	18	21	23	25	31	36	41	46	51	63	76	89	103	114	127	139	153	177	202						
2¾	20·8							10	12	14	16	17	19	23	26	31	34	38	43	57	67	76	86	95	105	114	133	152	171	183	228	266		
3	27·0	. .	. .	.	. .	.	. .	.	. .	.	. .	11	12	13	15	18	21	24	27	30	37	44	52	59	66	73	81	88	103	117	132	146	176	205	234
3½	34·3													12	14	16	19	21	23	29	35	41	47	52	58	64	69	81	93	104	115	138	161	185
3¾	42·9														11	13	15	16	19	23	28	33	37	42	46	51	55	65	74	83	92	110	129	147
3¾	52·7															11	12	14	15	19	23	26	30	34	37	41	45	53	60	68	75	89	105	120
4	64·0	. .	. .	.	. .	.	. .	.	. .	.	. .	.	. .	.	. .	.	. .	.	. .	.	. .	11	13	16	19	22	25	28	31	34	37	43	50	56	62	74	87	99
4¼	76·8																				13	16	18	21	24	26	29	31	36	41	46	51	62	72	82
4½	91·1																					13	16	18	20	22	24	26	31	35	39	44	52	61	70
4¾	107·2																						13	15	17	19	21	22	26	30	34	37	45	52	59
5	125·0	. .	. .	.	. .	.	. .	.	. .	.	. .	.	. .	.	. .	.	. .	.	. .	.	. .	.	. .	13	15	16	18	19	22	26	29	32	38	44	51
5½	166·4																								11	12	13	14	17	19	22	24	29	34	38
6	216·0																										10	11	13	15	17	19	22	26	30

RULE.—To find the least number of teeth in either wheel of a pair, which will insure sufficient strength, divide the horses power transmitted by the revolutions of the wheel per minute; under the nearest number to the quotient so obtained, and opposite the pitch selected, is the required least number of teeth.

are in this respect materially in a worse position than pulleys, because the teeth add considerably to the weight of the rim without adding to the section which resists bursting. No increase of the pitch or section of the rim renders the wheels safe, because the increase of weight increases the centrifugal tension in the same ratio as the increase of section. For very high velocity wheels must be made of a stronger material than cast iron.

From the equation previously given (§ 213), we have for the load on the teeth

$$P = 0{\cdot}046\, b\, p\, f$$

where f is the safe stress allowed for the breaking across of the teeth. The actual section of the rim is about $0{\cdot}5\, b\, p$. But if the teeth were thrown into the rim the section would be about $0{\cdot}85\, b\, p$. Hence the weight of the rim per foot of length (12 cubic inches of iron weighing 3·36 lbs.) is $0{\cdot}85 \times 3{\cdot}36\, b\, p = 2{\cdot}86\, b\, p$ lbs.

Each foot of rim has a radial centrifugal force of $\frac{2{\cdot}86\, b\, p}{g} \cdot \frac{v^2}{R}$ lbs. where v is the velocity of the rim in feet per second, and R its radius in feet. The resultant centrifugal force of half the rim is

$$\frac{2{\cdot}86\, b\, p}{g} \cdot \frac{v^2}{R} \times 2R = \frac{5{\cdot}72\, b\, p\, v^2}{g} \text{ lbs.}$$

This is balanced by the stress on two radial sections of the rim. Hence the stress due to rotation is

$$\frac{5{\cdot}72\, b\, p\, v^2}{b\, p\, g} = \frac{5{\cdot}72\, v^2}{g} \text{ lbs. per sq. in.}$$

The stress in the rim due to the pressure on the teeth will be on the average $\frac{1}{2}$ P ; the load being transmitted half to the arm in advance and half to the arm behind the teeth in contact. But as the proportion transmitted each way will depend on the relative nearness of the arms, it seems probable that the maximum stress due to the load may

amount to twice the mean value, or $\frac{P}{0{\cdot}5\,b\,p}$ lbs. per sq. in.
Putting in the value of P above, this becomes

$$0{\cdot}092\,f.$$

Consequently the whole stress per sq. in. in the rim is

$$f_2 = 0{\cdot}092\,f + \tfrac{5}{8}\frac{{\cdot}72\,v^2}{g}.$$

For wheels run at high speed, we may take $f=4{,}000$ lbs. per sq. in. The safe limit of tensional resistance for cast iron is about 3,000 lbs. per sq. in. ; but looking to the fact that there are initial stresses in wheels due to contraction in cooling, and bending stresses due to the oblique action of the teeth, which have been neglected, it does not appear safe to take f_2 at more than 2,000 lbs. per sq. in. Then the limiting safe velocity is

$$v = \sqrt{\left\{\frac{g}{5{\cdot}72}\,(2000 - 0{\cdot}092 \times 4000)\right\}}$$
$$= 96 \text{ feet per sec. nearly.}$$

In a case which came under the author's notice, a pair of very large and well-constructed wheels run at about this speed, actually broke up, apparently from the action of the centrifugal force. Hence it is doubtful if even this calculation allows quite margin enough in the case of heavy wheels.

216. *Wheels for high speeds.*—At high speeds the influence of shocks and vibrations becomes more serious. Reuleaux has proposed to allow for this by making the value of the stress f decrease inversely as the cube root of the velocity of the pitch line. Then for cast iron

$$f = \frac{10{,}000}{\sqrt[3]{v}} \quad . \quad . \quad . \quad . \quad (11)$$

217. *Strength of bevil wheels.*—In stating the size of bevil wheels, the pitch at the outer circumference of the wheel is always given, but in estimating their strength the pitch at the inner circumference of the rim should be taken.

Let p_i, p_o be the pitches at the inner and outer circumferences, r_i and r_o the corresponding radii of the smaller, and R_i, R_o those of the larger wheel. Width of face $= b$. Let $R_o + 0{\cdot}4\, r_o = m$. Then,

$$p_i = p_o \frac{r_i}{r_o} = p_o \frac{m-b}{m} \text{ nearly.}$$

It is p_i, not p_o, which should be taken in estimating the strength of the wheel. For all other purposes p_o is used.

218. *Shrouded wheels.*—The teeth of wheels are sometimes united at the ends by annular rings cast with the wheel, and the wheel is then said to be shrouded. The shrouding may extend the whole depth of the teeth of the pinion of a pair of wheels. In that case the shrouding has the effect of neutralising the weakness of the teeth, which in very small wheels are of a weak form. With the pinion shrouded, it is stronger than the wheel, but it wears more rapidly than the wheel, so that the shrouding may be regarded as a provision against the failure of the pinion in consequence of wear. If both wheel and pinion are shrouded to half the depth of the teeth, the strength of the pair of wheels is considerably increased. But this arrangement is seldom adopted, and the casting of the wheels is difficult.

219. *Width of face of wheel.*—The durability of wheels is increased by making the wheels wider. In practice, b is rarely less than $1\frac{1}{2}\, p$ in wheels used to transmit power, and that width answers well for wheels moving slowly or intermittently. For ordinary mill-gearing $b = 2\, p$ to $4\, p$.

Wear of wheels.—No exact data of the wear of wheels in given circumstances have yet been recorded. The following theory may be useful as a guide when there is a doubt as to the width to be given to a pair of wheels.

Let R_1, R_2 be the radii of a pair of wheels, making N_1, N_2 revolutions per minute, and transmitting H horses' power,

Let p be the pitch, and b the width of face of the wheels. Then the work lost in friction is proportional to

$$p\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \text{H ft. lbs. per min.} \quad . \quad . \quad (12)$$

The wearing surface of a tooth is proportional to bp, and the whole wearing surface of the pinion is proportional to $R_1 b$. Supposing the total wear to be proportional to the work expended, the depth worn away in the unit of time is proportional to

$$p\left(\frac{1}{R_1}+\frac{1}{R_2}\right) H \div R_1 b$$

Suppose the wheel to be worn out when the depth worn away is γp, where γ is some fraction varying in different circumstances, but constant for wheels in similar conditions. Then, for equal durability,

$$\gamma p \div \frac{p\left(\frac{1}{R_1}+\frac{1}{R_2}\right) H}{R_1 b} = \text{constant}$$

or

$$b = k_1 \frac{H}{R_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \quad . \quad . \quad (13)$$

where k_1 is a constant to be determined by experience. Since $P R_1 N_1$ is proportional to H, we have also

$$b = k_2 P N_1 \left(\frac{1}{R_1}+\frac{1}{R_2}\right) . \quad . \quad (13a)$$

or when the pinion is small compared with the wheel,

$$b = k_1 \frac{H}{R^2_1} = k_2 P \frac{N_1}{R_1} \text{ nearly} . \quad . \quad . \quad (13b)$$

Average values for these constants would be of little service, because the conditions in which wheels are employed are so variable. If k_1 or k_2 is deduced from a pair of wheels known to have worked well in given conditions,

the value so obtained may be applied to determine the minimum width of another pair of wheels which are to work in similar conditions.

Construction and Proportions of Wheels

220. *Rim of wheel.*—In iron wheels the teeth are cast on, and in mortice wheels they are tenoned into, a continuous

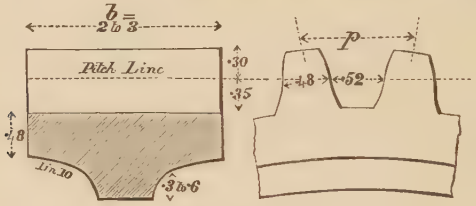


Fig. 236

rim. Fig. 236 shows the section of a spur-wheel rim, and fig. 237 that of a bevil-wheel rim. The unit for the proportional figures is the pitch. The proportional figures for the

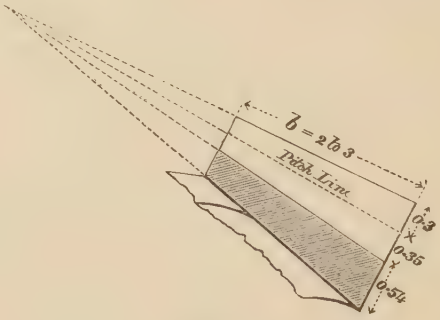


Fig. 237

teeth are approximate only, more exact proportions having been already given in § 190.

Fig. 238 shows the section of a mortice spur-wheel rim, the

end elevations indicating two ways of forming the tenons. The mortice teeth are either fixed by wood keys, or by

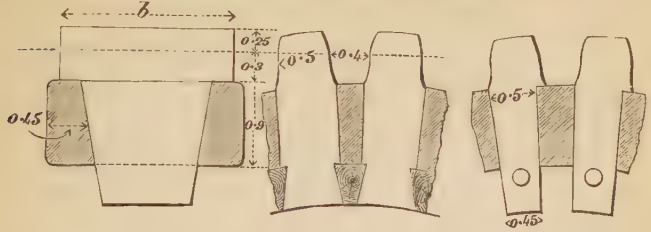


Fig. 238

round iron pins driven in behind the rim of the wheel. Both methods are shown in fig. 238. In fig. 239 the cogs are fixed by bolts, iron plates about 2 ft. long being fitted to the inside of the rim of the wheel. Fig. 240 shows a mortice bevil wheel.

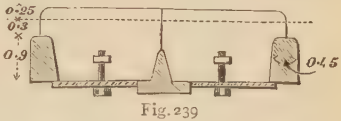


Fig. 239

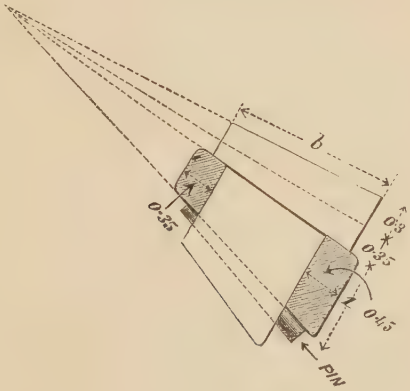


Fig. 240

The radiating lines in the figures of bevil wheels meet at the intersection of the shafts on which the wheels are placed.

221. *Arms of wheels.*—The arms of wheels are most commonly cross-shaped in section for spur-wheels, and T-shaped for bevil-wheels. For machine-moulded wheels, the arms are often I-shaped, the spaces between the arms being cored out in casting with loam cores. The number of arms in wheels is fixed very arbitrarily. Usually there are four arms for wheels not exceeding four feet diameter; six arms for wheels of from four feet to eight feet; and eight arms for wheels from eight feet to sixteen feet diameter.

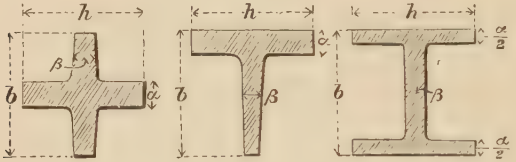


Fig. 241

The arms are subjected to bending, and when the arms and rim are cast in one piece, they are fixed at both ends. If the arms are attached to the rim by bolts, they are free at the rim, and fixed at the nave. It will be assumed that the arms are equally loaded, and that they may in all cases be treated as if they were fixed at one end and free at the other. This will give a slight excess of strength when the arms are cast in one with the rim, but such arms are at the same time weakened by contraction in cooling.

Let ν be the number of arms,
R, the radius of the wheel,
P, the total pressure transmitted (§ 211)

Then the bending moment on each arm is $\mathrm{P\,R} \div \nu$ nearly. The strength of the arm is almost entirely due to that part which is parallel to the plane of rotation. The ribs or feathers at right angles to this part add very little

to the resistance to the force acting on the wheel. They are necessary to give lateral strength and rigidity, and to resist accidental straining actions at right angles to the plane of rotation. Let h be the width, and a the thickness of the arm, exclusive of the feathers. The moment of resistance of that section is $\frac{1}{6} a h^2 f$. Equating this to the bending moment

$$a h^2 = \frac{6 \text{ P R}}{\nu f} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

In proceeding to design the arm either of the three following methods may be followed.

(1.) Given the ratio $\frac{h}{a}$, and the limiting stress on the arm.

If $\frac{h}{a} = 5$, we get from eq. 14,

$$h = \sqrt[3]{\frac{30}{f}} \quad \sqrt[3]{\frac{\text{P R}}{\nu}}$$

The limiting stress must be taken at a low value, partly to allow for unequal distribution of load on the arms, and partly because of the initial stresses due to contraction in cooling. If $f = 3{,}000$ lbs. per sq. in.,

$$h = \frac{0{\cdot}2154}{\sqrt[3]{\nu}} \sqrt[3]{\text{P R}} \quad . \quad . \quad . \quad . \quad (15)$$

$\nu =$	3	4	6	8	10	12
$\frac{0{\cdot}2154}{\sqrt[3]{\nu}} =$	·149	·136	·119	·108	·100	·094

(2.) Since the arm must be of equal strength with the teeth, we may replace P by its value in terms of the pitch in eq. 9, namely,

$$\text{P} = 0{\cdot}046\, b\, p\, f$$

Introducing this in eq. 14,

$$a h^2 = 0{\cdot}276 \frac{b\, p\, \text{R}}{\nu} \quad . \quad . \quad . \quad . \quad (16)$$

Let $a=0{\cdot}2\,h$

$$h=\frac{1{\cdot}113}{\sqrt[3]{\nu}}\sqrt[3]{(b\,p\,\mathrm{R})} \quad . \quad . \quad . \quad (17)$$

$\nu=$	3	4	6	8	10	12
$\frac{1{\cdot}113}{\sqrt[3]{\nu}}=$	·772	·701	·613	·557	·517	·486

A comparison of some existing wheels shows that the arms are sometimes one-fifth wider than is given by this rule, this additional width being required to meet the stresses due to contraction.

(3.) Given the thickness of the arm. It is desirable to make the different parts of the wheel nearly uniform in thickness to secure regularity in cooling and contraction. Let $a=0{\cdot}48\,p$, so that the arm is the same thickness as the teeth. Introducing this in eq. 16,

$$h=\frac{0{\cdot}758}{\sqrt{\nu}}\sqrt{b\,\mathrm{R}} \quad . \quad . \quad . \quad (18)$$

$\nu=$	3	4	6	8	10	12
$\frac{0{\cdot}758}{\sqrt{\nu}}=$	·438	·379	·309	·268	·240	·219

One-fifth may be added to the dimensions thus obtained to allow a margin against contraction, and for the unequal loading of the arms.

The dimensions given by the foregoing rules apply to the section of the arm produced to the centre of the wheel. Towards the rim the arm is usually tapered, the amount of taper being $\frac{1}{4}$ in. per foot of length on each side. The thickness of the arm a is constant.

The width of the cross feathers (marked b in fig. 241) may be b to $1\frac{1}{4}\,b$ at the centre, and $\frac{3}{4}\,b$ to $\frac{15}{16}\,b$ at the rim, where b=width of face of wheel. The thickness of the feathers may be $\beta=0{\cdot}3\,p$. The feathers must be slightly tapered at right angles to their length, so as to draw easily from the sand.

222. *Nave of the wheel.*- For the ordinary case of a cast-iron wheel of radius R and pitch p, bored to fit a shaft and fixed by a key, the nave thickness δ may be taken at $0{\cdot}4\sqrt[3]{(p^2R)} + \frac{1}{2}$, and its length should be at least three times its thickness.

It is more common in practice to roughly proportion the nave to the diameter on the shaft on which the wheel is fixed. If d is the diameter of the wrought-iron shaft, then the boss for the wheel should be $1{\cdot}17\,d$ in diameter. The nave thickness may be $\frac{1}{2}\,d$ and the nave length at least $1\frac{5}{8}d$.

Large wheels may be fixed by four keys. Then the eye of the wheel has a diameter $1\frac{1}{4}$ times that of the shaft at the place where it is fixed. Heavy wheels have the nave split to prevent fracture of the arms from contraction in casting. The nave is then gripped by two strong wrought-iron rings or hoops, fitted over the nave on each side and shrunk on.

223. *Weight of toothed gearing.*—Let p be the pitch, b the breadth of face, and n the number of teeth of a wheel. Then, its weight in lbs. is, approximately,

$$w = k\,n\,b\,p^2 \qquad . \qquad . \qquad . \quad (18a)$$

where $k = 0{\cdot}38$ for spur wheels, and 0·325 for bevil wheels. The weight of a pair of wheels is independent of the radii, and depends directly on the H.P. transmitted and the numbers of revolutions of the wheels. The weight of a train of wheels is smaller when the number of pairs of wheels is as small as possible, and when all the pairs, except the quickest-running pair, have the greatest practicable velocity ratio.

Mr. D. K. Clark gives the following formula for the weight of cast-iron spur wheels per inch of breadth in lbs.:

$$w = (5{\cdot}6 + 9\,p)\,(d + 0{\cdot}1\,d^2) \text{ Spur wheels,}$$
$$= (4 + 6{\cdot}3\,p)\,(d + 0{\cdot}1\,d^2) \text{ Bevil wheels,}$$

where d is the diameter in ft and p the pitch in ins.

CHAPTER XI

HELICAL AND SCREW GEARING

224. Helical gearing is a modification of ordinary toothed gearing in which the velocity ratio is inversely as the radii of the pitch surfaces. Screw gearing is gearing in which the velocity ratio is independent of the radii of the pitch surfaces. It has the great advantage that high velocity ratio can be obtained with comparatively small wheels. Its defect is that the friction and wear is greater than with toothed gearing.

In helical gearing the pitch surfaces may be cones or cylinders as in bevil or spur gearing. In screw gearing the pitch surfaces are usually cylinders. In both helical and screw gearing the teeth intersect the pitch surfaces in helical lines. A screw wheel may have one or any number of teeth. A one-toothed wheel corresponds to a one-threaded screw; a many-toothed wheel to a many-threaded screw. In screw gearing the axes may be at any angle.

Helical Gearing

225. *Helical gearing with parallel axes.*—Gearing of this kind was invented by Dr. Hooke. Let an ordinary spur wheel be cut into n slices by planes perpendicular to the axis. Let the slices be so arranged that, for example, in passing from left to right across the face of the wheel, each successive slice is $1/n$th of the pitch behind the previous one. Such a wheel is termed a stepped spur wheel. Two such wheels will work together, and they have the advantage compared

with ordinary wheels that one or other of the pairs of slices are always in contact at a distance not exceeding $1/n$th of the pitch from the pitch point.

As the slices come successively into gear, the motion of the wheels is very regular. Such wheels were at one time used for driving planing-machine tables, and in other cases where regularity of motion was important. If the slices are infinitely numerous, then the front of the tooth intersects the pitch cylinder in a helical line, and we get the helical wheels

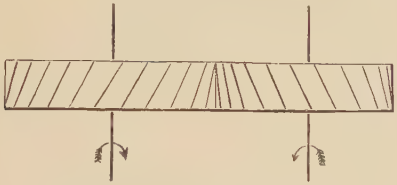


Fig. 242

shown in fig. 242. When accurately constructed there is always line contact between any two teeth in action. In two wheels of this kind which gear together the pitch measured circularly is equal; the obliquity is equal, but in opposite directions, and the velocity ratio is inversely as the radii of the wheels. Wheels of this kind are used in machine tools (in milling machines for example) where regular motion is important.

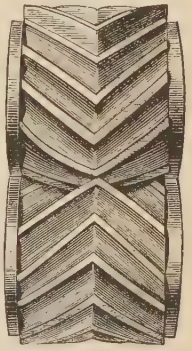


Fig. 243

The wheels just described are open to the objection that from the obliquity of the teeth the wheels tend to thrust each other apart and thus produce prejudicial end thrust on the supports of the wheels. By combining two pairs of wheels, constructed with right-handed and left-handed obliquity, this objection is obviated. Messrs. Jackson of Manchester appear to have overcome the diffi-

culty of casting these wheels, both in the case of spur and bevil gearing. The double helical teeth are shown in fig. 243. Wheels of this kind work very smoothly, as the teeth have always two points touching in the plane of the axes. It is claimed for them also that the teeth from their form are of very great strength. For pinions with few teeth helical gearing works more smoothly than ordinary gearing. Helical gearing is also so noiseless that it may in some cases take the place of mortice gearing. The cost is not much greater than that of ordinary gearing.

226. *Strength of helical gearing.*—As to the smoothness of action of helical gearing when accurately erected there seems no doubt. As to its strength a question has arisen. Mr. Longridge (Report 1888, p. 18) points out that the total pressure on a helical tooth, normal to the tooth, is greater than its component in the plane of rotation in the proportion of the length of the tooth to the width of face of the wheel. He infers that the helical tooth carries the same load per inch of length as the straight tooth, and therefore is no stronger. But there is here an oversight. At one moment in each contact, the whole load on a straight tooth is concentrated at its edge and acts with a leverage equal to the height of the tooth. With a helical tooth this never occurs. The points of contact at any moment are distributed at different parts of the width of the wheel over every portion of the working face and flank of the tooth, from root to point, or at any rate on such portions of two teeth simultaneously in contact. Hence the mean leverage at which the pressure acts at any moment to break the tooth cannot exceed half the height of the tooth. It may be inferred from this that a helical tooth is twice the strength of a straight tooth. This, however, probably somewhat exaggerates the advantage of the helical teeth, partly because it assumes the helical teeth to be of quite exact form, partly because even with straight teeth there is never only a single

pair of teeth in contact. No doubt, however, the helical teeth are stronger than straight teeth.

There appear, however, to be some difficulties in using helical gearing. For their correct action the middle plane of the two wheels must coincide. If it does not all the load is thrown on one half of the tooth. But to secure this permanently, it seems almost necessary that one of the wheels should have some small lateral play so that it can accommodate itself to the other wheel. Mr. Longridge appears to have recommended in one case, with good reason, that a sliding coupling should be placed on each side of one of a pair of helical wheels, so that a lateral shifting of the wheel should be possible.

Screw Gearing

227. *Screw gearing when the axes are not parallel.*— When the axes are not parallel the pitch cylinders touch at a single point, which may be termed the pitch point. Draw through that point a tangent to the pitch surfaces. If helices are traced on the pitch cylinders touching that tangent, they define the fronts of teeth which will drive each other.

The common tangent to the pitch surfaces and the teeth is termed the line of contact. It is shown at $a\,b$, fig. 245; the angles θ_1 θ_2 it makes with the axes are termed the angles of inclination of the teeth. The number of threads v, in a screw wheel, is equal to the number of helices which intersect any plane perpendicular to the axis. Let fig. 244 represent a series of helices (in this case four), intended to mark out the teeth of a screw wheel. The same screw thread intersects a line $a\,b$, parallel to the axis at a and b. Then $a\,b$ is the *axial pitch* of the screw and the distance $a\,c = p = \frac{a\,b}{v}$ is the *divided axial pitch*. Let a plane $d\,e$ perpendicular to

the axis intersect two successive threads in d and e. Then de is the circumferential pitch c, and is equal to $\frac{2\pi r}{\nu}$ where r is the radius of the pitch cylinder. Draw ef perpendicularly to the threads. Then the distance ef is the divided *normal pitch*, n. Let the tangent gh to a thread make the angle $ghx = \theta$ with the axis of the wheel. Then from the properties of helices the following relations obtain :—

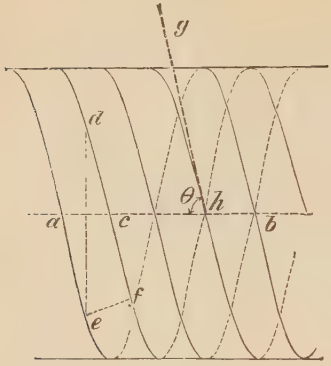


Fig. 244

$$\left.\begin{aligned} \tan\theta &= \frac{2\pi r}{\nu p} \\ p : c : n &:: 2\pi r \cot\theta : 2\pi r : 2\pi r \cos\theta \\ &:: \cot\theta \quad : \quad 1 \quad : \quad \cos\theta \end{aligned}\right\} \quad (19)$$

Let fig. 245 represent two screw wheels projected on the common tangent plane to the two pitch cylinders. Let the angle between the axes $= i$, and let the tangent to the teeth $a\,b$ make with the axes the angles θ_1, θ_2, so that

$$\theta_1 + \theta_2 = i$$

Let ω_1 ω_2 be the angular velocities of the wheels ; r_1 r_2 their radii ; and ν_1 ν_2 the number of threads of each. Let c_1 c_2 be the circumferential, n_1 n_2 the divided normal, and p_1 p_2 the divided axial pitches. Then,

$$\frac{\omega_1}{\omega_2} = \frac{\nu_2}{\nu_1} \quad . \quad . \quad (20)$$

Let r_1 and r_2 be decided upon. The surface velocities of the wheels are $\omega_1 r_1$ and $\omega_2 r_2$, and these are proportional to the circumferential pitches, because each wheel rotates a

distance equal to the circumferential pitch in the same time. Hence,

$$\frac{c_1}{c_2}=\frac{a_1 r_1}{a_2 r_2} \quad . \quad . \quad . \quad (21)$$

If the circumferential pitches are chosen so as to satisfy this relation, then the axial pitch and inclination of the

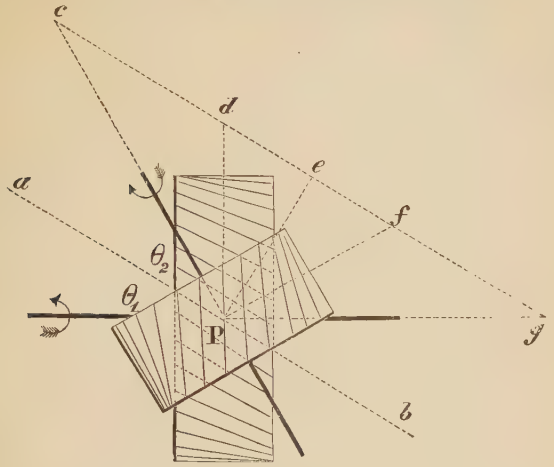


Fig. 245

threads are determined by the condition that in two wheels which work together the normal pitches must be equal. Hence,

$$n_1=n_2$$

and using the proportions in eq. 19

$$c_1 \cos\theta_1=c_2 \cos\theta_2$$

$$\left.\begin{aligned}\therefore \cos\theta_1&=\frac{c_2 \sin i}{\sqrt{(c_1{}^2-2c_1 c_2 \cos i+c_2{}^2)}}\\ \cos\theta_2&=\frac{c_1 \sin i}{\sqrt{(c_1{}^2-2c_1 c_2 \cos i+c_2{}^2)}}\end{aligned}\right\} \quad . \quad (22)$$

$$\left.\begin{aligned}p_1&=c_1 \cot\theta_1\\ p_2&=c_2 \cot\theta_2\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (23)$$

In fig. 245, set off from the pitch point P, the lines Pd, Pf perpendicular to the axes, in the directions the wheels are moving at the point P. Take Pd, Pf equal to the surface velocities $\omega_1 r_1$ and $\omega_2 r_2$ of the wheels; join df and produce it to meet the axes; then a P b parallel to cg is the line of contact, making angles θ_1 θ_2 with the axes. Draw Pe perpendicular to cg. Then Pe is the common component of the surface velocities, and cg the velocity of transverse sliding of the teeth.[1]

$$c_1 : c_2 :: p_1 : p_2 :: n_1 \text{ or } n_2$$
$$:: \quad \text{P}d : \text{P}f : \text{P}g : \text{P}c : \text{P}e$$

If Pd, Pf are set off equal to the circumferential pitches of the two wheels, and the same construction is made, then

$$p_1 = \text{P}g\,;\ p_2 = \text{P}c\,;\ \text{and } n_1 \text{ or } n_2 = \text{P}e.$$

228. *Screw gearing when the shafts are at right angles. Worm and Wheel.*—If $i = 90°$, then $\cos\theta_2 = \sin\theta_1$

$$\frac{c_1}{c_2} = \tan\theta_1$$

$$\frac{p_1}{p_2} = \cot\theta_1$$

Hence, $$p_1 = c_2 \text{ and } p_2 = c_1$$

or the axial divided pitch of one wheel is equal to the circumferential pitch of the other.

The most common form of screw gearing is that in which the shafts are at right angles, and a wheel of one thread, or sometimes of two or three threads, works with a wheel of many threads. Then the former is termed a *worm*, and the latter a *worm wheel*. With this arrangement, a high velocity ratio is obtained with a pair of small wheels. If N_1 N_2 are the numbers of revolutions of the worm and wheel, ω_1 ω_2,

[1] See Rankine's 'Millwork,' p. 216.

their angular velocities, and ν_1 ν_2 the number of threads on each,

$$\frac{N_1}{N_2}=\frac{a_1}{a_2}=\frac{\nu_2}{\nu_1}$$

Thus, if the worm has one thread and the wheel twenty-five, the velocity ratio is twenty-five. Spur wheels for that velocity ratio would have to be at least ten times larger in diameter. The disadvantage of screw gearing of this kind is that the friction and wear is excessive, hence it is rarely used for the continuous transmission of power. If the obliquity of the helices exceeds a certain amount, the wheels are no longer reciprocal; that is, one wheel will drive the other but the second will not drive the first. In that case the motion is prevented by the friction at the point of contact of the teeth. The worm and wheel are commonly so constructed that the worm will drive the wheel, but the wheel will not drive the worm. This is often advantageous, because the gearing remains stationary in any position after being moved.

229. *Friction of worm and wheel.*—Suppose the worm drives the wheel, and that a force P acts at the pitch line of the worm, in the plane of rotation, overcoming a resistance Q acting at the pitch line of the worm wheel in the plane of its rotation. Let θ_1, as before, be the inclination of the worm thread, μ the coefficient of friction, r_1 r_2 the radii of the worm and wheel, and ν_1 p_1 the total axial pitch of the worm :—

$$\frac{P}{Q}=\frac{1+\mu\tan\theta_1}{\tan\theta_1-\mu} \quad . \quad . \quad . \quad (24)$$

or if ϕ is the angle of repose of metal on metal, so that $\mu=\tan\phi$,

$$\frac{P}{Q}=\cot(\theta_1-\phi) \quad . \quad . \quad (25)$$

When the worm drives the wheel this ratio must be positive. Hence θ_1 must be less than $90°+\phi$. The ratio of

the useful work done to the power expended, or the efficiency of the pair of wheels is

$$\eta = \frac{\cot \theta_1}{\cot (\theta_1 - \phi)} = \frac{1 - \mu \frac{v_1 p_1}{2\pi r_1}}{1 + \mu \frac{2\pi r_1}{v_1 p_1}} \quad . \quad . \quad (26)$$

For $\mu = 0{\cdot}15$, we get

$$\eta = \frac{v_1 p_1}{v_1 p_1 + r_1} \text{ nearly} \quad . \quad . \quad (27)$$

Hence the efficiency is greater the less the radius of the worm. Generally $r_1 = 1{\cdot}5$ to $3\ p_1$. For a one-threaded worm, therefore, the efficiency is only $\frac{2}{3}$ to $\frac{1}{4}$; for a two-threaded worm, $\frac{4}{7}$ to $\frac{2}{3}$; for a three-threaded worm, $\frac{2}{3}$ to $\frac{1}{2}$. Since so much work is wasted in friction it is not surprising that the wear is excessive.

The radius of the pitch surface of the worm is very variable in practice. The least value (which gives the greatest efficiency) is about $r_1 = p_1$. More commonly, especially if the worm is to be cast and keyed on the shaft, $r_1 = 1\frac{1}{4}$ to $1\frac{1}{2} p_1$. Sometimes for special reasons $r_1 = 4$ to $6 p_1$.

Let $r_1 = x p_1$, where x may vary from 1 to any larger value. Then

$$\eta = \frac{v_1}{v_1 + x} \quad . \quad . \quad . \quad (27a)$$

where v_1 is the number of threads in the worm. The following table gives the values of the efficiency η :—

Number of threads v_1	$x =$								
	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3	4	6
1	·50	·44	·40	·36	·33	·28	·25	·20	·14
2	·67	·62	·57	·53	·50	·44	·40	·33	·25
3	·75	·70	·67	·63	·60	·55	·50	·43	·33
4	·80	·76	·73	·70	·67	·62	·57	·50	·40

Worm gearing has been used for reducing the speed of electric motors by Mr. Reckenzaun, who gives the following particulars of a case of this kind :—

Steel worm polished, 6 ins. in diameter, 6 ins. long, three threads, 2 ins. pitch. Worm wheel of phosphor bronze, trimmed teeth 15·3 ins diameter, $3\frac{1}{2}$ ins. face, 24 teeth. Velocity ratio, 8 to 1. The worm ran in an oil-bath, and the efficiency was 81 to 87 per cent.

Here $r_1 = 3$, $v_1 = 3$, $x = 1{\cdot}5$.

Hence equation (27*a*) gives

$$\eta = \frac{3}{3 + 1{\cdot}5} = \frac{2}{3} = 66 \text{ per cent.}$$

The difference between this and Mr. Reckenzaun's result is probably due to the good condition of the surfaces of the worm and wheel teeth, and the oil bath. Possibly in trimming the worm wheel some approach was made to the form which secures line contact.

230. *Form of worm-wheel rim.*—Fig. 246 shows the forms adopted for the rims of worm wheels. The simplest form is shown at A, but it is only suitable for those worm wheels in which contact is confined to a point on the

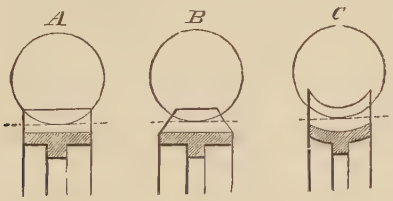


Fig. 246

median plane of worm wheel. B and C show forms suitable when line contact is secured across the face of the worm wheel, and especially for those worm wheels in which the teeth are cut by a hob of the form of the worm. Greater wearing surface is obtained in C than in A.

231 *Form of worm-wheel teeth and threads of worm. Point and line contact.*—In the cases which most commonly occur, the axes of the worm and wheel are at right angles. Hence, the axial divided pitch of one is equal to the circumferential pitch of the other. For this pitch the common symbol p will be used. In such cases it is convenient to design, at first, the teeth of worm and wheel on a plane passing through the worm axis and normal to the wheel axis as shown in fig. 247. For shortness the plane of this section will be called the median plane of the worm wheel. For sections on this plane we may take (p = axial pitch of worm or circumferential pitch of worm wheel).

Thickness of tooth on pitch line	$0{\cdot}48\,p$
Height outside pitch line	$0{\cdot}3\,p$
Depth below pitch line	$0{\cdot}4\,p$
Width of face of worm wheel usually	1·5 to 2·5 p
Length of worm	3 to 6 p
„ „ usually	$4\,p$

For worms in which the threads make at least a complete turn, this fixes completely the form of the worm threads, for all radial sections must be identical, since all in turn come into the median plane of the worm wheel. But the form of the worm-wheel teeth requires further investigation.

Willis pointed out that, if the section of the teeth of the worm wheel by the median plane be made like those of the spur wheel of the same radius and pitch, and the threads of the worm like the teeth of a rack suitable for working with such a spur wheel, the worm and wheel will gear correctly together so far as contact in that plane is concerned. Any of the wheel and rack constructions given in the table above, therefore, may be used in designing the section of worm and wheel on the median plane of the worm wheel.

It has commonly been assumed and acted on that all radial sections of the worm threads must be of the

form so fixed, which is right and necessary; and also that all sections of the worm-wheel teeth on sections parallel to the median plane should be similar, which is unnecessary and, as will be seen, gives a bad form of worm-wheel teeth. If that proceeding is adopted, the worm-wheel teeth become merely twisted spur-wheel teeth, which only touch the worm teeth at a point in the median plane. The point of contact moves from root to point of the teeth along the intersection of the median plane with the tooth, the rest of the surface of the tooth is never in contact at all, and the whole of the pressure and wear is concentrated at a single line on the face of the tooth. It is probably in part due to this imperfect construction of the worm wheel that screw gearing owes its bad reputation for friction and rapid wear.

Fig. 247 shows a worm and wheel, the teeth of which are drawn in this way. The worm here shown is of wrought

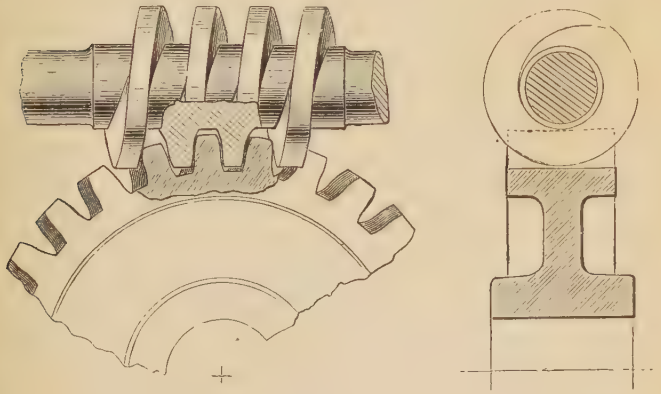


Fig. 247

iron or malleable cast iron, formed in one piece with its shaft. Usually the worm is of cast iron, and when small may be fixed by a pin passing through both worm and shaft. When larger its rotation on the shaft may be prevented by

a key, and its tendency to slide along the shaft by collars, one of which may be fixed and the other a loose collar fixed by a set screw. Sometimes the bearings which support the worm shaft are so arranged as to prevent the endways motion of the worm.

232. *Screw gearing with line contact.*—In the earlier editions of this book a method was given for drawing sections of the worm-wheel teeth on a series of parallel planes such that in each of these planes there should always be a point of contact with the worm thread. There is then always contact between worm and wheel along a line across the wheel face, and the whole surface of the worm-wheel tooth comes into action during the rotation of the wheel through the arc of contact. Such wheels naturally wear longer, and probably have less friction. They are, however, somewhat difficult to construct by ordinary processes of moulding and casting Probably they could be moulded without difficulty in a wheel-moulding machine. An old plan occasionally used in making worm wheels for dividing engines, when great accuracy was required, was to make a steel worm like the worm to be used, to cut grooves along it to form cutting edges, then to harden it and to use it as a cutting tool, to cut the worm-wheel teeth on a worm wheel cast with a blank rim. A worm wheel cut in this way has teeth of exactly the form given by the method mentioned above. There is always line contact between the worm and wheel. More recently Messrs. Browne and Sharpe have extended this method, and steel hobs ready prepared as chasers for worm wheels of almost any usual pitch are obtainable.[1]

233. *Mr. Briggs's account of the mode of designing screw gearing.*—When originally writing the chapter on worm gearing the author received from the late Mr. Briggs of Philadelphia a paper which led him to re-examine the sub-

[1] 'Treatise on Gearing.' Browne & Sharpe Manufacturing Company. Providence, 1886.

ject. Mr. Briggs's statements are so interesting that they may be repeated here. After reverting to current opinion that the friction and wear of worm gearing is disastrous, Mr. Briggs says :—

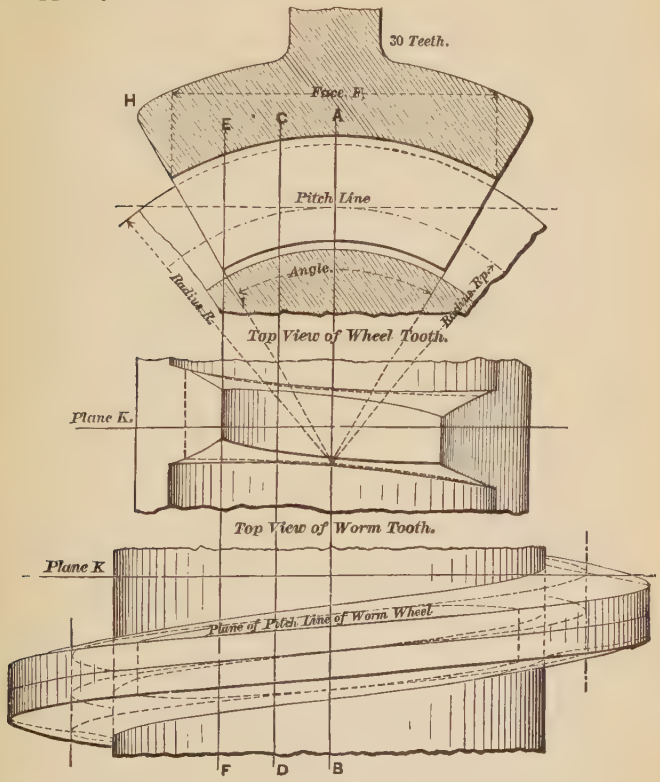


Fig. 248

'Now, the fact is, that the use of worm gearing for hoists, cranes, boring-bars, lathes, &c., has been growing in favour, and it is found that neither excessive loss of power nor excessive wear of gearing ensues. In regard to friction, it is established that for ordinary ratio of wheel to worm, say not to exceed 60

or 80 to 1, well-fitted worm gear will transmit motion backward through the worm, exhibiting a lower coefficient of friction than is found in any other description of running machinery. For the text-books, also it should be remarked that Weisbach (1850 about), in his illustration of a worm gear, shows (without description) the pitch-lines located to give an equality of bearing face to both the teeth of wheel partly encircling the worm and to those of the worm thus segmentally surrounded.

'It remains to be shown how to lay out a worm gear and worm so that this result will be reached, and to exhibit this, the accompanying figures of a worm in position have been prepared

'Accept the teeth on the worm to be 0·65 of the pitch, radially, of which 0·60 p is to be the line of contact with the teeth of the wheel (on the radius and also on the plane through the middle of wheel), with 0·05 p for clearance between the roots and points of worm and wheel teeth.

'Let the teeth of the wheel follow the circle of the worm through the arc $2a$, which ought not to exceed 60°, and is shown as 60° in the figures. Let R = outside radius of worm; R_p = radius of pitch line of worm; p = pitch; F = width of face of wheel at the root of the teeth. Then

$$R_p = \tfrac{1}{2}\{R + (R - 0{\cdot}6p) \operatorname{Cos} a\}$$
$$F = 2\,(R + 0{\cdot}05p) \operatorname{Sin} a.$$

'To simplify the process of drawing worm wheels, it has been usual to make $R = 2p$ and $2a = 60°$. Then

$$R_p = 1{\cdot}606p.$$
$$F = 2{\cdot}05\ p.$$

'It will be found better to limit the number of teeth in the worm wheel to not less than 30; and if any less ratio of speed than 1 in 30 be demanded, to employ double or treble threaded screws.

'The figures consist of fig. 248, (1) a cross-section of worm wheel and worm; (2) a top view of wheel teeth; (3) a top view of worm teeth; fig 249, (4) a side view development on the line or plane H I, or on the inclined face of the wheel teeth, and forming a radial section through the worm; (5, 6, and 7) horizontal sections on the planes or lines A B, C D, and E F, where

the plane A B passes through the middle of the wheel and on the axis of the worm, and the planes C D and E F are parallel to A B, as shown on cross-section of worm wheel and worm.'

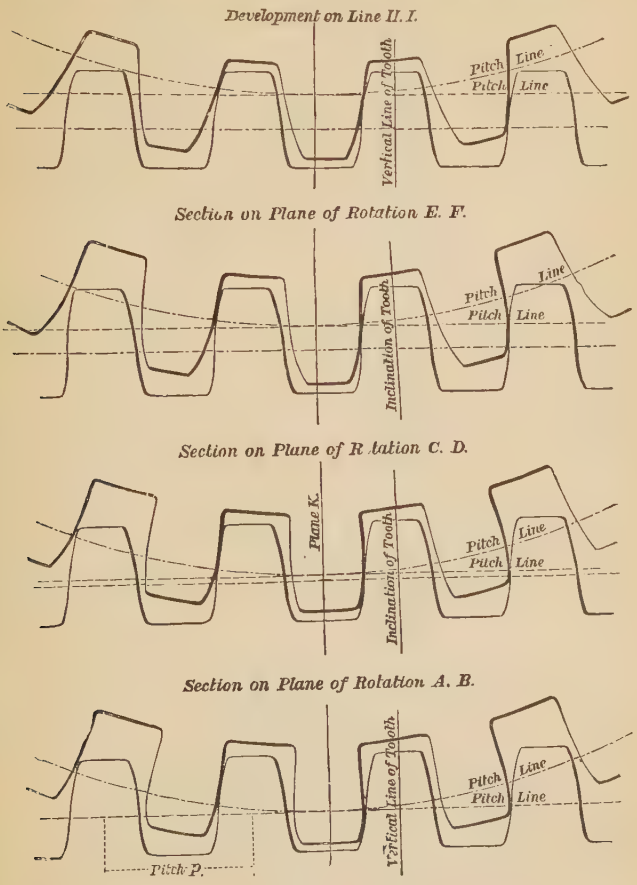


Fig. 249

It will be seen that Mr. Briggs does not explicitly state how the curves of the worm-wheel teeth are to be drawn.

No doubt he did perceive that line contact could be obtained instead of point contact, but probably he had not definitely found a method of designing the worm-wheel teeth to secure that end. He appears to have taken a series of pitch lines for the sections A B, C D, &c., of the worm wheel by scaling off the radii to the circle or radius R p in the cross section (fig. 248) and to have measured off the heights and depths of the teeth also. Then the tooth curves were taken as for ordinary cycloidal spur gearing for these assumed pitch lines.

234. *Accurate method of designing worm-wheel teeth.*— Mr. Briggs's statements led to a re-examination of the mode of designing worm-wheel teeth. The following method is believed to be new. A form of worm thread is assumed, and a number of parallel sections of the worm-wheel tooth are obtained which will work with it. The case has been chosen so as to show in a marked way the difference between the proper sections and those ordinarily adopted.

To obtain correct forms of teeth, the rules applied to the sections in the plane passing through the worm axis normal to the wheel axis must be applied to all sections on planes parallel to that plane ; that is, all the sections of the worm threads and wheel teeth, on planes normal to the wheel axis, must be of forms suitable for a spur wheel and rack of the same pitch. To obtain these sections proceed as follows :—

Draw first the two views, A, B, fig. 250, of the worm and wheel on planes passing through each axis normal to the other axis. With the proportions given above mark off the root and addendum of the teeth, and design the section of the worm threads and wheel teeth on the plane $c\,c$ in accordance with Prof. Willis's principle mentioned above. The sections chosen in the present example are shown at $c\,c$ in fig. 251. The wheel teeth are involute teeth, and the worm threads are similar to those of a rack suitable to work with such teeth. They are bounded by straight lines normal to

Fig. 250

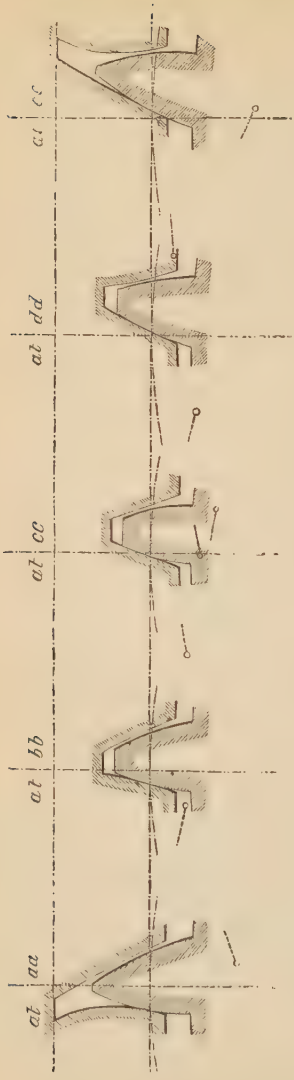


Fig. 251

the base circle tangent. The section of the worm teeth is shown again in fig. 250.

Next in fig. 250, B, draw helices corresponding to any points in the worm-thread section. In the figure three helices are shown corresponding to the point, root, and pitch point of the thread. Then the true section of the worm thread on, for example, the plane *e e*, fig. A, is found in fig. B by projecting the points $\alpha\,\beta\,\gamma$, in fig. A to the corresponding helices in fig. B. We thus get the points $\alpha\,\beta\,\gamma$, $\alpha\,\beta\,\gamma$ marking the section in fig. B, and the same section has been transferred to *e e*, fig. 251. The other sections in that figure were obtained in the same way. It remains to find the corresponding worm-tooth sections, of which only that for *c c* section is as yet determined.

In fig. 252, let *o* be the centre of the worm wheel, A A the worm pitch line, B B the worm-wheel pitch line, and of the worm thread forms let *o* be the one corresponding to the section at *e e* in fig. 251. It will make no difference in the relative motion of the worm and wheel if we suppose the

worm-wheel pitch line to roll along the worm pitch line.

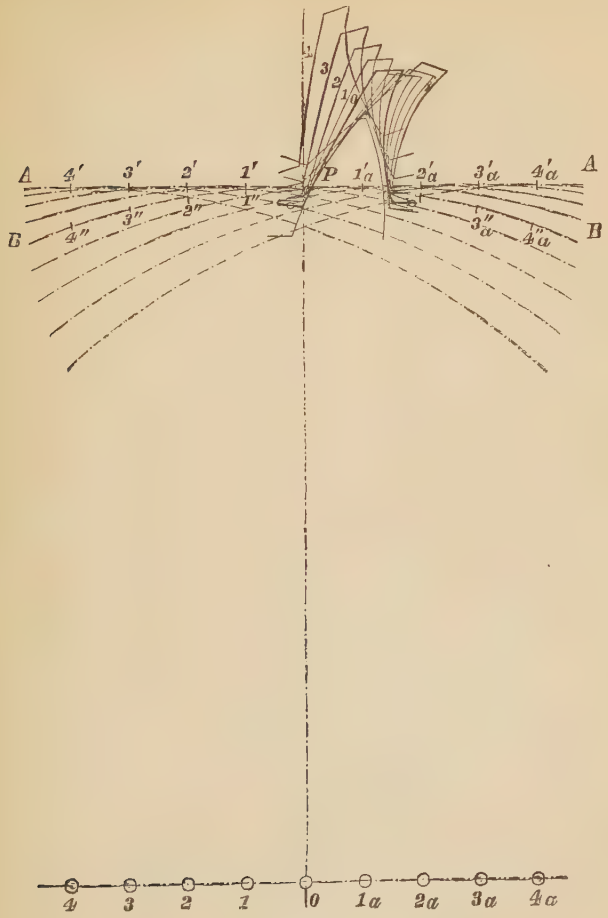


Fig. 252

On a piece of tracing-paper mark off the centre *o*, and the pitch line B B. In order to roll this traced pitch line on A A,

mark off equidistant positions of the centre of the worm wheel, 0, 1, 2, 3, 4, 1*a*, 2*a*, 3*a*, 4*a*, and also the corresponding touching points P, 1′, 2′, 3′, 4′, ... along the worm pitch line. On the traced worm-wheel pitch line take P 1″ = P 1′ ; 1″ 2″ = 1′ 2′ ; and so on. The traced pitch line can then be easily placed in the successive positions shown by the dotted arcs touching the worm pitch line. In each of these positions trace off on the tracing-paper the worm-thread form 0. We shall thus obtain on the tracing-paper the figure of the threads shown at 0, 1, 2, 3, 4. The envelope of these positions is the proper form of the worm-wheel thread. In drawing this, however, the top and side clearance must be left as shown. The side clearance may be 0·04 of the pitch.

In fig. 251, the result of applying this method to four sections other than the central one is shown. It may be remarked that a circular arc can be found by trial for each side of the worm tooth, which agrees accurately enough for any practical purpose with the required envelope of the worm threads. The teeth in fig. 251 are drawn with such circular arcs, the centres of the arcs being marked on the figure. The pattern of the wheel would have to be made by cutting the teeth to templets of the forms found by the construction. The worm wheel in the figure is shown with the front of the teeth concave. But they may be cut off parallel to the rim without altering the correct action of the teeth.

235. The process shown in fig. 251 seems to have suggested to Mr. Last the tracing-paper method of drawing wheel teeth, described in Chapter X. It is easy to see that Mr. Last's mode of rolling the pitch circle described on tracing-paper, along a straight line A A, on which has been drawn the worm-thread section, facilitates the process of finding the worm-wheel tooth section. If a worm wheel is to be cast of the form here described, templets would have to be made of the tooth sections found in fig. 251, and a wood

tooth carved to fit the templets. This could then be used in a wheel-moulding machine. It appears that Messrs. Browne and Sharpe ('Treatise on Gearing:' Providence, 1886) have adopted exactly the involute form shown above for the hobs for cutting worm wheels. Their cut worm wheels, therefore, must be exactly of the form found by construction above.

236. *Strength of worm wheels.*—The resultant pressure on the teeth (friction being neglected) is in the direction of the normal to the faces of the teeth at the point of contact, or in the direction in which the normal pitch is measured. Hence it is the tendency to break under the action of the force acting in that direction, which has to be considered in estimating the strength of the teeth.

The worm is usually at least as strong as the worm wheel, hence it is only necessary to consider the strength of the latter. Let n_2 be the normal divided pitch of the worm wheel, c_2 its circumferential pitch, r_2 its radius, and θ_2 the angle between the threads and the axis :

$$n_2 = c_2 \cos \theta_2 = \frac{2 \pi r_2}{r_2} \cos \theta_2, \quad . \quad . \quad (28)$$

where $\cos \theta_2 = \frac{P e}{P f}$ in fig. 245. Let Q be the resistance to rotation at the circumference of the worm wheel. Then the pressure acting normally to the teeth is $Q_n = \frac{Q}{\cos \theta} = Q \frac{P f}{P e}$. The worm wheel is equivalent to a spur wheel resisting the force Q_n at the pitch line, and having the pitch n_2. Hence the normal pitch n_2 may be obtained by the rules for the teeth of spur wheels. Then $c_2 = \frac{n_2}{\cos \theta} = n_2 \frac{P f}{P e}$.

When the shafts are at right angles, the angle θ_2 is often small, so that $\cos \theta = 1$ nearly. Then the worm wheel is approximately equivalent to a spur wheel resisting the force Q, and having the pitch c_2. Hence, when θ_2 is small, the obliquity of the teeth may be neglected in calculating the

pitch. The width of the face of the worm wheel is about $1\frac{1}{2}$ times the pitch. In calculating the size of the worm shaft, from the resistance Q overcome, friction should not be neglected. The twisting moment acting on the worm shaft is $\frac{Q\,r_2}{\eta}\,\frac{r_1}{r_2} = Q\,r_2\,\frac{r_1\,p_1 + r_1}{r_2\,p_1}$ nearly.

CHAPTER XII

BELT GEARING

The term *belt*, *band*, or *strap* is applied to a flexible connecting piece, which drives a rotating piece termed a *pulley* by its frictional resistance to slipping. The belt always acts by tension.

Material of belting.—Belts are most commonly of leather tanned by oak bark. The best part of the hide is cut into strips, which are united into lengths by cementing, lacing, or riveting. Special kinds of leather are used where great strength is required. Pure vulcanised india-rubber, or more often india-rubber with interposed plies of strong canvas, is often used, especially in wet places where leather is unsuitable. Gutta-percha has been used, but it stretches permanently too much. Waterproofed cotton woven belting is now a good deal used, and can be made of great width (up to 60 inches). It may be applied in the open air and where exposed to damp. Llama-hair belts have also been introduced.

Ordinary belts are flat belts, and are used on pulleys with flat or slightly rounded rims. But round belts are also used running in pulleys with V-shaped grooves. Rope belting will be treated in another chapter. The smaller round belts are of catgut, or of twisted leather, which is cheap and effective if the tension is small.

237. *Velocity ratio in belt transmission.*—A belt is not used in cases where a very exact velocity ratio is necessary, hence it is generally accurate enough to regard the belt as inextensible. If also there is no slipping of the belt on the pulley, the velocity of the belt and the surface velocities of

the pulleys must all be equal. Let v be the velocity of the belt, d_1 d_2 the diameters of the pulleys, and N_1 N_2 their revolutions per minute—

$$\left.\begin{aligned}\pi\, d_1\, N_1 &= V\\ \pi\, d_2\, N_2 &= V\end{aligned}\right\} \therefore \frac{d_1}{d_2} = \frac{N_2}{N_1} \quad . \quad . \quad (1)$$

These equations are in strictness only true when the belt is infinitely thin. When the belt has a thickness δ, the effective diameters of the pulleys are $d_1+\delta$, and $d_2+\delta$. Then,

$$\frac{N_2}{N_1} = \frac{d_1+\delta}{d_2+\delta} \quad . \quad . \quad . \quad (1\,a)$$

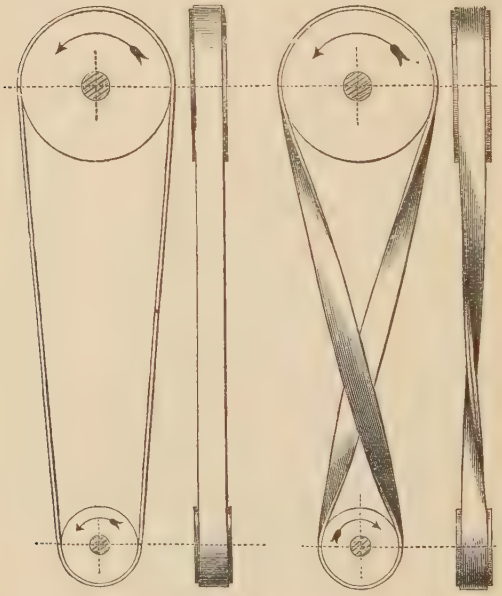


Fig. 253 Fig 254

As the belt thickness is generally small compared with the pulley diameter, δ may be neglected without any great

error, but it should be remembered that, in all questions of velocity ratio in belting, the virtual diameter of the pulley is the diameter measured to the centre of the belt.

238. *Endless Belt.*—When one shaft is driven from another, a pulley is placed on each shaft, and an endless belt is strained over the two pulleys. The belt may be an open belt (fig. 253) or a crossed belt (fig. 254). In the former case the two shafts rotate in the same direction. In the latter case they rotate in opposite directions.

239. *Length of belts.*—Let D and d be the diameters of the two pulleys in inches; c, their distance apart, from centre to centre; L, the length of the belt. Also, let $D+d=\Sigma$, and $D-d=\Delta$.

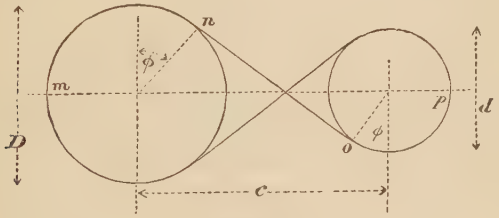


Fig. 255

For a crossed belt (fig. 255) the total length—

$$=L=2(m\,n+n\,o+o\,p)$$

$$=\left(\frac{\pi}{2}+\phi\right)D+2\,c\cos\phi+\left(\frac{\pi}{2}+\phi\right)d$$

$$=\left(\frac{\pi}{2}+\phi\right)\Sigma+2\,c\cos\phi \quad . \quad . \quad . \quad (2)$$

$$\sin\phi=\frac{D+d}{2\,c}=\frac{\Sigma}{2\,c} \quad . \quad . \quad . \quad . \quad (3)$$

The length of the belt is obtained thus:—Calculate the value of $\sin\phi$. From a table of natural sines and cosines find the nearest values of $\cos\phi$ and ϕ, the latter being ex-

pressed in circular measure. Then eq. (2) gives the belt length. If ϕ is found or measured off the drawing in degrees, the circular measure of the angle is obtained by multiplying by 0·0175.

With a crossed belt ϕ depends only on $D+d$. Hence, if Σ and c are constant for two or more pairs of pulleys, the same belt will run on any pair of pulleys of the set.

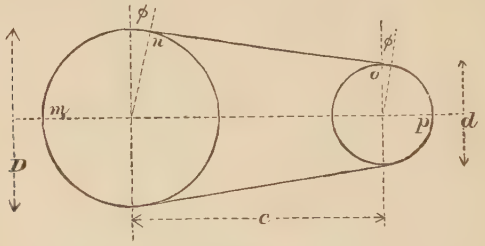


Fig. 256

When the belt is an open one (fig. 256) the equations are rather less simple—

$$L=2\,(m\,n+n\,o+o\,p)$$

$$=\left(\frac{\pi}{2}+\phi\right)D+2\,c\cos\phi+\left(\frac{\pi}{2}-\phi\right)d$$

$$=\frac{\pi}{2}\Sigma+\phi\Delta+2\,c\cos\phi \quad . \quad . \quad . \quad (4)$$

$$\text{Sin}\ \phi=\frac{\Delta}{2\,c};\ \cos\phi=\sqrt{\left(1-\frac{\Delta^2}{4\,c^2}\right)} \quad . \quad (5)$$

For an open belt ϕ is generally small, so that,

$$\phi=\sin\phi, \text{ nearly}$$

$$\therefore L=\frac{\pi}{2}\Sigma+2\,c\left\{\frac{\Delta^2}{4\,c^2}+\sqrt{\left(1-\frac{\Delta^2}{4\,c^2}\right)}\right\}$$

$$=\frac{\pi}{2}\Sigma+2\,c\left\{1+\frac{1}{8}\,\frac{\Delta^2}{c^2}\right\} \text{ nearly} \quad . \quad (6)$$

Hence, if an open belt runs on a pair of pulleys, the sum and difference of whose diameters are Σ_1 and Δ_1 and the same belt is also to run on another pair of pulleys, the sum and difference of whose diameters is Σ_2 and Δ_2, since the length of the belt is the same in the two cases,

$$\frac{\pi}{2}\Sigma_1 + 2c\left\{1 + \frac{1}{8}\frac{\Delta_1^2}{c^2}\right\} = \frac{\pi}{2}\Sigma_2 + 2c\left\{1 + \frac{1}{8}\frac{\Delta_2^2}{c^2}\right\}$$

$$\Sigma_2 = \Sigma_1 + \frac{\Delta_1^2 - \Delta_2^2}{2\pi c} \quad . \quad . \quad (7)$$

It is accurate enough for practical purposes to calculate the diameters D_2 and d_2 as if the belt were a crossed belt. Then, taking Δ_2 = the difference of these diameters, find the value of Σ_2. From that value of Σ_2 recalculate the diameters D_2 and d_2, using eq. (1) or eq. (1*a*).

240. *Speed cones.*—When a shaft running at a constant speed has to drive a machine at several different speeds, sets of pulleys are used which are termed stepped speed cones.

The speed cones (fig. 257) are placed opposite one another, so as to form a series of pairs of pulleys, and by shifting the belt from one pair to another the speed of the machine is altered. In designing these speed cones the ratio of the diameters of each pair depends on the speeds of the shafts, and the sum of the diameters should be so arranged that the same belt will work on any pair of the set without alteration of length.

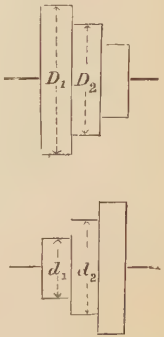


Fig. 257

Let D_1 d_1 be the diameters of one pair; D_2 d_2 the diameters of another pair. Let N be the number of revolutions of the shaft on which D_1 and D_2 are placed; n_1 and n_2 the revolutions of the other shaft, when the belt is on d_1 and d_2 respectively.

If the belt is a crossed belt, from eq. (1)—

$$\frac{D_1}{d_1}=\frac{n_1}{N} \qquad \frac{D_2}{d_2}=\frac{n_2}{N}$$

also, $$D_1+d_1=D_2+d_2=\Sigma.$$

Hence,

$$\left.\begin{aligned} D_2&=\frac{n_2}{N+n_2}\Sigma \\ d_2&=\frac{N}{N+n_2}\Sigma \end{aligned}\right\} \quad . \quad . \quad (8)$$

If the belt is an open belt, the diameters will be slightly different. Let $\Delta_1=D_1-d_1$; $\Delta_2=D_2-d_2$; $\Sigma_1=D_1+d_1$; $\Sigma_2=D_2+d_2$. If the belt were a crossed belt, we should have,

$$D_2=\frac{n_2}{N+n_2}\Sigma_1\ ;\ d_2=\frac{N}{N+n_2}\Sigma_1$$

and since the diameters for an open belt are but little different,

$$\Delta_2=D_2-d_2=\frac{n_2-N}{N+n_2}\Sigma_1 \text{ nearly.}$$

Then from eq. (7),

$$\Sigma_2=\Sigma_1+\frac{\Delta_1^2-\Delta_2^2}{2\pi c}$$

And from eq. (1),

$$\frac{D_2}{d_2}=\frac{n_2}{N}$$

Hence, more exactly,

$$\left.\begin{aligned} D_2&=\frac{n_2}{N+n_2}\Sigma_2 \\ d_2&=\frac{N}{N+n_2}\Sigma_2 \end{aligned}\right\} \quad . \quad . \quad (9)$$

Hence, the process of designing a set of speed cones is

this :—Having given the speed N of the driving shaft, decide on the speeds n_1, n_2, n_3 . . . . of the driven shaft. Choose a diameter for one of the pulleys of the first pair, and find the diameter of the other by equation (1). The values of Σ_1 and Δ_1 can then be found. From these the diameters of any other pair of pulleys can be found by the equations above.

241. *Resistance to slipping of a belt on a pulley.*—Let fig. 258 represent a belt strained over a pulley and on the point of slipping from T_1 towards T_2. Then the tension T_2 must be greater than the tension T_1, by the amount of the frictional resistance to slipping at the surface of the pulley.

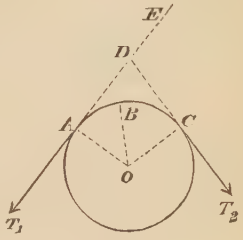


Fig. 258

Let B be any point of contact, and let the tension at B=T. Let the angle A O B in circular measure be θ; the arc A B=s; the radius A O=r; the normal pressure of the belt on the pulley estimated per unit of arc =p; and the coefficient of friction =μ.

Consider a small length, ds of the belt at the point B. The tensions at the ends of that small length are T and T+dT, so that $\frac{d\,T}{d\,s}$ is the increase of tension, per unit length of the arc of contact. But in unit length of belt the friction is μp,

$$\therefore \frac{d\,T}{d\,s}=\mu\,p$$

Let $d\theta$ be the small angle at the centre corresponding to the arc $a\,b=d\,s$ (fig. 259). Then the pressure R on the arc $a\,b$ is the resultant of the tensions T and T+dT acting tangentially to the pulley at the extremities of the arc. Neglecting dT, we have

$$R=2\,T\sin\frac{d\,\theta}{2}=T\,d\,\theta \text{ nearly.}$$

But $R = p\,ds.$

$$\therefore p = T\frac{d\theta}{ds} = \frac{T}{r}.$$

Combining these equations and remembering that $ds = r\,d\theta$,

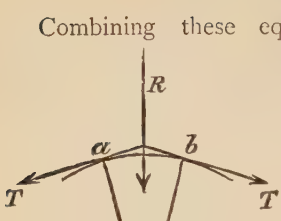


Fig. 259

$$\frac{dT}{ds} = \frac{dT}{r\,d\theta} = \mu\frac{T}{r}$$

$$\frac{dT}{T} = \mu\,d\theta$$

$$\int_{T_1}^{T_2}\frac{dT}{T} = \mu\int_{o}^{\theta} d\theta$$

$$hyp.\ log.\ \frac{T_2}{T_1} = \mu\theta \quad . \quad . \quad . \quad (10)$$

where θ is the angle A O C, or, what is the same thing, the angle C D E, expressed in circular measure, or the arc A C ÷ radius A O. If the angle θ is measured in degrees, it can be reduced to circular measure, by multiplying by $\frac{\pi}{180}$ or by 0·0175.

This equation may be put in the form

$$k = \frac{T_2}{T_1} = \epsilon^{\mu\theta} \quad . \quad . \quad (11)$$

where $\epsilon = 2{\cdot}71828$, the base of the system of natural logarithms; and for simplicity k is put for the ratio of the tensions in the two parts of the belt. Hence $\mu\theta$ is the hyperbolic or natural logarithm corresponding to the number $T_2 \div T_1$. As common logarithms are more convenient,

Common log. $\frac{T_2}{T_1} = 0{\cdot}434\ \mu\theta$ if θ is in circular measure.

$= 0{\cdot}007578\ \mu\theta$ if θ is in degrees.

$= 2{\cdot}729\ \mu n$ if n is the fraction of the circumference embraced by the belt.

Hence, if the right-hand member of either of these equations is calculated, the value obtained is the logarithm of $T_2 \div T_1$ or k. The natural number corresponding to that logarithm, found by means of a table of logarithms, is the value of

$T_2 \div T_1$ or k. This value is the greatest value of the ratio of the tensions consistent with the belt not slipping.

242. *The coefficient of friction.*—The value of μ for belts varies from 0·15 to 0·56 in different cases. For leather belting on iron pulleys, in an ordinary condition of working, $\mu = 0{\cdot}3$ to 0·4. The experiments of Messrs. Briggs and Towne appear to show that the latter value may safely be taken.[1] For wire rope running on the bottom of a grooved pulley, $\mu = 0{\cdot}15$, and if the pulley is bottomed with leather or gutta percha, $\mu = 0{\cdot}25$.

The following table will give the values of $T_2 \div T_1$ or k for all cases likely to occur, with accuracy enough for most practical purposes :—

Greatest Value of the Ratio of Tensions on Tight and Slack Sides of Belting from eq. 11

Angle embraced by belt $\theta =$			Ratio of Tensions $k =$			
In degrees	In circular measure	In fraction of circumference	$\mu = 0{\cdot}2$	$\mu = 0{\cdot}3$	$\mu = 0{\cdot}4$	$\mu = 0{\cdot}5$
30	·524	·083	1·110	1·170	1·233	1·299
45	·785	·125	1·170	1·266	1·369	1·481
60	1·047	·167	1·233	1·369	1·521	1·689
75	1·309	·208	1·299	1·481	1·689	1·924
90	1·571	·250	1·369	1·602	1·874	2·193
105	1·833	·319	1·443	1·733	2·082	2·500
120	2·094	·334	1·521	1·875	2·312	2·851
135	2·356	·375	1·602	2·027	2·565	3·247
150	2·618	·417	1·689	2·194	2·849	3·702
165	2·880	·458	1·778	2·372	3·163	4·219
180	3·142	·500	1·875	2·566	3·514	4·808
195	3·403	·541	1·975	2·776	3·901	5·483
210	3·665	·583	2·082	3·003	4·333	6·252
240	4 188	·666	2·311	3·514	5·340	8·119
270	4·712	·750	2·566	4·112	6·589	10·55
300	5·236	·833	2·849	4·808	8·117	13·70

243. *Tensions in an endless belt.*—Let an endless belt be strained over two pulleys with an initial tension T_0. At the

[1] 'Journal of Franklin Institute,' 1868.

moment the driving pulley begins to move the belt is stretched on the driving side and the tension increased, whilst the other side of the belt is shortened and the tension diminished. Since the lengthening of the tight and the shortening of the slack side must be equal in amount, the average tension remains unaltered. That is,

$$\frac{T_2+T_1}{2}=T_0 \quad . \quad . \quad . \quad (12)$$

This process goes on till the force T_2-T_1, tending to rotate the driven pulley, is sufficient to overcome its resistance to motion. The driven pulley then rotates, the condition of the belt remaining permanent till the motion ceases again. It is necessary, however, that the initial tension should be sufficient to prevent slipping on either of the pulleys.

Let P be the resistance at the circumference of the driven pulley. Then

$$P=T_2-T_1 \quad . \quad . \quad . \quad . \quad . \quad (13)$$

But if H is the number of horses power transmitted, V the velocity of the circumference of the pulley or of the belt,

$$P\,V=550\,H$$

$$P=\frac{550\,H}{V} \quad . \quad . \quad . \quad . \quad . \quad (13a)$$

If N = number of revolutions of pulley per minute, and d=diameter of pulley in inches, then $V=\frac{\pi d N}{12\times 60}$, and consequently

$$P=126{,}000\,\frac{H}{d\,N} \quad . \quad . \quad . \quad . \quad . \quad (13b)$$

244. *Tensions in a belt transmitting a given horse-power.*—Suppose that the value of P is obtained from the equations just given, and the value of k from equation 11 or the table corresponding to it. Then from equation 13

$$\left.\begin{aligned} T_2&=P\frac{k}{k-1}=x\,P\\ T_1&=P\frac{1}{k-1}=y\,P\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (14)$$

Table to Facilitate the Calculation of the Belt Tensions

$\theta =$			Values of x for				Values of y for			
In degrees	In circ. measure	In fractions of circumference	$\mu = 0{\cdot}2$	$\mu = 0{\cdot}3$	$\mu = 0{\cdot}4$	$\mu = 0{\cdot}5$	$\mu = 0{\cdot}2$	$\mu = 0{\cdot}3$	$\mu = 0{\cdot}4$	$\mu = 0{\cdot}5$
30	·524	·083	10·09	6·89	5·29	4·35	9·09	5·88	4·29	3·34
45	·785	·125	6·89	4·76	3·71	3·08	5·88	3·76	2·71	2·08
60	1·047	·167	5·29	3·71	2·92	2·45	4·29	2·71	1·92	1·45
75	1·309	·208	4·35	3·08	2·45	2·08	3·34	2·08	1·45	1·08
90	1·571	·250	3·71	2·66	2·14	1·85	2·71	1·66	1·14	·840
105	1·833	·319	3·26	2·37	1·93	1·67	2·26	1 36	·924	·667
120	2·094	·334	2·92	2·14	1·77	1·54	1·92	1·14	·762	·541
135	2·356	·375	2·66	1·98	1·64	1·44	1·66	·984	·649	·444
150	2·618	·417	2·45	1·84	1·54	1·37	1·45	·840	·541	·370
165	2·880	·458	2·29	1·73	1·47	1·31	1·29	·730	·462	·311
180	3·142	·500	2·14	1·64	1·40	1·26	1·14	·638	·398	·262
195	3·403	·541	2·03	1·56	1·35	1·22	1·03	·563	·345	·223
210	3·665	·583	1·93	1·50	1·30	1·19	·926	·499	·300	·190
240	4·188	·666	1·76	1·40	1·23	1·14	·763	·398	·230	·140
270	4·712	·750	1·64	1·32	1·18	1·10	·639	·322	·179	·105
300	5·236	·833	1·54	1·26	1·14	1·08	·541	·262	·140	·079

245. *Strength of leather belting.*—The ultimate strength of the leather used for belting is 3,000 to 5,000 lbs. per sq. in., or, stating it more conveniently, 750 to 1,200 lbs. per inch width of belt. At the laced joints the strength is reduced to 0·3 of this, or, say, 250 to 400 lbs. per inch of width. The following table gives some tests of belting and of different kinds of joint fastening :—

Strength of Leather Belts and Fastenings

	Tenacity in lbs. per inch of width	Authority
Single Leather Max. . . .	1,600	Riehle Bros.
,, ,, Min. . . .	700	,, ,,
,, ,, Mean . . .	1,280	,, ,,
,, ,, Max. . . .	1,272	Unwin
,, ,, Min. . . .	616	,,
,, ,, Mean . . .	964	,,
Double Belt, copper sewn . . .	1,110	,,
Single Belt, ordinary laced joint . .	473	,,
,, ,, butt laced	265	,,
,, ,, joint scarfed and glued .	544	,,
,, ,, grip fastener . . .	242	,,
,, ,, Crowley's fastener . .	635	,,
,, ,, hose riveted, rivets in two rows . . .	394	Watertown

The greatest safe working tension (since the belt is subject to only one kind of stress) is about $\frac{1}{4}$th of these values. Usually a belt has cemented and riveted joints made at the belting factory, and a laced joint, which is made when the belt is put in place, and which serves for tightening up the belt if it wears slack. Hence, the greatest working tension is that corresponding to the laced joint, and may be taken at 320 lbs. per sq. in. of the belt section.

The thickness of the belt varies from $\frac{3}{16}$ to $\frac{5}{16}$ inch, if the belt is a single one ; and from $\frac{3}{8}$ to $\frac{3}{4}$ inch, if the belt is a double one. Hence, calling f the safe working tension per inch width of belt, and δ the belt thickness,

$$f = 320\,\delta \quad . \quad . \quad . \quad . \quad (15)$$

Thickness of Belt$=\delta=$

$\frac{3}{16}$	$\frac{7}{32}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$

Working Tension in lbs. per inch width$=f=$

60	70	80	100	120	140	160	180	200	220	240

246. *Width of belt for a given working stress.*—The greatest tension on the belt is the tension T_2 on the driving side. Then, if f is the safe working stress per inch width, the width of the belt must be

$$\beta = \frac{T_2}{f} = \frac{k}{k-1} \cdot \frac{P}{f} = x\frac{P}{f} \quad . \quad . \quad . \quad (16)$$

where the values of x are given in the preceding Table and P is obtained from eq. (13*a*). In practice the working stress is commonly taken at 80 lbs. per inch width for single belts and 140 lbs. per inch width for double belts.

247. *Horses' power per inch width of belt.*—From (16) we get for a belt one inch wide, $P = f/x$. From eq. (13*a*), taking f at the values just given, and putting H for the horses' power transmitted and V for the velocity of the belt in feet per second,

$$H = \frac{8}{55} \cdot \frac{V}{x} \text{ for single belts.}$$

$$= \frac{14}{55} \cdot \frac{V}{x} \text{ for double belts.}$$

Hence the horses' power transmitted per inch width is $\frac{8}{55} \cdot \frac{V}{x}$ and $\frac{14}{55} \cdot \frac{V}{x}$ for single and double belts respectively. The following Table gives values of this expression for different values of x. For any given pair of pulleys look out the value of x in the preceding Table. Then the number of horses' power in the following Table, opposite the nearest value of x, multiplied by the velocity of the belt in feet per

second, gives the horses' power transmitted per inch width of belt.

Horses' Power transmitted per Inch Width of Belt for each Foot of Velocity per Second

$x=$	Horses' power per foot per second		$x=$	Horses' power per foot per second	
	Single Belt	Double Belt		Single Belt	Double Belt
1·0	·1455	·2546	2·74	·0528	·0924
1·25	·1166	·2040	3·00	·0485	0848
1·50	·0970	·1690	3·25	·0448	·0784
1·75	·0831	·1460	3·50	·0416	·0728
2·00	·0727	·1272	3·75	·0389	·0680
2·25	·0644	·1130	4·00	·0363	·0636
2·50	·0582	·1020			

The most common value of x in ordinary cases is 2. Then in round numbers the H. P. transmitted per inch width of belt is v/14 for single belt and v/8 for double belt.

248. *Rough calculations of the size of belts.*—In a great many cases in practice the belt embraces about 0·4 of the circumference of the pulley on which it is most liable to slip,[1] and the coefficient of friction is at least 0·3. Then, $T_2/T_1 = 2$. When this is the case, the following simple rules may be used:—

$$\left.\begin{aligned} &\text{Driving force} && = P = \frac{550\ H}{V} \\ &\text{Greatest tension} && = T_2 = 2\ P \\ &\text{Initial tension} && = T_0 = 1\tfrac{1}{2}\ P \\ &\text{Width of belt} && = \beta = \frac{2\ P}{f} \end{aligned}\right\} \quad . \quad . \quad . \quad (17)$$

The following approximate Table gives the width of belt calculated by these rough rules, the belt being assumed to be $\frac{1}{32}$nds of an inch in thickness, and carrying safely 70 lbs. tension per inch of width:—

[1] That is, the pulley having the smaller arc of contact.

Velocity of belt in ft. per sec.	Width of belt in ins. $\frac{1}{4}$ inch thick when the horses' power transmitted is									
	1	2	3	4	5	$7\frac{1}{2}$	10	15	20	25
1	15·7	31·4	47·0	63·0	...	..	...	...	...	...
$2\frac{1}{2}$	6·3	10·6	18·8	25·2	31·2	46·8	...	...	...	...
5	3·1	6·3	9·4	12·6	15·6	23·6	31·4	47·2	...	...
$7\frac{1}{2}$	2·1	4·2	6·3	8·4	10·4	15·6	21·0	31·2	42·0	52·4
10	1·5	3·2	4·7	6·4	7·8	11·8	15·7	23·6	31·4	39·2
$12\frac{1}{2}$	1·3	2·5	3·7	5·0	6·4	9·4	12·6	18·8	25·2	31·2
15	1·1	2·1	3·1	4·2	5·2	7·8	10·5	15·6	21·0	26 2
20	·79	1·6	2·4	3·2	3·9	5·9	7·9	11·7	15·7	19·6
25	·63	1·3	1·9	2·6	3·1	4·7	6·3	9·4	12·6	15·6
30	...	1·1	1·6	2·2	2·6	3·9	5·2	7·8	10·5	13·1
35	...	...	1·3	1·7	2·2	3·4	4·5	6·8	9·0	11·2
40	...	...	...	1·5	2·0	2·9	3·9	5·9	7·8	9·8
45	...	...	...	...	1·8	2·6	3·5	5·2	7·0	8·8
50	...	...	..	...	1·6	2·4	3·2	4·7	6·3	7·8
60	...	...	...	...	1·3	2·0	2·6	3·9	5·2	6·5
70	...	...	...	...	1·1	1·7	2·2	3·4	4·5	5·6
80	...	...	...	...	...	1·5	2·0	2·9	3·9	4·9
90	...	...	...	...	...	1·3	1·8	2·6	3·5	4·4
100	...	...	...	...	...	1·2	1·6	2·4	3·1	3·9

249. *American high-speed belting.*—In America belts are used for transmitting very great amounts of power, in conditions somewhat different from those described above. If the belt is very wide a partial vacuum is formed between the belt and pulley at a sufficiently high velocity. The pressure between the belt and the pulley is then greater than that calculated from the tensions in the belt, and the resistance to slipping is greater. This has the advantage not only of permitting a greater power to be transmitted by a given belt, but also of diminishing the strain on the shafting. For belts used in this way the speed is usually made from 4,000 to 6,000 feet per minute, by using sufficiently large pulleys. The belts are of single thickness, and as the roughness of the belt and pulley is disadvantageous rather than otherwise, the hair side of the belt is placed next the pulley. The widths which the Americans give to belts put up on this principle are such

that the driving pressure P is 50 to 67 lbs. per inch width of the belt Consequently the greatest strain in the belting T_2 is about 156 to 185 lbs. per sq. in. of section. The belts are generally not more than $3\frac{1}{2}$ to 4 feet wide. These details are taken from Achard, 'Proc. Inst of Mechanical Engineers,' 1881, p. 60.

250. *Influence of the elasticity of the belt on the velocity ratio.*—Let s be the length of belt which runs off either pulley in the unit of time, the belt being measured in its unstrained condition. In working, the length s is extended to s_2 by the tension T_2 on the driving side, and to s_1 by the tension T_1 on the slack side. Since the elongation is proportional to the straining force (§ 26),

$$s_2=(1+a\,T_2)\,s, \text{ and } s_1=(1+a\,T_1)\,s,$$

where a is the elongation of one foot of belt by one pound of tension. The driving pulley receives s_1 feet of belt in the unit of time, and the driven pulley s_2 feet. Hence the velocities of the pulley circumferences are not exactly the same (as assumed in § 210) but are equal to s_2 and s_1 respectively,

$$\left.\begin{aligned}\pi\,d_1\,N_1&=s_1\\ \pi\,d_2\,N_2&=s_2\end{aligned}\right\}\ \therefore\ \frac{N_1}{N_2}=\frac{d_2}{d_1}\cdot\frac{s_1}{s_2}=\frac{d_2}{d_1}\left(\frac{1+a\,T_1}{1+a\,T_2}\right). \qquad (18)$$

where N_2 is the number of revolutions of the driving, and N_1 the number of revolutions of the driven pulley. According to M. Kretz, $\frac{1+a\,T_1}{1+a\,T_2}=0{\cdot}975$ for new, and $0{\cdot}978$ for old belts. Hence,

$$a=\frac{{\cdot}00011}{\beta\,\delta} \text{ for old belts}$$

$$=\frac{{\cdot}00015}{\beta\,\delta} \text{ for new belts.}$$

$\beta\,\delta$ being the area of section of the belt in sq. ins. The

velocity of the driven pulley is about 2 per cent. less than it would be if the belt were inelastic. If motion is transmitted through several belts, the loss of velocity due to this cause would become important. This loss of velocity may be termed the *slip* due to elasticity of the belt.

251. *Effect of centrifugal tension on the strength of belts.*—When belts run at high speeds, part of the belt tension is expended in deviating the belt as it passes over the curved surface of the pulley. Hence, a given belt tension produces a less normal pressure on the pulley, and less resistance to slipping, in consequence of the centrifugal force of the belt. The weight of belting is about $w = 0{\cdot}43\ \beta\ \delta$ lbs. per foot of length, where β and δ are in inches. The centrifugal force of one foot length of belting is $w\frac{v^2}{gr}$ lbs.

The normal pressure on the pulley is $p = \frac{T}{r} - \frac{w v^2}{g r}$ where the second term becomes unimportant at small velocities, as has been assumed above.

Hence the greatest tension in the belt is,

$$T_2 + \frac{w v^2}{g} \quad . \quad . \quad (19)$$

and the belt width must be calculated for that tension instead of for T_2.

Hence, if β is the width of the belt when the centrifugal tension is neglected, its width when centrifugal tension is allowed for will be

$$\beta_1 = \frac{743\ \beta}{743 - \frac{v^2}{g}} = \frac{23924\ \beta}{23924 - v^2} \quad . \quad (19.a)$$

The influence of centrifugal tension was first pointed out by Professor Rankine ('Millwork,' p. 532).

252. *Single, double, and combined belting. Joints in belting.*—The leather used for belting is of ox-hide tanned

with oak bark, and only the best part of the hide, termed

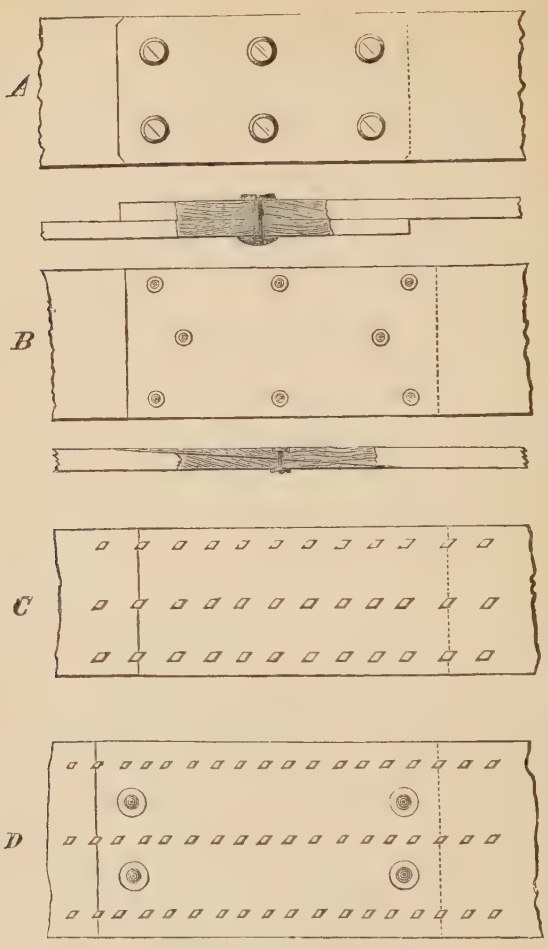


Fig. 260

the butt, is used. The butts are cut into strips of the width

required, and joined together to form a belt of any required length. The joints are made by paring down the ends of the strip, overlapping them, and cementing them with glue. They are then either sewn, laced, or riveted as an additional precaution. Fig. 260 *C* shows a cemented and laced joint; the overlap is about 7 inches long, and the laces $1\frac{1}{2}$ inch apart, extending an inch beyond the overlap at each end. Sometimes a few rivets are used in addition to the lacing. Fig. 260 *B* shows a cemented and riveted joint, the overlap 6 to 7 inches long, and having about one rivet to $2\frac{1}{4}$ or $2\frac{1}{2}$ sq. ins. of overlap. Fig. 260 *D* shows a laced and riveted joint.

In an endless belt one joint must be uncemented, so that it can be easily broken when the belt requires to be tightened. This joint may be a laced joint, like that previously described, or it may be made with belt screws shown in fig. 260 *A*. These belt screws are of iron with a very flat nut. The length of overlap may be 6 ins., and there may be one screw to 6 or 8 sq. ins. of overlap. This joint is more clumsy than a laced joint, but is very easily broken or made. The laces commonly used are strips of white leather tanned with alum.

Fig. 261

A very convenient belt screw with a right- and left-hand screw thread has been introduced lately (Sonnenthal's Patent), and is shown in fig. 261. The screws are made of steel

or gun-metal, and are less likely to work loose than ordinary screws.

Another convenient belt fastening intended to replace laces is that shown in fig. 262. The fastener is shown at *a*, the belt in process of fastening at *b*, and the belt in running condition at *c*.

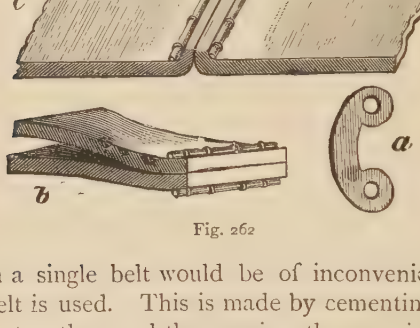


Fig. 262

When a single belt would be of inconvenient width, a double belt is used. This is made by cementing two strips of leather together, and then sewing them or riveting them. There may be about one rivet to 3 to 4 sq. ins. of belt. The double belt is more rigid than a single belt, and does not work satisfactorily unless there is ample distance between the pulleys, and the pulleys are not less than 3 feet diameter.

When a very broad belt is required to connect two shafts which are not parallel (that is, when the belt has a half or quarter twist), it does not work well, because its rigidity prevents its lying down in contact with the pulleys. It comes in contact with the pulleys on one side only. Messrs. Tullis, of Glasgow, have in such cases employed several narrow belts instead of a single wide one. These run side by side on the same pulleys, and are kept parallel by cross strips of leather riveted to them. Thus, for instance, instead of a 12-inch belt, three four-inch belts may be used, connected by cross strips $1\frac{1}{2}$ inch wide, at intervals of about

12 inches. A combined belt of this kind runs parallel and fits the pulleys better than a very wide belt.

Leather has a smooth or grain side and a rough or flesh side. Usually belts are run with the flesh side next the pulley, and care is taken that in twisted belts the same side of the leather is kept next all the pulleys. In America it seems to have been found that the driving power of the belt is greater with the grain side next the pulley. Castor-oil applied to the grain side of a belt makes it more supple. Dubbin and boiled linseed oil are sometimes applied to the flesh side.

Of late an excellent leather belting has been manufactured, under the name Victoria Belting. In making this the spongy and weak inside part of the leather is pared away by machinery, and the belt reduced to a uniform thickness. Although thinner and more flexible than ordinary belting, the Victoria Belting is equally strong. Usually two thicknesses of pared leather are cemented together and riveted with copper rivets. The belt is then virtually a double belt, though little thicker than ordinary single belting. With this belting the hair side of the belt runs in contact with the pulley.

253. *Cotton belting* can now be obtained, made of 4 to 10 thicknesses of American cotton duck stitched together. It is waterproof, and cheaper and stronger than leather. The ordinary widths are, for 4-ply, $1\frac{1}{2}$ to 6 ins.; for 6-ply, 3 to 12 ins.; for 8-ply, 6 to 30 ins.; and for 10-ply, 12 to 60 ins. According to a test made for the manufacturers, 8-ply cotton belting is twice as strong as double leather belting, the breaking stress being 1,135 lbs. per inch of width. The best way of making the joints is by butting the ends of the belts and using a special metal fastening. A test of this gave a breaking strength of 330 to 540 lbs. per inch of width (thickness of belt not stated). Ordinarily 4-ply is taken as equivalent to single-leather, and 8-ply to double-leather belting.

254. *Special forms of belting.—Chain belting or link belting* is made of a series of short links of leather strung together on wire pins (fig. 263). It is more flexible than ordinary belting, can be made of great width, and is very easily shortened by taking out a set of links and rejoining

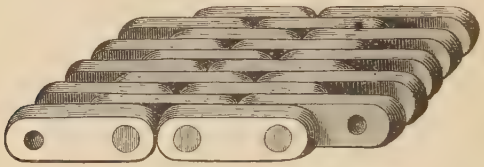


Fig. 263

with a wire pin. Messrs. Tullis make this belt with an arched section to suit the curve of the pulley. It then has an even bearing, and the wire pins are not bent. It is said to transmit 25 per cent. more power than an ordinary flat belt of the same width. Chain belting is often made of tapered section for half-twist driving. Chain belts have been made up to 33 ins. wide.

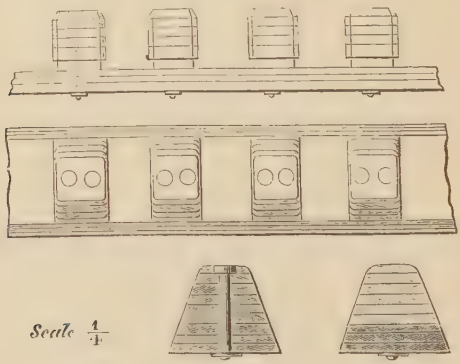


Fig. 264

Fig. 264 shows a peculiar leather belt introduced by Messrs. Tullis, of Glasgow, and intended to work on pulleys

having V-shaped grooves round their circumference. When the grooves have sides inclined at 45°, the adhesion of the belt to the pulley is increased about 2·6 times, so that the grooved pulley is equivalent to a cylindrical pulley with a coefficient of friction, $\mu = 0{\cdot}8$ to $1{\cdot}0$. The V-shaped belt shown in fig. 264 has been used for some years in America. It is made of slices of leather riveted together. The continuous part of the belt consists of three strips about $\frac{3}{8}$ths of an inch in total thickness, and 2 ins. in average width. Hence the belt section is about $1\frac{1}{4}$ sq. in. Several of these belts may be used side by side, precisely in the same way as the rope belts which are described in the next chapter. Messrs. Tullis state that the driving power of the belt is considerably greater than that of an ordinary rope belt.

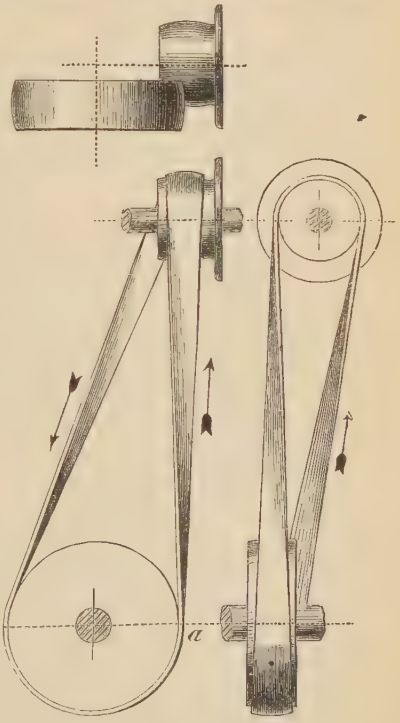


Fig. 265

255. *Belts connecting shafts which are not parallel.* — When two shafts are not parallel and do not intersect, they may still be connected by an endless belt, provided the pulleys are properly placed. The single and sufficient condition that the belt may run properly is this: The point at which the belt is delivered from each pulley

must be in the plane of the other pulley. This condition can only be fulfilled for a belt which always runs in one direction.

Fig. 265 shows three views of this arrangement of belting applied to two shafts at right angles or quarter twist. The arrows show the direction of the motion of the belt. If this be followed, it will be found that the point at which the belt runs off each pulley is in the plane passing through the centre of the other pulley. The belt would in this case be said to have a quarter twist.

256. *Guide pulleys.*—When two shafts are not parallel, and whether their directions intersect or not, they may be connected by a single endless belt if intermediate guide pulleys are used. These guide pulleys alter the direction of the belt without modifying the velocity ratio of the shafts. Fig. 266 shows an elevation and plan of an arrangement of pulleys and guide pulleys: *a b* is the intersection of the middle planes of the principal pulleys. Select any two points *a* and *b* on this line and draw tangents, *a c*, *b d*, to the principal pulleys. Then *c a c* and *d b d* are suitable directions for the belt. The guide pulleys must be placed with their middle planes coinciding with the planes *c a c* and *d b d*. The belt will run in either direction.

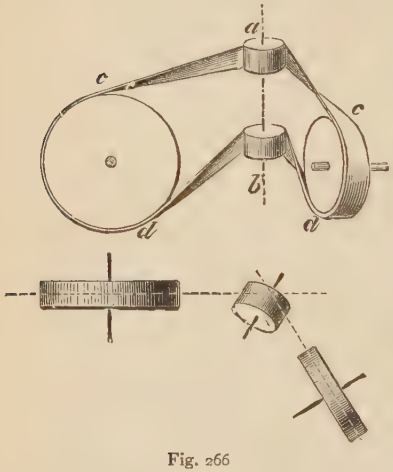


Fig. 266

Guide pulleys are sometimes used merely to lengthen the belt between two shafts, which are too close together to

be connected direct. Fig. 267 shows an arrangement of this kind. The middle planes of the guide pulleys are determined by the method just mentioned. It is, however, possible to place the guide pulleys with their axes parallel. Then the belt must be delivered from each pulley in the plane of the pulley on to which it is running. When this is provided for, it will be found that the belt will only run in one direction.

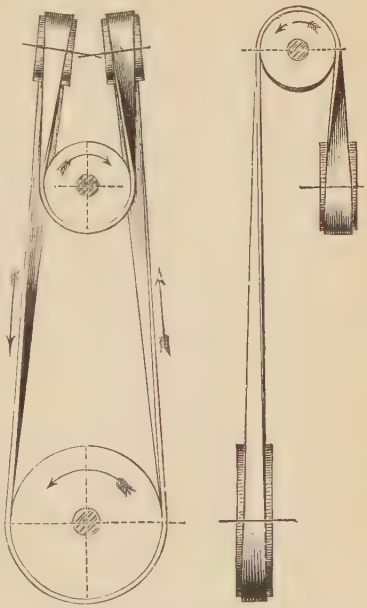


Fig. 267

Figs. 268 and 269 show two arrangements of belting and guide pulleys for shafts at right angles. If the belts be traced round, it will be found that the rough side of the belt is always next the pulleys. It is to secure this that the belts have a quarter or half twist between the pulleys as shown.

257. *Rounding of pulley rim.*—When a flat belt is placed on a conical pulley, it tends to climb toward the larger end. If the pulley is made of a double conical form, or, still better, with a rounded rim a little larger at the centre than at the sides, the flat belt keeps its place on the pulley and has no tendency to slip off. The rounding of the rim may be $\frac{1}{8}$ inch to $\frac{1}{4}$ inch for pulleys up to 12 inches broad, and $\frac{1}{8}$ inch to $\frac{1}{4}$ inch per foot of width for wider pulleys, or the section of

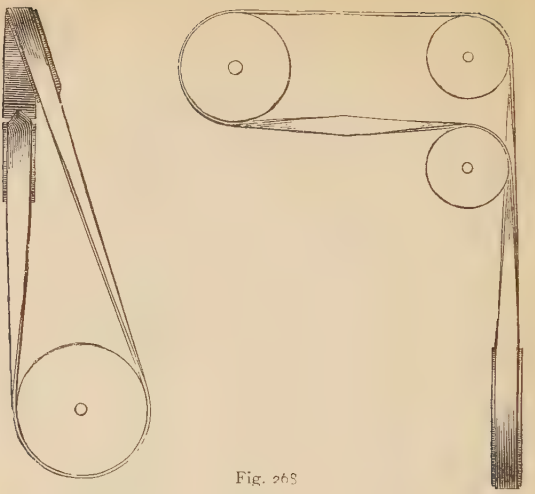


Fig. 268

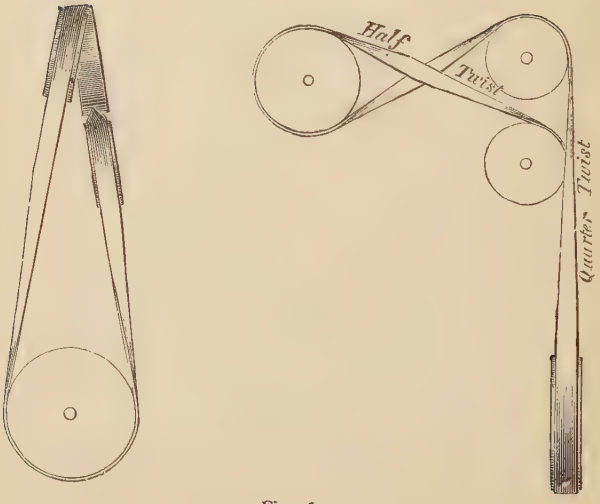


Fig. 269

the rim may be struck with a radius equal to from three to five times the width of the rim.

258. *Proportions of pulley. Rim of pulley.*—The pulley rim is a little wider than the belt it is intended to carry. Let B = width of rim, β = width of belt. Then,

$$B = \frac{9}{8}(\beta + 0{\cdot}4)$$

$\beta =$	2	3	4	5	6	8	10	12
B =	2·7	3·82	4·95	6·08	7·2	9·45	10·7	13·95
=	$2\frac{3}{4}$	$3\frac{7}{8}$	5	6	$7\frac{1}{4}$	$9\frac{1}{2}$	$11\frac{3}{4}$	14

The form of the rim in section is shown in fig. 270 ; at the edge the thickness may be

$$t = 0{\cdot}7\,\delta + {\cdot}005\,D$$

where D is the diameter of the pulley and δ the thickness of the belt.

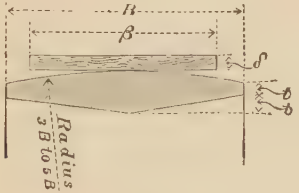


Fig. 270

The diameter of pulleys should not be less than 6 to 8 times the diameter of a wrought-iron shaft suitable for transmitting the power transferred to the belt, and the diameter of the smaller of two pulleys should not be less than about 18 times the belt thickness.

259. *Centrifugal tension in rim of pulley.*—Pulleys run at high speeds are liable to burst from the tension in the rim. Let w = weight of a bar 1 sq. in. in section and 12 ins. long

($w = 3{\cdot}36$ lbs.). Then the weight of one foot length of a pulley rim of section ω sq. ins. is $w\,\omega$ lbs. If v = velocity in feet per second, and r = radius in feet of pulley rim, then centrifugal force of one foot length of rim $= \frac{w\,\omega\,v^2}{g\,r}$ (See p. 37.) Suppose the pulley divided by a diametral plane. Then the resultant centrifugal force of each half of the pulley rim, acting normally to the dividing plane, is

$$\frac{w\,\omega\,v^2}{g\,r} \times 2\,r = \frac{2\,w\,\omega\,v^2}{g}.$$

This force is balanced by the tensions on the two sections of the rim by the diametral plane. Consequently the whole tension in the rim is $\frac{w\,\omega\,v^2}{g}$, and the intensity of the stress is $\frac{w\,v^2}{g}$ lbs. per sq. in.

v in ft. per sec.	=	70	80	100	150	200
Centrifugal tension in lbs. per sq. in.	=	511	668	1043	2349	4175

The stresses due to the pull of the belt and those due to contraction in casting are additional to these stresses. Hence in practice the speed of pulley rims rarely is allowed to exceed 80 to 100 feet per second.

260. *Arms of the pulley.*—The arms of pulleys are of elliptical or segmental section, as shown in fig. 271. The latter form of section looks lighter than the elliptical section and is preferable. For a segmental arm the thickness $h_2 = \frac{1}{2}h_1$. For an elliptical arm the thickness $h_2 = 0{\cdot}4h_1$. The arms are either straight or curved. The curved arms are rather less liable to fracture from contraction in cooling, but in other respects the straight arms are preferable, being lighter and stronger. The section of the arms is diminished

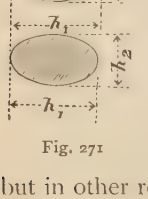


Fig. 271

from the nave to the rim, so that if we put $h_1\ h_2$ for the breadth and thickness of the arm, supposed produced to the

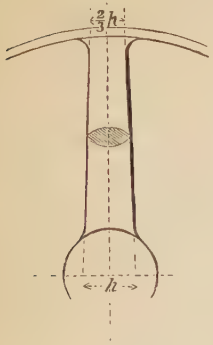


Fig. 272

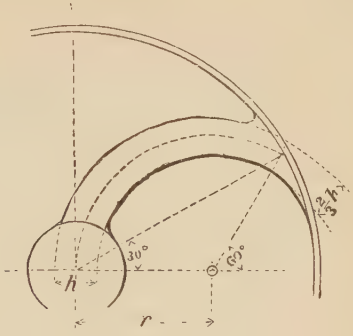


Fig. 27

centre of the shaft, the breadth and thickness at the rim will be $\frac{2}{3}\ h_1$ and $\frac{2}{3}\ h_2$.

Fig. 272 shows an ordinary straight arm, fig. 273 a

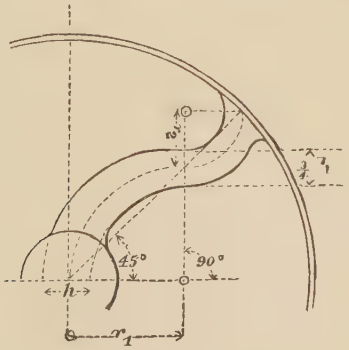


Fig. 274

curved arm, and fig. 274 an S-shaped or doubly curved arm. The figures indicate sufficiently the way in which the centre line of the arm is drawn. Let R be the radius of the pulley

measured to the inside of the rim. Then in fig. 273, $r = 0{\cdot}577$ R; and in fig. 274, $r_1 = 0{\cdot}471$ R, and $r_2 = 0{\cdot}236$ R.

Let ν be the number of arms, B the breadth, and D the diameter of the rim. Then,

$$\nu = 3 + \frac{B\,D}{150}$$

the nearest whole number being taken.

Width of Pulley B	Diameter of pulley in inches when the number of arms is				
	4	5	6	8	10
3	50	100	150	...	...
6	25	50	75	125	175
12	12	24	36	62	87
18	8	16	24	42	58
24	6	12	18	31	44

The number of arms is really arbitrary, and may be altered if necessary. In calculating the strength of the arms it will be assumed that each arm is equally loaded, and also that each arm may be considered to be fixed at the nave and free at the rim. As these assumptions are only in a rough sense true, a large factor of safety must be allowed. Pulley-arms are also liable to be considerably strained by contraction in cooling. Hence a margin of strength must be allowed to meet this contingency. For these reasons the working stress on the cast-iron will be taken at $f = 2250$ lbs. per sq. in.

If P is the driving force transmitted by the belt, determined by eq. 13, and D is the diameter of the pulley, the greatest bending moment on each arm is—

$$M = \tfrac{1}{2}\,\frac{P\,D}{\nu}$$

For an elliptical section of width h (measured at the

centre of the pulley) and thickness 0·4 h, the section modulus (Table V., p. 56) is

$$\frac{\pi}{32} \times h^2 \times 0{\cdot}4\, h = 0{\cdot}0393\, h^3 \text{ nearly,}$$

and for a segmental section of width h and thickness 0·5 h, the modulus may be taken to be the same. Equating the bending moment and moment of resistance

$$\tfrac{1}{2} \frac{P\,D}{\nu} = 0{\cdot}0393\, f h^3$$

$$h = \sqrt[3]{\left(\frac{1}{0{\cdot}0786 f} \cdot \frac{P\,D}{\nu}\right)}$$

and putting $f = 2250$

$$h = 0{\cdot}17811 \sqrt[3]{\frac{P\,D}{\nu}}. \qquad . \quad (20)$$

Since in designing pulleys the driving force P will often be unknown, we may design the arms to resist the maximum driving force which is likely to be transmitted by a belt, the width of which is $\frac{1}{5}$ B. The driving force will be very often half the greatest tension in the belt, and will rarely exceed $\frac{1}{3}$th that tension, except when the belt embraces an unusually large arc. The greatest belt tension may be taken at 70 lbs. per inch width of the belt for single belting, and 140 lbs. for double belting. Hence, P will not exceed 56 and 112 lbs. per inch width of the belt, or 45 and 90 lbs. per inch width of the pulley.

$$P = 45\, B \text{ for single belts.}$$
$$= 90\, B \text{ for double belts.}$$

Inserting this value in the equations above,

$$\left.\begin{aligned} h &= 0{\cdot}6337 \sqrt[3]{\frac{B\,D}{\nu}} \text{ for single belts.} \\ &= 0{\cdot}798 \sqrt[3]{\frac{B\,D}{\nu}} \text{ for double belts.} \end{aligned}\right\} \quad (21)$$

These equations agree well with practice. If the arms are of wrought-iron f may be taken equal to 9,000 lbs. per sq. in. If the section of the arms is different, the proper section modulus must be substituted for that assumed above.

261. *The nave of the pulley.*—The thickness of the nave may be

$$\delta = 0{\cdot}14\sqrt[3]{B D} + \tfrac{1}{4} \text{ (single belt)}$$
$$= 0{\cdot}18\sqrt[3]{B D} + \tfrac{1}{4} \text{ (double belt).}$$

The length of the nave, λ, should not be less than $2\frac{1}{2}\,\delta$, and is often $\frac{2}{3}\,B$. The key is to be proportioned by the rules in § 98. When the pulley is to run loose on the shaft the nave should be bushed with brass, and the length of the nave should be equal to B.[1] Provision must also be made for lubrication. In large pulleys the nave may be strengthened by wrought-iron rings shrunk on.

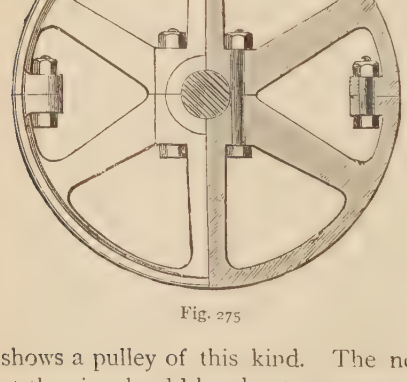


Fig. 275

262. *Split pulleys.*—When the pulleys are intended to be fixed on shafts which are bossed at the ends, they are often cast in halves. The two halves can then be bolted together on the shaft without dismounting the shaft and without having recourse to cone keys. Fig. 275 shows a pulley of this kind. The net section of the bolt at the rim should be about a quarter the section of the rim,

[1] Box, 'Millgearing,' p. 75.

plus $\frac{1}{4}$ sq. in. and that of bolt at the nave about $\frac{1}{4}$ sq. in. plus a quarter the section of the nave calculated as above. The two half-pulleys may be made to grasp the shaft so tightly that relative motion is prevented by friction, and no keys are necessary.

If it is undesirable to cast the pulley in halves, the eye of the pulley must be bored out large enough to pass over the bosses at the ends of the shaft and slightly conical. Then three cone keys, described in § 95, are fitted in the space between the pulley-eye and the shaft. Another plan is to use a conical sleeve, split on one side, like that shown in the drawing of the Sellers's coupling, fig. 150; this is drawn into the eye of the pulley by bolts. In either of these plans the pulley is fixed on the shaft by friction only.

Wrought-iron pulleys of ordinary size and of exceptionally large dimensions are made, and are preferable in many cases to cast iron. They are safer at high speeds because they are entirely free from strains due to contraction in cooling, and because, if they should break, their toughness would prevent them from flying to pieces.

Wrought-iron pulleys are generally made as shown in fig. 276. They are then virtually split pulleys. The split edges of the rim are joined by a lapping piece and screws.

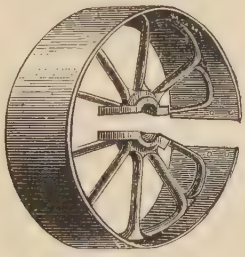


Fig. 276

263. *Weight of pulleys in lbs. per inch of width.*—The diameter being d in ft., Mr. D. K. Clarke gives

$w = 7{\cdot}6\ d - 1{\cdot}5$ to $12\ d - 9{\cdot}5$ for rough castings
$= 7\ d - 1{\cdot}75$ to $11{\cdot}6\ d - 9{\cdot}25$ for finished pulleys.

264. *Management of belting.*—In fixing, repairing, or splicing a belt, it must be thrown off the pulley, and it then rests chiefly on the upper of the two connected shafts. If the shaft on which it rests is the driven shaft, no great danger is incurred ; but if the belt rests on the driving shaft there is danger of the belt getting entwined round the shaft and so causing injury to the machinery and perhaps to the workman.[1] The danger is greater the more flexible the belt, and depends to some extent on the direction of motion of the shaft. The danger is greatest when the lower side is the tight side, and the slack side is liable to rest on it, as in fig. 277. The shaft may then grip the belt and roll up the two sides together. In fig. 278 the arrow is placed on the driving pulley. A and C are comparatively safe, B and D dangerous arrangements.

Fig. 277

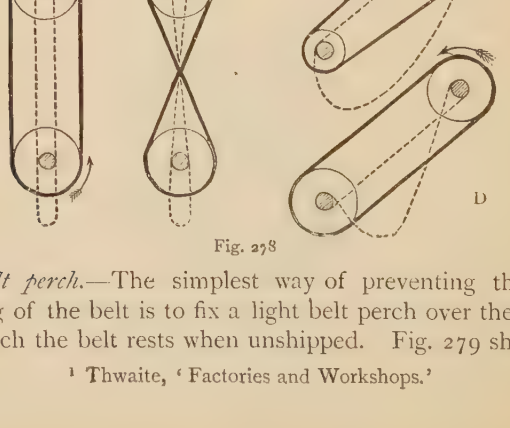


Fig. 278

Belt perch.—The simplest way of preventing the entwining of the belt is to fix a light belt perch over the shaft on which the belt rests when unshipped. Fig. 279 shows a

[1] Thwaite, 'Factories and Workshops.'

simple perch of this kind. Where a pulley is placed close to a hanger it is desirable to fix a light guard to prevent the belt falling between the pulley and hanger, and it should be

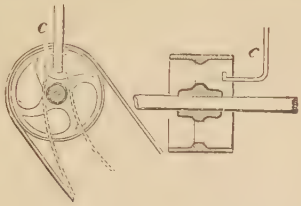


Fig. 279

placed near the point where the belt advances towards the pulley in running.

CHAPTER XIII

ROPE GEARING

At the present time, ordinary hemp ropes are being used to replace leather belting and toothed gearing in the transmission of power. For special purposes, similar ropes, made of cotton, are also used. The pulleys for belts of this kind are made with V-grooves round their circumference, each groove having its own rope-belt. When only a small amount of power is to be transmitted, the rope rests on the bottom of the groove, but in most cases the rope rests against the sides of the groove, and is wedged between them, so that the frictional resistance to slipping is very great. The ropes most commonly used are patent ropes of three strands (fig. 280), white or untarred, and from 1 to 2 inches in diameter. They are placed on the pulleys with very little initial tension, and the joint is made by splicing the rope. The pressure of the rope on the pulley is chiefly due to its weight. Hence, to secure sufficient frictional adhesion the pulleys should be large, and at a sufficient horizontal distance apart. If the pulleys are vertically over each other the rope must be strained more tightly, and its durability is impaired. Usually the horizontal distance between the pulleys is 20 to 60 feet, whatever their vertical distance may be. The ropes are never strained so tightly as to draw them nearly straight. They hang between the pulleys in catenary curves which approximate to parabolas. It is

Fig. 280

advisable to have the driving side of the rope on the lower side of the pulleys and the slack side above. Then, in driving, the two sides approach each other, and the arc of contact on the pulleys is increased. The slacker the ropes are, consistently with obtaining sufficient frictional resistance to slipping at the pulleys, the better, because the ropes are less squeezed in the grooves and wear longer.

265. *Strength of ropes.*—The breaking strength of white or untarred rope varies from 7,000 to 12,000 lbs. per sq. in., and is to some extent dependent on the amount of twist given to the rope. The twist diminishes the strength of the rope, but makes it more solid and durable. The working strength may be taken at about $\frac{1}{8}$th of the breaking strength. Hence, the working strength is $f=875$ to 1500 lbs. per sq. in. In the following calculations it is assumed that $f=1200$ lbs. per sq. in.

Let c be the diameter; γ, the girth; and G, the weight per lineal foot of the rope. The section of hawser-laid rope is about $\frac{9}{10}$ths of the area of the circumscribing circle. Hence,

$$\left.\begin{aligned} \text{Area of section} &= 0{\cdot}9 \times \tfrac{\pi}{4} \times c^2 = 0{\cdot}707\, c^2 = 0{\cdot}0716\, \gamma^2 \\ \text{Working strength} &= 0{\cdot}707 f c^2 = 850\, c^2 = 86\, \gamma^2 \end{aligned}\right\} \quad (1)$$

When the rope is wet or tarred, the strength is reduced by about one-fourth.

The weight of ropes, G, in lbs. per foot of length, is given by the following equations:[1]—

$$\left.\begin{aligned} G &= 0{\cdot}2812\, c^2 = 0{\cdot}0285\, \gamma^2 \quad \text{dry} \\ &= 0{\cdot}3376\, c^2 = 0{\cdot}0342\, \gamma^2 \quad \text{wet or tarred} \end{aligned}\right\} \quad . \quad (2)$$

Hence, for dry ropes, the weight of 3016 feet of rope is equal to the working strength.

[1] Karl Von Ott, 'Proc. Inst. of Civil Engineers,' xlv. p. 270.

266. *Ordinary driving force of rope belts.*—In order to ensure durability, the tension in the belt when at work is only a small fraction of the working strength. From data furnished by Messrs. Pearce Brothers, of Dundee, who have erected rope belting extensively, it appears that the difference of tension on the two sides of the belt, or driving force is:[1]—

$$T_2 - T_1 = P = 7{\cdot}81\ \gamma^2 \text{ lbs.} \quad . \quad . \quad (3)$$

It will be shown presently that when the belt embraces 0·4 of the circumference of the smaller pulley, the greatest tension is $T_2 = 1{\cdot}208\ P = 9{\cdot}43\ \gamma^2$.

Hence the greatest tension is less than $\frac{1}{9}$th of the working strength of the rope.

Table of Weight, Strength, and Driving Force of Rope Belts

Girth of Rope in ins. γ	Diameter of rope in ins. δ	Weight per foot in lbs. G	Working strength in lbs.	Driving force in lbs.		
				P	K	K_1
$3\frac{1}{8}$	1	·279	842	$76\frac{1}{2}$	0·14	·00061
$4\frac{1}{4}$	$1\frac{5}{16}$	·513	1548	140	0·25	·00110
$4\frac{3}{4}$	$1\frac{1}{2}$	·643	1940	176	0·32	·00140
$5\frac{1}{4}$	$1\frac{11}{16}$	·862	2602	236	0·43	·00188
$6\frac{1}{2}$	$2\frac{1}{16}$	1·204	3633	330	0·60	·00262

267. *Work transmitted by rope belts.*—Since the power which any given rope will transmit is limited, and it is not convenient to use very large ropes, it is necessary in most cases to use several ropes. The pulleys have parallel grooves in which the ropes are placed, sometimes to the number of 20 or 25. Let n be the number of ropes on a pulley; V, the velocity of the rope in feet per second; d, the diameter of the pulley in inches; N, the number of revolutions of the pulley per minute. Then the work transmitted by each rope is

$$P\,V \text{ foot lbs. per second.}$$

[1] In different cases in practice, $P = 6\gamma^2$ to $8\gamma^2$.

Let H be the number of horses' power transmitted,

$$H = \frac{n\,P\,V}{550} = K\,n\,V \quad . \quad . \quad . \quad (4)$$

Also since $V = \frac{\pi d N}{12 \times 60}$

$$H = \frac{n\,P\,d\,N}{126100} = K_1\,d\,N\,n \quad . \quad (5)$$

where K and K_1 are constants, the values of which are given in the table above.

268. *Friction of rope belting.*—The coefficient of friction for a rope on a metal pulley is $\mu = 0{\cdot}28$. In rope transmission, however, the rope is wedged in the groove of the pulley, and the normal pressure between the rope and the sides of the groove is greater than the force pressing the rope into the groove, in the ratio of cosec. $\frac{\theta}{2}$: 1, where θ is the inclination of the sides of the groove. Hence, the resistance to slipping is the same as on a cylindrical pulley having a coefficient of friction,[1]

$$\mu = 0{\cdot}28 \text{ cosec. } \frac{\theta}{2}$$

In practice $\theta = 45°$, and then $\mu = 0{\cdot}7$.

From the equations in the previous chapter—

$$\left.\begin{array}{ll} \text{Greatest tension} & = T_2 = P\,x \\ \text{Tension on slack side} & = T_1 = P\,y \\ \text{Initial tension} & = \frac{1}{2}(T_1 + T_2) = P\,z \end{array}\right\} \quad . \quad (6)$$

And using the above value of μ :—

[1] A reduction of the results of some experiments by Messrs. Pearce, gives $\mu = 0{\cdot}57$ to $0{\cdot}88$ for ropes on ungreased grooved pulleys, and $\mu = 0{\cdot}38$ to $0{\cdot}41$, when the pulleys were greased. The former values agree fairly with $\mu = 0{\cdot}7$ assumed above.

Fraction of Circumference of Pulley embraced by Rope.

	0·1	0·2	0·3	0·4	0·5	0·6
$\frac{T_2}{T_1} =$	1·55	2·41	3·74	5·81	9·02	14·00
$x =$	2·82	1·71	1·73	1·21	1·13	1·08
$y =$	1·82	·71	·37	·21	·13	·08
$z =$	2·32	1·21	·87	·71	·63	·58

269. *Pulleys for rope belting.*—The pulleys are usually of cast iron, and when motion is taken from a steam-engine grooves are turned in the rim of the fly-wheel. The diameter of the smallest pulley should not be less than thirty times the diameter of the rope carried by it. The larger the pulley the less is the injury done to the rope by bending and unbending.

Fig. 281 shows the form of the grooves in the pulley-rim and the proportions adopted. The unit for the proportional

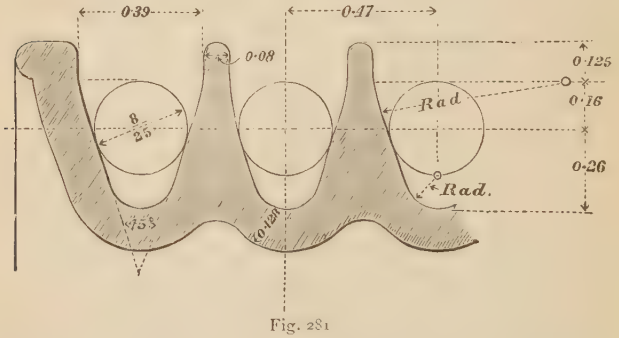


Fig. 281

figures is γ, the girth of the rope. If the pulley is a guide-pulley merely, the rope should rest on the bottom of the groove. The sides of the groove are usually inclined at 45°. The pulleys are cast in one piece, when they are less than 8 feet diameter, unless they have to be fixed on shafts which

have bosses at the ends, or require to be fixed after the shafts are in position. When this is the case they are cast in halves, and they are also usually cast in halves when they are from 8 to 12 feet diameter. Larger pulleys are cast in segments and bolted together. The grooves in each pulley must be accurately turned to the same gauge, and of the same diameter. The splices in the rope should be 9 or 10 feet long.

Very great care must be taken to have the pulley grooves of the same form and the ropes of the same diameter. If these conditions are not secured, the ropes will be virtually running on pulleys of different diameters, and some of the ropes will be severely strained. All the ropes intended to work on a given pair of pulleys should be put on at the same time. They then stretch and decrease in diameter equally. The velocity of rope belting is generally 3,000 ft. to 5,000 ft. per minute.

The amount of power which may be transmitted by rope gearing is very great when the conditions are suitable. Ropes usually last for 3 to 5 years and sometimes 10 years. They should occasionally be treated with castor-oil to prevent the fibres becoming too dry and brittle.

Relative merits of rope and toothed gearing in mills.— It is difficult to give any conclusive opinion on the relative merits of toothed gearing and rope gearing when used in large mills. Unquestionably rope gearing has made great progress since its introduction a few years ago. It is noiseless and free to a great extent from risk of serious breakdown. On the other hand, toothed gearing, especially if badly constructed or badly fixed, may give much trouble, and, if teeth break, serious delay is occasioned. It appears that an engine flywheel fitted with ropes must be heavier than a spur flywheel, so much heavier in fact that in large engines the bearings of the flywheel shaft sometimes give trouble by heating. The ropes appear to take more space in the mill than gearing, and in the opinion of many engineers absorb ten per cent. more power than gearing.

Wire-rope Gearing

270. Wire ropes have been occasionally used for transmission of power. Sometimes they have been used for direct haulage, and sometimes like ordinary belts, to connect rotating pieces. A method of wire-rope transmission has been in use, on the Continent chiefly, which is perfectly successful, and by which large amounts of motive power can be transferred to great distances, with an efficiency impossible with any other mode of transmission. This system, due to M. C. F. Hirn, is termed 'telodynamic transmission.'

In belt transmission, we may increase the amount of power transmitted in three ways: by increasing the frictional bite of the pulleys, by increasing the strength of the belt, and by increasing the velocity of the belt. The first principle is applied in ordinary rope transmission, by wedging the ropes in V-shaped grooves in the rim of the pulley. With wire ropes these wedge-grooves cannot safely be adopted, because of the injury done to the rope. On the other hand, wire ropes are enormously stronger than hemp ropes, and if in addition they are run at the highest practicable velocity, a very great amount of power can be transmitted, with comparatively light gearing. The principle of telodynamic transmission is, therefore, to use flexible belts of very great strength on ordinary pulleys, and to work them at very high velocities. Various expedients are necessary in the application of this principle and in securing the greatest possible efficiency, or the least waste of work in friction.

The pulleys are of large diameter, which tends to the preservation of the ropes by diminishing the bending action, and reduces the influence of the stiffness of the ropes and the loss of work in journal friction. If the distance to which power is transmitted is very great, the transmissions are divided into relays with a separate rope for each. The relays are separated by stations. Each station is provided with

a horizontal shaft upon which a double grooved pulley is fixed, which is the driven pulley as regards the relay terminating there, and the driving pulley as regards the succeeding relay. The stations are usually arranged on masonry pillars, more or less raised according to the configuration of the ground, for it is necessary that the rope should not touch the ground. Sometimes the power has to be partially distributed in its course; under these circumstances the shafts at the stations are made use of for the purpose. Frequently also intermediate pulleys are placed along a relay serving merely to support the rope. Occasionally a relay has been made 650 feet in length. Usually the length is 400 to 500 feet.

The system has proved so successful that power is now frequently transmitted to very great distances with comparatively little loss. That loss is estimated at only $2\frac{1}{2}$ per cent. + 1 per cent. in addition, for every 1,000 yards of distance. The method is not suitable when the distance to which the power is to be transmitted is short, and 130 feet has been fixed as the minimum distance for which transmission by wire rope is applicable.[1] At less distances the wire rope is subject to considerable oscillations, which, however, it is possible may be prevented.

271. *Form, strength, and weight of wire ropes.*—The rope used consists of six or more strands wound upon a hemp core. Each strand consists of six or more wires also twisted round a hemp core. The strands are wound in the opposite direction to the wires in each strand. Fig. 282 shows the section of a rope, the shaded circles being sections of the wires, and the unshaded portions hemp. The angles of twist are usually 8° to 15° for the strands, and 10° to 25° for the rope. The wire diameter varies usually from $\frac{1}{50}$ to $\frac{1}{12}$ of an inch.

The ropes most commonly used have six strands, each

[1] Vigreux, 'Proc. Inst. of Civil Engineers,' xlv. p. 266.

containing six wires and a hemp strand at the centre. For these ropes with 36 wires the diameter of the rope is nearly $9\frac{1}{4}$ times the diameter of the single wires. Ropes of 42 wires are used with the middle hemp core replaced by a strand of six wires, and their diameter is about $10\frac{1}{2}$ times the diameter of a single wire.[1] The number of strands and of wires in each strand is, however, arbitrary, and ropes of 8 strands, each of 10 wires, of 10 strands, each of 9 wires, and various other proportions, are adopted. The relation between the diameter of the rope Δ, the diameter of the wires δ, and the number of wires ν, is given very approximately by the formula

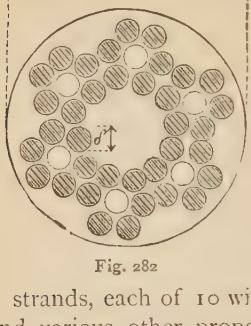


Fig. 282

$$\frac{\Delta}{\delta} = \frac{\nu}{13} + 7 \quad . \quad . \quad . \quad . \quad (7)$$

The breaking strength of iron wire varies from 85,000 to 108,000 lbs. per sq. in., and the greatest working stress has been fixed at 25,600 lbs. per sq. in. Steel wire has a greater tenacity, and might be assumed to be capable of bearing a still higher working stress. At first steel wire ropes did not answer so well as ropes of iron wire. But according to M. Naville steel wire ropes are now preferred to those of iron. They are, however, worked only up to the same limiting stress, and in such conditions they last longer than iron. A rope running night and day lasts about 200 to 250 days, if of iron; and 250 to 300 days, if of steel. Moreover the steel ropes stretch less in working.[2] An iron rope

[1] According to Stahl, the ropes used in America consist of six strands having seven wires in each strand and no hemp core in the separate strands. The strands are wound round a central hemp core. In these ropes $\Delta = 9\delta$ and $G = 1{\cdot}43\ \Delta^2$.

[2] Achard, 'Transmission of Power,' Proc. Inst. of Mech. Eng., January 1881.

requires tightening once in 60 days ; a steel rope only once in 120 days.

The weight of wire rope per lineal foot is very nearly

$$= G = 3{\cdot}268\, v\, \delta^2 = 1{\cdot}341\, \Delta^2 \text{ lbs.} \quad . \qquad . \quad (7a)$$

Splicing wire ropes.—The following directions are abbreviated from those given by Messrs. Roebling.[1] Overlap the ropes for a distance of 20 feet. Unlay the strands for a length of 10 feet of each rope, and cut away for that distance the central hemp core. Now let a, b, c, ... be the strands of one rope taken in order, and a', b', c' . . . . those of the other. Unlay a for a further distance of 10 feet, and lay into the spiral groove so formed a', and cut off a and a' so as to leave two short ends about 6 inches long. Next unlay about 10 feet of d', and lay in the corresponding strand d, cutting off as before. Proceed by unlaying b and laying in b', and by unlaying e' and laying in e, stopping about 4 feet short of the previous cut ends. Lastly, unlay c and lay in c', and unlay f' and lay in f, again stopping 4 feet short of the ends of the previously cut strands. To dispose of the cut ends, nip the rope about 6 inches on each side of the ends. Insert a stick and untwist the rope, cut out 6 inches of the hemp core, and force in the cut end into its place. Close the rope and hammer it even with a wooden mallet.

272. *Stresses in a wire rope belt.*—Used as a belt the wire rope is subjected to three different straining actions. (1) There is the longitudinal tension, due to the tightness with which the belt is strained over the pulleys, to the weight of the rope, and to the power transmitted. (2) There are stresses of tension and compression in the part of the belt which at any moment is bent to the curve of the pulley due to the bending. (3) There is a stress due to the centrifugal action of the part of the belt which is being bent. This last stress, though not insignificant, is sometimes left out of consideration.

Let f_t be the greatest working stress due to the longi-

[1] Stahl, 'Transmission of Power by Wire Ropes.'

tudinal tension of the belt, and f_b the stress due to bending. For those wires which lie on the stretched side of the belt in passing over the pulley, the total stress is

$$f = f_t + f_b \quad . \quad . \quad . \quad . \quad (8)$$

When a cylinder of diameter δ is bent to a radius R, the bending moment at any point is [1]

$$M = \frac{E\,I}{R} = \frac{E\,Z\,\delta}{2\,R} \quad . \quad . \quad . \quad . \quad (9)$$

where I is the moment of inertia, and Z the modulus of the section of the rope.

The moment of resistance to bending of a circular section of diameter δ (§ 28) is,

$$f_b\,Z.$$

Equating these values,

$$f_b = \frac{E\,\delta}{2\,R} \quad . \quad . \quad . \quad . \quad (10)$$

If T is the total longitudinal tension in a rope having ν wires, each of δ inches diameter,

$$f_t = \frac{T}{\frac{\pi}{4}\delta^2\nu} \quad . \quad . \quad . \quad . \quad (11)$$

Hence the total stress in the most strained wires is

$$f = \frac{E\,\delta}{2\,R} + \frac{T}{\frac{\pi}{4}\delta^2\nu} \quad ;$$

hence

$$T = \left(f - \frac{\delta}{2\,R}E\right)\frac{\pi}{4}\,\delta^2\nu \quad . \quad . \quad (12)$$

For a given value of the limiting stress f, T will be a maximum for pulleys of a given radius, when δ is so chosen that

$$\frac{d\,T}{d\,\delta} = o,$$

or when

$$\frac{R}{\delta} = \frac{3}{4}\frac{E}{f} \quad . \quad . \quad . \quad . \quad (13)$$

[1] Compare equation, p. 52, and the values of Z given at p. 58.

Putting $f=25{,}600$, and $E=29{,}000{,}000$ for wrought iron,

$$\frac{R}{\delta}=850.$$

That is, the longitudinal tension will be a maximum when the diameter of the wires is $\frac{1}{850}$th of the pulley radius.

When the ratio $\frac{R}{\delta}$ varies from these proportions, we have for the greatest safe working stress due to the longitudinal tension,

$$f_t=f-f_b=f-\frac{\delta E}{2R} \quad . \quad . \quad . \quad . \quad (14)$$

The deflection, when the rope is not working, should not be less than 18 inches. Sometimes when the pulleys are near together, the deflection of the rope will be too small with this tension. If this is the case, a lower value of the working tension should be adopted.

Direct Stress, and Stress due to bending in Wire Rope Belts of Iron or Steel

Ratio $\frac{R}{\delta}$	Bending stress f_b	Longitudinal stress f_t	Total stress f
650	22,310	3,290	
700	20,710	4,890	
750	19,330	6,270	
800	18,120	7,480	
850	17,060	8,540	
900	16,120	9,480	25,600
950	15,270	10,330	
1,000	14,500	11,100	
1,100	13,180	12,420	
1,200	12,090	13,510	
1,350	10,740	14,860	
1,400	10,360	15,240	

The proportions most commonly adopted are those corresponding to $\frac{R}{\delta}=1350$, so that the working longitudinal

stress is only 14,860 lbs. per sq. in. The reason of this apparently low working stress will be obvious, if the bending action is considered.

273. *Total longitudinal tension of rope.*—Let T be the greatest tension in any part of the rope, exclusive of the bending stress. Then since the section of iron in the rope is

$$\frac{\pi}{4}, \delta^2,$$

$$T = \frac{\pi}{4} \nu \delta^2 f_t.$$

Hence the size of wire for a given total tension is

$$\delta = \sqrt{\frac{4}{\pi f_t}} \sqrt{\frac{T}{\nu}} \quad . \quad . \quad . \quad (15)$$

$f_t =$	8000	9000	10000	12000	14000	16000
$\sqrt{\frac{4}{\pi f_t}} =$	·01262	·01190	·01128	·01030	·00954	·00892.

It has been stated already that δ is usually between $\frac{1}{50}$th and $\frac{1}{12}$th of an inch.

274. *Tension due to centrifugal force.*—The tension due to centrifugal force in a rope weighing G lbs. per foot, and travelling at v feet per second, is

$$C = G \frac{v^2}{g}.$$

Inserting the value found in eq. (7*a*) for G,

$$C = 3{\cdot}268 \, \nu \delta^2 \frac{v^2}{g} \quad . \quad . \quad . \quad (16)$$

and dividing this by the section of the rope, the intensity of centrifugal tension is—

$$c = 0{\cdot}1293 \, v^2 \text{ lbs. per sq. in.} \quad . \quad . \quad (16a)$$

$v =$	50	60	70	80	90	100
$c =$	323	466	634	828	1047	1293

These stresses must be deducted from the stresses in the

table, § 272, p. 415, in order to find the safe tension due to the power transmitted.

275. *Driving force of belt, and power transmitted.*—The equations for the friction of a belt on a pulley given in Chapter XI. are equally applicable for an iron wire rope, if proper values are taken for the coefficient of friction.

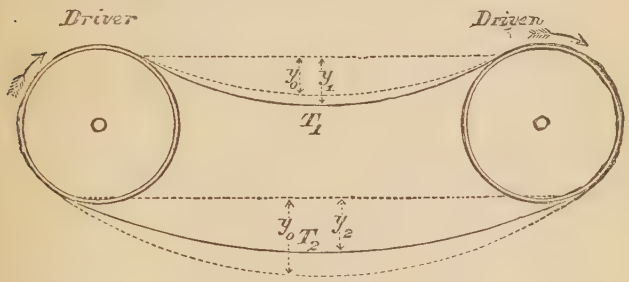


Fig. 283

Taking $\mu=0{\cdot}24$, and supposing that the belt embraces nearly a semicircle of the pulley, so that $\theta=3$,

$$\epsilon^{\mu\theta}=2 \text{ nearly.}$$

The ratio of the tensions on the tight and slack sides of the belt due to the resistance to slipping is,

$$\frac{T_2}{T_1}=e^{\mu\theta}=2 \quad . \quad . \quad . \quad . \quad (17)$$

The driving force of the belt is the difference of the tensions, that is,

$$\left.\begin{aligned} P&=T_2-T_1 \\ \therefore T_2&=2P \\ T_1&=P \end{aligned}\right\} \quad . \quad . \quad . \quad (18)$$

The work transmitted in foot pounds per second is $P\,v$, and if H is the horse-power transmitted,

$$P=\frac{550\,H}{v} \quad \text{or} \quad H=\frac{P\,v}{550} \quad . \quad . \quad (19)$$

We may put equation 15 in the form for calculating the size of rope from the driving force, instead of from the total tension. The tension, apart from the bending stress, must not exceed $f_t - c$ lbs. per sq. in., and the total tension due to the work transmitted and the initial tension is T_2 or $2P$. Hence,

$$\delta = \sqrt{\frac{2 \times 4}{\pi(f_t - c)}} \sqrt{\frac{P}{v}} \quad . \quad . \quad (20)$$

$f_t - c$	$\sqrt{\frac{8}{\pi(f_t - c)}}$	$f_t - c$	$\sqrt{\frac{8}{\pi(f_t - c)}}$
6000	·0206	10000	·0160
7000	·0191	12000	·0146
8000	·0178	14000	·0135
9000	·0168	16000	·0126

276. *Tightened belt.*—In some cases the diameter of the rope calculated by this rule will prove to be very small. Then it may be convenient to adopt a larger rope than is absolutely necessary. If this is done, either the size of the pulleys may be reduced, if desirable, the rope being capable of bearing a greater bending stress, or the tension in the rope may be increased beyond what is necessary to prevent slipping at the pulleys, with a view of reducing the deflection of the rope between the pulleys.[1] In this latter case, the tension T_2 may be calculated from the size of rope adopted; then T_1 is $T_2 - P$, and from these tensions the curves of the rope may be determined.

277. *Weight of ropes.*—The weight of wire ropes per lineal foot may be taken to be—[2]

$$G = 3{\cdot}268\, v\, \delta^2 = 1{\cdot}341\, \Delta^2 \text{ lbs.} \quad . \quad . \quad (21)$$

278. *The catenary curve.*—No important error is likely to

[1] Reuleaux, 'Der Constructeur,' p. 398.

[2] Karl Von Ott, 'Proc. Inst. of Civil Engineers,' xlv. p. 271.

be introduced by considering the rope to be perfectly flexible and of uniform section. In that case, the funicular curve in which the rope hangs between the pulleys is known to be the catenary, and the tensions in the rope are due to a distribution of load, vertical and constant per unit length of arc.[1]

[1] The curve in which a perfectly flexible cord of uniform weight hangs when suspended from two points is called the 'catenary.' The curve is obviously symmetrical with respect to a vertical axis which may be assumed as the axis of Y and cuts it at the distance m above the origin. Its transcendental equation is

$$y = \frac{m}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right)$$

where $e = 2{\cdot}7183$. The following note on the geometry of the curve, and the modern way of expressing its equation, is due to Prof. A. G. Greenhill, F.R.S.

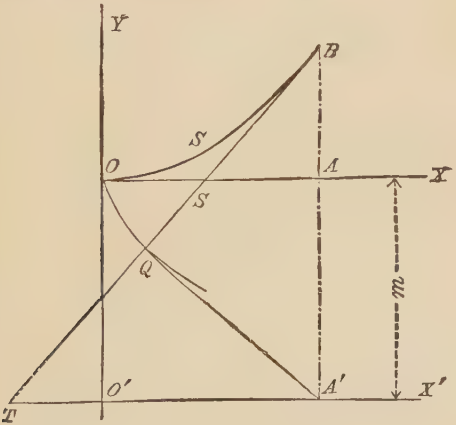


Fig. 285

Let O B be the catenary B Q T, its tangent at B. Make B Q = arc BO. Draw Q A′ perpendicular to B Q T, meeting the vertical from B in A′. Through A′ draw the horizontal O′ A′ X′. This is the directrix of the catenary and Q A′ = O O′ = m. The locus of Q is the tractrix.

Denoting $\frac{1}{2}(e^x + e^{-x})$ by cosh x and $\frac{1}{2}(e^x - e^{-x})$ by sinh x, then

Let fig. 284 show the form of the curve in which the rope hangs, and let O be the lowest point of the curve. Take O for origin of co-ordinates, and let $x = O$ A, and $y =$ A B be the abscissa and ordinate of any point B of the curve. Since

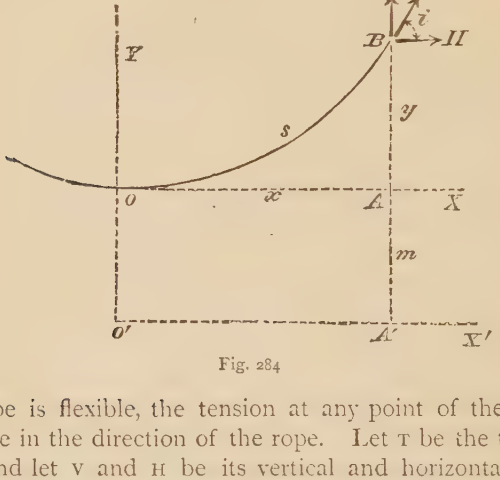


Fig. 284

the rope is flexible, the tension at any point of the curve must be in the direction of the rope. Let T be the tension at B, and let V and H be its vertical and horizontal components. Let the length of the arc O B $= s$, and let the inclination of the curve at B to the horizontal be denoted by i.

Since G is the weight of a unit length of rope, G s is the weight of O B, and this is equal to the vertical component of the tension at B; hence, G $s =$ V. The other tensions H and T are equivalent to the weight of lengths m and n of rope, m and n being at present undetermined, so that H $=$ G m and T $=$ G n.

$$\cosh^2 x - \sinh^2 x = 1.$$

$$s = \text{arc O B} = m \sinh \frac{x}{m}.$$

$$y = \text{ordinate A' B} = m \cosh \frac{x}{m} = \sqrt{(s^2 + m^2)},$$

and the ordinate A B $= 2m \sinh^2 \frac{1}{2} \frac{x}{m}$.

The inclination of the rope at B is given by the equations—

$$\left.\begin{aligned} \cos i &= \frac{dx}{ds} \\ \sin i &= \frac{dy}{ds} = \sqrt{1-\frac{dx^2}{ds^2}} \\ \tan i &= \frac{dy}{dx} = \frac{\sqrt{\left(1-\frac{dx^2}{ds^2}\right)}}{\frac{dx}{ds}} \end{aligned}\right\} \quad . \quad (22)$$

$$\text{But } \tan i = \frac{V}{H} = \frac{s}{m} \quad . \quad . \quad . \quad (23)$$

Hence,

$$\frac{dx}{ds} = \frac{m}{\sqrt{m^2+s^2}}$$

Integrating, and putting $x=o$, when $s=o$,

$$x = m \text{ hyp. log.} \left\{ \frac{s}{m} + \sqrt{\left(1+\frac{s^2}{m^2}\right)} \right\} \quad . \quad (24)$$

That is,

$$\frac{s}{m} + \sqrt{\left(1+\frac{s^2}{m^2}\right)} = e^{\frac{x}{m}}$$

$$s = \frac{m}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right) \quad . \quad . \quad . \quad . \quad (25)$$

$$\frac{s}{m} = \tan i = \frac{dy}{dx}$$

Inserting the value of $\frac{s}{m}$ in equation 25,

$$\frac{dy}{dx} = \tfrac{1}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right)$$

Integrating,

$$y = \frac{m}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right) + C$$

The constant is determined by the condition that $y=o$, when $x=o$,

$$\therefore y=\frac{m}{2}\left(e^{\frac{x}{m}}+e^{-\frac{x}{m}}-2\right)=\sqrt{s^2+m^2}-m \quad . \quad . \quad (26)$$

This is the equation to the curve termed the catenary, and m is its parameter. For the relation between the tensions at B we have

$$H=G\,m \qquad\qquad V=G\,s$$

$$T=\sqrt{(H^2+V^2)}=G\sqrt{(m^2+s^2)}=\frac{G\,m}{2}\left(e^{\frac{x}{m}}+e^{-\frac{x}{m}}\right)$$

$$=G\,(y+m) \quad . \quad . \quad . \quad . \quad (27)$$

From this last equation it is seen that the tension, at any point of the rope, is equal to the weight of a length of the rope, $y+m$, equal to the ordinate y of the point added to the parameter m. If $O\,O'=m$ (fig. 284) and $O'\,X'$ is drawn horizontally through O', then the tension at any point B is the weight of a length B A′ of the rope.

At the vertex O of the curve the tension is horizontal, and equal to the weight $G\,m$ of a length $O\,O'$ of the rope. But this is the same as the horizontal component of the tension at B. Hence the horizontal component of the tension at any point is equal to the horizontal tension at the vertex of the curve.

279. *Approximate equations.* — Introducing the value $\tan i=\frac{s}{m}$ in eq. 24, we get[1]

$$x=m \text{ hyp. log. } \left(\frac{\sin i+1}{\cos i}\right)$$

$$\frac{x}{m}=\text{hyp. log. } (1+\sin i)-\tfrac{1}{2} \text{ hyp. log. } (1-\sin^2 i)$$

$$=\tfrac{1}{2} \text{ hyp. log. } \frac{1+\sin i}{1-\sin i} \quad . \quad . \quad . \quad . \quad (28)$$

But hyp. log. $\frac{1+\sin i}{1-\sin i}=2\left(\sin i+\frac{1}{3}\sin^3 i+\ldots\right)$

[1] Keller's 'Treibwerke,' p. 201.

Or when i is small, neglecting the terms containing powers higher than the first—

$$\text{Hyp. log. } \frac{1+\sin i}{1-\sin i}=2 \sin i$$

$$\therefore x=m \text{ Sin } i \quad . \quad . \quad . \quad (29)$$

Using this value in equations 23 and 24,

$$s=\frac{x}{\cos i}=y \frac{\sin i}{1-\cos i} \quad . \quad . \quad (30)$$

$$y=x \frac{1-\cos i}{\sin i \cos i} \quad . \quad . \quad . \quad . \quad (31)$$

$$y+m=\frac{m}{\cos i} \quad . \quad . \quad . \quad . \quad (32)$$

$$\frac{2x}{y+m}=\sin 2i \quad . \quad . \quad . \quad . \quad (33)$$

These equations enable all problems relating to the form of the rope to be solved.

280. *Case I. Horizontal transmission.*—Let the supporting points of the rope be at the same level, and at a distance l apart, and let the total tension $T=T_2+C=2P+C$ be known.

From equation 27, we get $y+m=T \div G$ at the points of support. Since in this case the curve is symmetrical about its lowest point, $x=\frac{1}{2}l$. Hence at the points of support,

$$\left.\begin{aligned} &\text{Sin } 2i=\frac{2x}{y+m}=\frac{Gl}{T} \\ &\text{The parameter}=m=(y+m) \cos i=\frac{T}{G} \cos i \\ &\text{The deflection}=y=\frac{T}{G}-m \\ &\text{The length of the rope}=2s=2y \frac{\sin i}{1-\cos i} \end{aligned}\right\} \quad . \quad (34)$$

Conversely if the deflection y at the centre is given in place of the greatest tension, and also the half span x,

$$y = \frac{m}{\cos i} - m = \frac{m}{\sqrt{1 - \frac{x^2}{m^2}}} - m$$

$$m = \frac{x^2}{4y} + x\sqrt{\frac{x^2}{16\,y^2} + \tfrac{1}{2}} \text{ nearly} \quad . (35)$$

Then the other values can be found as before.

281. *Case II. Inclined transmission. The points of support* B′ B″ *not at the same level.*—This case is best solved by approximation. We may assume the length $s' + s''$ of the rope to be the same as if the points B′ B″ were at the same level and the same distance apart.[1] Let T′, the tension at B′, be given, and also the length B′ B″ $= l$, and the difference of level $y' - y'' = h$. Calculate first the length of rope $2s$ from the equations above, putting T′ for T, and assuming the pulleys to be l feet horizontally apart.

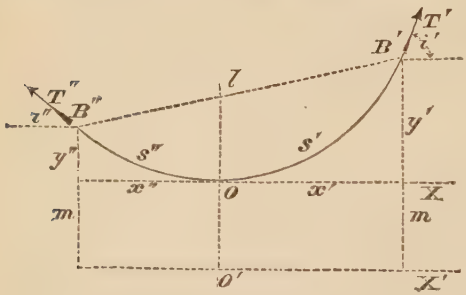


Fig. 286

Then on the assumption above,

$$s' + s'' = 2s$$

$$y' + m = \frac{T'}{G}$$

$$y'' + m = \frac{T'}{G} - h = \frac{T''}{G}$$

[1] Reuleaux, 'Der Constructeur': Keller, 'Treibwerke.'

By equation 26,

$$(y'+m)^2=m^2+s'^2$$

$$(y''+m)=m^2+s''^2$$

$$s'^2-s''^2=(y'+m)^2-(y''+m)^2$$

$$s'-s''=\frac{(y'+m)^2-(y''+m)^2}{2\,s}$$

Having now obtained $s'+s''$ and $s'-s''$, it is easy to find s' and s''. Let i' i'' be the inclinations of the ropes at the points of support. The vertical forces at B' B'' are

$$V'=G\,s' \text{ and } V''=G\,s''$$

$$\text{Sin } i'=\frac{V'}{T'} \text{ and } \sin i''=\frac{V''}{T''} \quad . \quad . \quad (36)$$

The value of T'' being given above. Hence i' and i'' can be found,

$$\left.\begin{aligned} m&=\frac{s'}{\tan i'}=\frac{s''}{\tan i''}\\ x'&=m\sin i'\\ y'&=\frac{m}{\cos i'}-m \end{aligned}\right\} \quad . \quad . \quad (37)$$

From these values of x' and y' the position of the vertex of the curve can be found.

282. *Deflection for which the longitudinal tension is a minimum.*—From the equations $y=\frac{m}{\cos i}$ and $\sin i=\frac{x}{m}$ we get

$$y=\tfrac{1}{2}\,\frac{x^2}{m} \text{ nearly}$$

But

$$T=G\,(y+m)=G\left(y+\frac{x^2}{2\,y}\right) \text{ nearly.}$$

This will be a minimum for different values of the deflection, when,

$$\frac{dT}{dy}=1-\frac{x^2}{2y^2}=0$$

or when,

$$y=\frac{x}{\sqrt{2}}=0.7\,x$$

The tension increases to infinity for $y=0$ and for $y=\infty$.

283. *Tensions in the sloping wire rope.*—Let T'_2 T'_1 be the tensions on the tight and slack sides of the rope at the upper pulley, T''_2 and T''_1 those at the lower pulley,

$$\left.\begin{aligned} T'_2&=2P+C\\ T'_1&=P+C\\ T''_2&=2P+C-Gh\\ T''_1&=P+C-Gh \end{aligned}\right\} \quad \cdot \quad \cdot \quad (38)$$

where h is the difference of level of the pulleys.

284. *To draw the curve of the rope.*—In drawing the curve of the rope, which is often necessary to determine the space it will occupy, it is sufficiently accurate to substitute for the catenary curve a common parabola. Divide the abscissa O A and the ordinate A B, of any point, into an equal number of equal parts. Join o_1 o_2 o_3, and from 1′ 2′ 3′ draw verticals. These verticals will intersect the corresponding sloping lines, in points situated in a parabola.

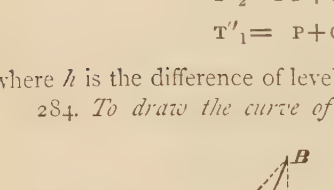


Fig. 287

285. *Approximate simpler equations.*—M. Achard has pointed out that a parabola may be substituted for the cate-

nary curve in calculating the tensions, without introducing any serious error. Suppose the two pulleys on the same level, then the lowest point of the curve is midway between them, and the tension at that point is

$$H=\frac{G\,l^2}{8\,v} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

G being the weight of the rope per unit of length; l its horizontal projection, which is approximately equal to the distance between the pulleys; and v the versed sine of the curve in which the rope hangs or the deflection of the rope at the centre of the span. From the property of the catenary curve described above, the difference of tension at any two points of the curve is equal to the weight of a portion of rope of length equal to the difference of level of the two points. Consequently the tensions at the ends of the rope will be

$$T=H+G\,v=\frac{G\,l^2}{8\,v}+G\,v \quad . \quad . \quad . \quad (2)$$

Now let T_2 and T_1 be the tensions at the ends of the rope in the driving and slack sides of the belt, and v_2, v_1, the corresponding deflections. From equation 17 we have already $T_2=2\,P$ and $T_1=P$. Putting these values in eq. 2 above, we get for the deflections,—

$$v_2=\frac{P}{G}\pm\sqrt{\left\{\frac{P^2}{G^2}-\frac{l^2}{8}\right\}} \quad . \quad . \quad (3)$$

$$v_1=\frac{P}{2\,G}\pm\sqrt{\left\{\frac{P^2}{4\,G^2}-\frac{l^2}{8}\right\}} \quad . \quad . \quad (4)$$

The deflection common to the two portions of the rope when not transmitting power is

$$v_0=\sqrt{\{\tfrac{1}{2}(v_1^2+v_2^2)\}} \quad . \quad . \quad . \quad (5)$$

These equations determine the deflections of the span; the driving force and the size of rope are given.

The greatest tension in the rope is

$$H_2+G\,v_2=\frac{G\,l^2}{8\,v_2}+G v_2. \quad . \quad . \quad . \quad (6)$$

Let $f_t - c$ be the greatest permissible working stress, from the table p. 415, and the values of the centrifugal stress on p. 416; then

$$(f_t - c)\frac{\pi}{4} v \delta^2 = \frac{G l^2}{8 v_2} + G v_2. \quad . \quad . \quad (7)$$

But $G = 3{\cdot}268\, v \delta^2$, and hence

$$f_t - c = 4{\cdot}16 \left(\frac{l^2}{8 v_2} + v_2 \right). \quad . \quad . \quad (7a)$$

This gives $f_t - c$, if l and v_2 are assumed, or conversely determines v_2 in terms of the stress, if $f_t - c$ is assumed. Commonly δ is so chosen that $f_t - c = 14{,}000$ lbs. per sq. in. Then

$$\frac{l^2}{8 v_2} + v_2 = 3364$$

$$v_2 = 1682 - \sqrt{\left(1682^2 - \frac{l^2}{8}\right)}. \quad . \quad . \quad (8)$$

This gives for

l =	420	500	600	feet
v_2 =	7	9	14	feet.

286. *Efficiency of wire rope transmission.*—The experiments of M. Ziegler on the transmissive machinery erected at Oberursel give for the efficiency of a single relay

$$\eta_1 = 0{\cdot}962.$$

Hence if there are m intermediate stations, the efficiency is approximately

$$\eta = \eta_1^{\frac{m+2}{2}} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

No. of intermediate stations	0	1	2	3	4	5
Efficiency	0·962	0·944	0·925	0·908	0·890	0·873
Proportion of work wasted in per cent.	3·8	5·6	7·5	9·2	11·0	12·7

287. *Pulleys for wire rope transmission.*—Wire ropes will not support without injury the lateral crushing which occurs when the rope rests against the sides of V-shaped grooves. Hence it is necessary to construct the pulleys with grooves so wide, that the rope rests on the rounded bottom of the pulley. It was found by Hirn that the wear of the rope was greatly diminished, and at the same time the frictional resistance to slipping was increased, by lining the bottom of the groove of the pulley with gutta-percha or wood. The gutta-percha is softened and hammered into the groove, which is dovetailed in section. The wood may be inserted in short blocks, through a lateral opening, which is afterwards covered by a metal plate.

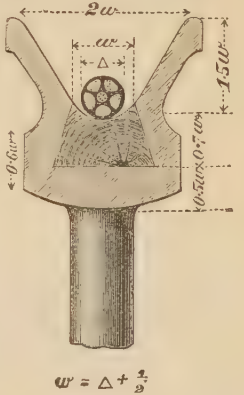


Fig. 288

More recently, leather has been found to succeed better than either wood or gutta-percha. The leather is cut into pieces the shape of the notch, and placed in it edge upwards. When these pieces are filled in all round, the pulley is placed in the lathe, and the bottom of the groove is turned to the section required. This lagging of leather lasts on the average three years.

Fig. 288 shows the section of a pulley rim. The unit for the proportional figures is $w = \Delta + \frac{1}{2}$, where Δ is the diameter of the rope.

The pulleys are often of cast iron, with cross-shaped arms, which may be calculated in the same way as the arms of toothed wheels. Sometimes they have oval curved arms like those of ordinary pulleys, and sometimes the arms are of round bar iron. These are cut to the right length and tinned at the ends. They are then placed in the sand mould, and the rim and nave cast round them. Such arms are usually

placed sloping in the plane of the axis of the pulley, the slope being alternately in opposite directions. The pulley is thus rendered rigid enough to resist accidental lateral forces.

It has already been proved (eq. 14) that the radius of the pulley must not be less than

$$R=\frac{2(f-f_t)}{\delta E}.$$

Or, when $f=25{,}000$, and the tension f_t, due to the work transmitted and the centrifugal force, does not exceed 8,000 lbs. per sq. in.,

$$R=900\ \delta \text{ nearly.}$$

The pulleys commonly used are 12 to 15 ft. diameter.

When the distance to which the power is transmitted is great, intermediate guide or supporting pulleys are introduced to lessen the deflection of the rope. The supporting pulleys for the tight side of the belt must be of the same size as the principal pulleys, those for the slack side may be smaller, in the ratio

$$\frac{R'}{R}=\frac{f-\frac{1}{2}f_t+\frac{3}{2}c}{f-f_t}$$

where f is the total stress in the rope, f_t the stress due to the longitudinal tension, including centrifugal force, c the stress due to centrifugal force.

The pulleys are supported on shafts which rest in pedestals on masonry piers or timber trestles.

The weights of the most ordinary sizes of pulleys employed, including their shafts, are on the average as follows (Achard):

Diameter	Weight in lbs.	
	Single pulley	Double pulley
18 ft. 0 in.	6,232	8,267
14 ,, 9 ,,	5,180	6,988
12 ,, 4 ,,	2,425	4,078
7 ,, 0 ,,	798	1,164

288. *Velocity of the rope.*—The rope is run at the highest safe velocity. That velocity is determined by the liability of the pulleys to burst, under the action of the centrifugal force, when the speed exceeds a certain limit. Let G be the weight of a cubic foot of cast iron, v the velocity of the pulley rim in feet per second, a its sectional area in square feet, R its radius in feet, and f its tensile strength in lbs. per sq. ft. The tension in the rim due to centrifugal force is,

$$\frac{G a v^2}{g}$$

The resistance of the rim is fa. Equating these,

$$v = \sqrt{\frac{gf}{G}}$$

Thus, putting $G = 450$ lbs., $f = 4500 \times 144$,

$$v = 215 \text{ ft. per sec.}$$

The actual speed is never as high as this, a larger margin of safety being necessary. Usually the speed of the rope is from 60 to 100 feet per second.

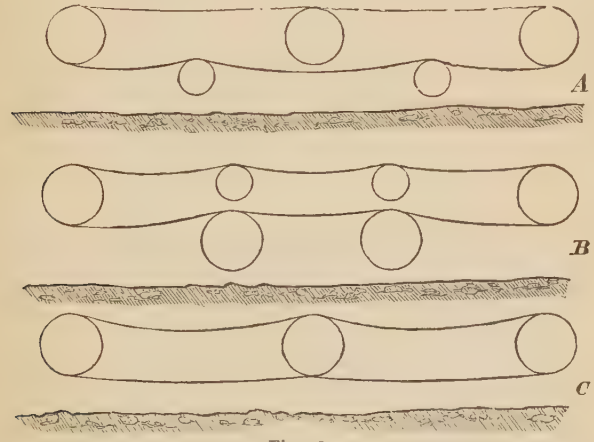


Fig. 289

Fig. 289 shows three arrangements of a wire-rope transmission.

In A and B guide or supporting pulleys are used. The upper part of the rope is the driving side in A, and the lower part in B; C is the arrangement adopted by Ziegler at Frankfort for transmitting 100 H P a distance of 984 metres.

289. *Duration of ropes.*—The ropes appear to last about a year. To preserve them from oxidation and improve their adhesion, they are coated with a mixture of grease and resin applied hot.

The following table contains data taken from a paper by Achard :[1]—

Wire Rope Transmission

Locality	Rope			Pulleys		H.P. transmitted	Total distance		Velocity of belt ft. per sec.
	Diam. Δ ins.	Diam. of wires δ in ins.	No. of wires ν	Diam. 2 R in ft.	Distance apart in ft.		Horiz.	Vert.	
Oberursel	0·59	·06	36	12·3	394	94	3,153	146	73·8
Schauffhausen [2]	0·95	·072	80	14·75	333 to 456	326	1,997	...	61·87
Fribourg	0·97	·070	90	14·75	502	300	2,510	269	65

290. *Change of direction of the rope.*—When the direction of a rope requires to be changed either at a right angle or otherwise, two plans may be adopted. A horizontal pulley may be used, in which case the pulley must have the same diameter as the other pulleys used in order that the bending stress may not be increased. More commonly, however, two vertical guide pulleys, in the required directions in plan, are connected by bevil gearing. The splitting of the power

[1] 'Annales des Mines,' viii. p. 229; 'Proc. Inst. of Civil Engineers,' xlv. p. 267.

[2] There are two cables. If one breaks, the other is capable of transmitting the power.

transmitted to different points of application may be effected in the same way.

Vertical rope.—In the case of a vertical rope the initial tension on the lower pulley due to the weight of the rope would vanish, and that on the upper pulley would generally be insufficient unless special devices were used for producing the initial tension of the rope. Then tension or tightening pulleys may be used, like those for leather or rope belts.

Stretching of ropes.—In course of time the ropes stretch, and especially in summer sag so much that they become incapable of transmitting the required power. This may be remedied by resplicing the ropes, but this must be avoided as long as possible because it injures the ropes. Of late a mode of laterally compressing and stretching the ropes before use has been adopted which diminishes the stretching while working. According to Stahl the simplest way of neutralising the stretching which occurs in the working of the ropes, is to increase a little the diameters of the pulleys by nailing wood strips to the material filling the bottom of the groove. Poplar or willow is used in pieces $1\frac{1}{4}$ inches thick and 45 to 70 inches in length. They are half cut through on one side with saw cuts, and steeped in water for two days to render them flexible. They are then nailed to the groove filling by wrought nails, long enough to pass through it and clinch themselves against the iron.

Cost of wire rope transmission.—The cost is estimated in France at about £330 per mile, exclusive of the terminal stations. These cost about £1 per H. P. transmitted.

Lateral swaying of the ropes.—When a rope transmission is running well there should be little lateral swaying of the ropes, except the unavoidable motion produced by the wind. If swaying occurs it may be due to the pulleys being unbalanced or untrue, or to their not being in the plane of the rope; or it may be due to the pulley filling, or the rope being too much worn, or to bad splicing of the rope.

The following short table, abbreviated from one calcu-

lated by Roebling and given by Stahl, may serve as a general guide to the size of ropes and amount of power transmitted by wire ropes :—

Power Transmitted by Wire Ropes.

Diam. of Rope	Diam. of Pulleys	No. of Revs. per min.	Breaking Strength of Rope	Horses Power Transmitted	Velocity of Belt in ft. per sec.
ins.	ft.		lbs.		
$\frac{7}{16}$	5	100	4260	8·6	26
$\frac{15}{32}$	6	100	5660	13·4	31
$\frac{1}{2}$	7	100	8200	21·1	36
$\frac{5}{8}$	8	100	11600	27·5	42
$\frac{5}{8}$	8	120	11600	33·0	50
$\frac{5}{8}$	9	100	11600	51·9	47
$\frac{5}{8}$	9	120	11600	62·2	56
$\frac{11}{16}$	10	100	15200	73·0	52
$\frac{11}{16}$	10	120	15200	87·6	62
$\frac{11}{16}$	10	140	15200	102·2	73
$\frac{11}{16}$	12	100	15200	116·7	63
$\frac{3}{4}$	12	120	17600	148·9	75
$\frac{3}{4}$	12	140	17600	173·7	87
$\frac{3}{4}$	14	100	17600	185·0	73
$\frac{3}{4}$	14	120	17600	222·0	87
$\frac{3}{4}$	15	120	17600	300·0	94

The ropes have each 42 wires.

CHAPTER XIV

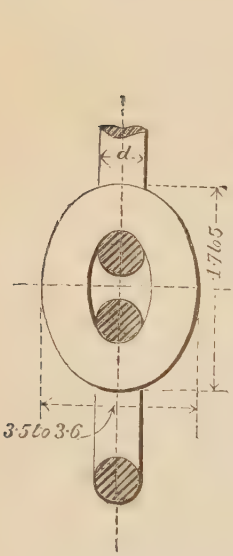
CHAINS AND GEARING CHAINS

291. Chains are used both as flexible transmitters of energy, like belts, and as simple fastenings. As, however, the two modes of use are more or less connected, it is convenient to treat chains generally without respect to their special application. Chains may be divided into :

Round iron chains, open links.
„ „ stud links.
Flat bar chains.
Gearing chains.

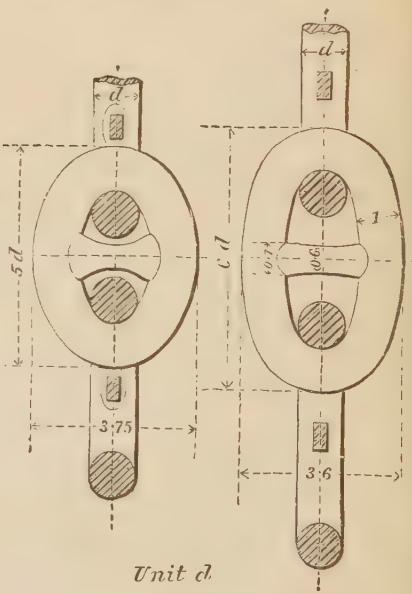
Round iron chains are those most commonly used, and they are forged out of round iron bars of the best quality. When a tension is applied to such a chain, each link is subjected to a bending action additional to the tension, the bending being greatest at the extremities of the longer diameter of the link. Hence, on purely theoretical grounds the link should be stronger at the ends of the link. On the other hand, it would involve excessive expense to vary the section of the link, and the question of the best theoretical section is complicated by the uncertainty as to the strength at the weld. Welds in bars have been found to be from 10 to 40, or on the average 20, per cent. weaker than the bars themselves. Further, chains are constantly used on pulleys, and may then be subjected to bending action, perpendicular to the plane of the link. In designing a chain the links should be as small as possible, (1) because the

greater the number of links in a given length the more flexible is the chain; (2) because the less the transverse dimensions of the link the less is the bending action. The inside radius at the ends of the link must be a little greater than the radius of the iron of which the chain is made. Let d be the diameter of the iron. Then $2{\cdot}6d$ and $1{\cdot}5d$ are about the smallest possible internal diameters of the chain link, and $4{\cdot}6d$ and $3{\cdot}5d$ the least outside diameters.



Unit d.

Fig. 290



Unit d

Fig. 291

292. Very common proportions for ordinary chains are given in fig. 290. Such chain is termed close-link chain. Cheaper but weaker chain is made with longer links. Such chain may have the inside diameters $1{\cdot}5d$ and $4d$, and consequently the outside diameters $3{\cdot}5d$ and $6d$. In fig. 291, two forms of studded chain are shown. The stud resists the

tendency of the link to collapse, and renders the chain less liable to kink. The proportions of studded chain cables differ a good deal. The following are probably extreme proportions :—

Length of link, outside . .	$5d$ to $6\frac{1}{2}d$
„ „ inside . . .	$3d$ „ $4\frac{1}{2}d$
Width of link, outside. . .	$3\frac{1}{2}d$ „ $3\frac{3}{4}d$
„ „ inside . . .	$1\frac{1}{2}d$ „ $1\frac{3}{4}d$
Stud, diameter at ends . .	$0{\cdot}7d$ „ d
„ „ centre . .	$0{\cdot}6d$

The end links of a length of cable are usually made of iron of $1{\cdot}2d$ in diameter ; they are a little larger than the other links, say $6\frac{1}{2}d$ in length and $4{\cdot}1d$ in width outside. Chain cable is often made in lengths of $12\frac{1}{2}$ or 25 fathoms. The lengths are joined by swivels and shackles.

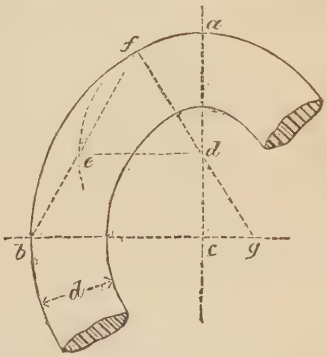


Fig. 292

A convenient method of drawing the elliptical form of chain links is shown in fig. 292. Set off $a\ c$, $c\ b$, the semi-diameters. Take the radius $d\ a$ a little greater than $1{\cdot}5d$, and draw the circular curve $a\ f\ e$ for the end of the link. Draw $d\ e$ parallel to $c\ b$. Join $b\ e$ and produce it to meet the arc in f. Join $f\ d$ and produce it to meet the smaller diameter of the link in g. Then g will be the centre for the arc $f\ b$, and $g\ b$ or $g\ f$ will be its radius.

293. *Strength of chains.*—The strength of the iron of which chains are made is about 21 tons per sq. in. for large sizes (2 ins. diameter), to 25 tons per sq. in. for small sizes ($\frac{1}{2}$ inch diameter). Deducting one-fifth for the reduction of strength due to the weld, and to the presence of bending

stress, the ultimate strength of chain might be expected to be from $16\frac{1}{2}$ to 20 tons per sq. in., the strength being greater for the smaller sizes. Experiments at Woolwich in 1842–3 showed that studded chain cable broke with a mean tension of 15·9 tons per sq. in., and crane chain (two sizes only were tested) with a mean tension of $17\frac{1}{2}$ tons per sq. in. The reduction of strength of the chain compared with the bar iron of which it is manufactured is partly due to the welds and partly to the bending stress.

The Admiralty rule for the proof stress of studded chain cables is—

$$\text{Test load in tons} = 18d^2,$$

corresponding to a stress of $11\frac{1}{2}$ tons per sq in. of section. For close-link crane chains without studs,

$$\text{Test load in tons} = 12d^2,$$

corresponding to a stress of 7·7 tons per sq. in. of section.

294. *Working stress for chains.*—It is difficult to give definite rules for the working stress of chains because the circumstances in which they are used vary so greatly. Looking to the fact that they are always liable to be subjected to some bending action, the working stress is often stated at half the proof load. This gives for the greatest load P in tons on a chain of iron, d inches in diameter,—

$$P = 9d^2 \text{ for studded link chain}$$
$$= 6d^2 \text{ for unstudded close link chain,}$$

corresponding to working stresses of 5·7 and 3·8 tons per sq. in. For much-used chain, subject frequently to the maximum load, it is better to limit the stress to $3\frac{1}{4}$ tons per sq. in. Then

$$P = 5d^2 \text{ tons.}$$

Towne gives loads for ordinary crane chains which require a large margin of security, which agree with the rule—

$$P = 3{\cdot}3\ d^2 \text{ tons.}$$

This corresponds to a stress of about 2 tons per sq. in.

Strength and Weight of Close-Link Crane Chains, and Size of Equivalent Hemp Cable

Diameter of iron *d* in inches	Weight of chain per fathom	Breaking strength in tons	Testing load in tons	Girth of equivalent rope in inches	Weight of rope in lbs. per fathom
$\frac{1}{4}$	3·5	1·9	0·75	2	$1\frac{3}{8}$
$\frac{5}{16}$	6·0	3·0	1·10	$2\frac{1}{2}$	$1\frac{1}{2}$
$\frac{3}{8}$	8·5	4·3	1·6	$3\frac{1}{4}$	$2\frac{3}{4}$
$\frac{7}{16}$	11·0	5·9	2·3	4	$3\frac{3}{4}$
$\frac{1}{2}$	14·0	7·7	3·0	$4\frac{3}{4}$	5
$\frac{9}{16}$	18·0	9·7	3·8	$5\frac{1}{2}$	7
$\frac{5}{8}$	24·0	12·0	4·6	$6\frac{1}{4}$	$8\frac{1}{2}$
$\frac{11}{16}$	28·0	14·6	5·6	7	$10\frac{1}{2}$
$\frac{3}{4}$	31·5	17·3	6·8	$7\frac{1}{2}$	12
$\frac{13}{16}$	37·0	20·4	7·9	$8\frac{1}{4}$	15
$\frac{7}{8}$	44·0	23·1	9·1	9	$17\frac{1}{2}$
$\frac{15}{16}$	50·0	26·1	10·5	$9\frac{1}{2}$	$19\frac{1}{2}$
1	56·0	29·3	12·0	10	22
$1\frac{1}{8}$	71·0	36·3	15·3	$11\frac{1}{4}$	$27\frac{3}{4}$
$1\frac{1}{4}$	87·5	44·1	18·8	$12\frac{1}{2}$	$34\frac{1}{2}$
$1\frac{3}{8}$	105·8	52·8	22·6	$13\frac{3}{4}$	$41\frac{1}{2}$
$1\frac{1}{2}$	126·0	62·3	27·0	15	$49\frac{1}{2}$

Strength and Weight of Studded Link Cable

Diameter of iron *d* in inches	Weight in lbs. per fathom	Breaking strength in tons	Test load in tons	Girth of equivalent rope in inches	Weight of rope in lbs. per fathom
$\frac{5}{8}$	24	9·5	7	$6\frac{1}{2}$	9
$\frac{11}{16}$	28	11·4	$8\frac{1}{2}$	$7\frac{1}{2}$	12
$\frac{3}{4}$	32	13·5	$10\frac{1}{2}$	8	14
$\frac{7}{8}$	44	20·4	$13\frac{3}{4}$	$9\frac{1}{2}$	$19\frac{1}{2}$
1	58	24·3	18	$10\frac{1}{2}$	$22\frac{1}{2}$
$1\frac{1}{8}$	72	29·5	$2\frac{3}{4}$	12	$30\frac{3}{4}$
$1\frac{1}{4}$	90	38·5	$28\frac{1}{2}$	$13\frac{1}{2}$	$39\frac{1}{2}$
$1\frac{3}{8}$	110	48·5	34	15	$48\frac{1}{4}$
$1\frac{1}{2}$	125	59·5	$40\frac{1}{2}$	16	55
$1\frac{5}{8}$	145	66·5	$47\frac{1}{2}$	17	62
$1\frac{3}{4}$	170	74·1	$55\frac{1}{8}$	18	$68\frac{1}{4}$
$1\frac{7}{8}$	195	92·9	$63\frac{1}{4}$	20	86
2	230	99·5	72	22	104
$2\frac{1}{8}$	256	112	$81\frac{1}{2}$	24	124
$2\frac{1}{4}$	285	126	$91\frac{1}{8}$	26	145

Mr. Towne gives the following as the safe working loads for crane chains [1] :—

Diameter of iron	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{9}{32}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{13}{16}$
Load on chain in tons	·06	·25	·5	·75	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	4	5

295. *Weight of chains.*—The weight of chains in lbs. per fathom (of six feet) is :

$$w = 54d^2 \text{ to } 58d^2.$$

The stowage room required for chains is about $35d^2$ cubic feet for each 100 fathoms.

In all the above equations d is to be taken in inches.

296. *Chain barrels.*—Sometimes a chain is coiled completely on to a chain barrel. Such barrels should have a spiral groove just wide enough to receive the edges of the links, and so deep that the alternate links lie flat on the cylindrical part of the barrel. The diameter of a chain barrel should be at least $24d$ to $30d$, and is usually determined with reference to the amount of chain to be coiled on it. In good machinery the barrel should take the chain in one layer. If one layer is coiled on another the chain is injured.

Pocketed chain wheels.—Some chain wheels are made with pockets which fit the chain. On such wheels the chain passing over half or three-quarters of a circumference is so gripped that it will not slip, and the loose end may be run off on to the ground. Such wheels are narrow and moderate in diameter, and remain in the line of the pulling part of the chain. They occupy much less space, and injure the chain less than a chain barrel. The pitch of the pockets in the chain wheel must agree exactly with the pitch of the chain. The best material for the chain wheel is soft cast-iron. To insure the engagement of the chain and wheel a chain guide is provided. Fig. 293 shows a pocketed chain wheel and guide.[2] A is the chain wheel mounted on

[1] 'Treatise on Cranes.' Towne, 1883.

[2] Taken from Towne on 'Cranes.'

the frame B. C is the chain guide surrounding the lower half of the chain wheel, and bolted to B. Its inner side is

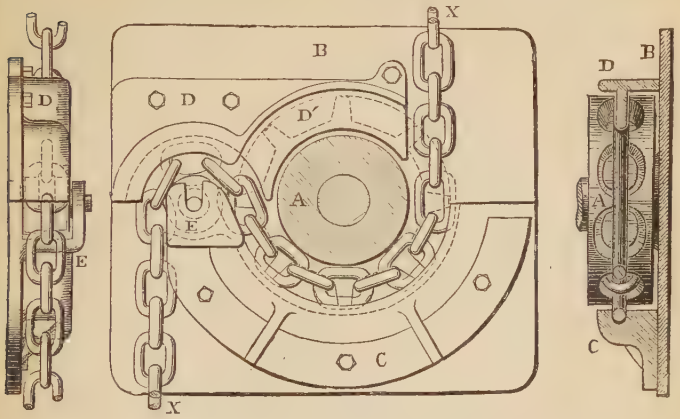


Fig. 293

grooved, and only a small space is left between it and the chain wheel. At E the chain guide carries a small roller, over which the chain passes down to a suitable receptacle. D is a chain stripper.

297. *Straining action on a crane hook.*—Consider a section of the crane hook through *o a b*. Let P be the load supported by the hook, and let *l* be the distance to a line *c d* through the centre of figure of the section *a b*. Let y_1 y_2 be the distances from *c d* to the extended and compressed edges of the section.

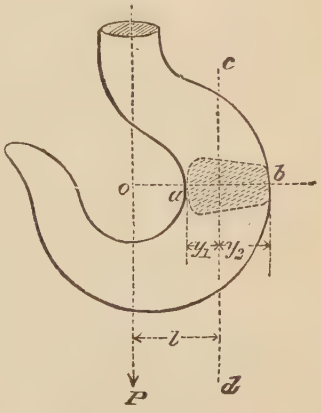


Fig 294

Then the section $a\,b$ is subject to the action of a bending moment $P\,l$, and a tension P (see Case I., § 43, page 89). Let A be the area of the section $a\,b$, and I its moment of inertia about the axis $c\,d$. Then the stress at b is

$$f_2 = P\left\{\frac{1}{A} - \frac{ly_2}{I}\right\}$$

and the stress at a is

$$f_1 = P\left\{\frac{1}{A} + \frac{ly_1}{I}\right\}$$

298. *Proportions of crane hook.*—A crane hook is most often required to receive a rope sling. The opening of the hook should therefore have a width c=the diameter of the

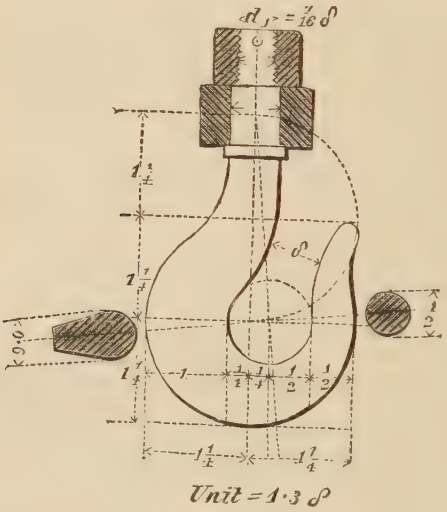


Fig. 295

rope. The inside of the hook should have a broad rounded surface which will not injure the rope; and the section of the metal of the hook should be as deep as convenient in those parts where the bending is greatest. Fig. 295 gives proportions for such a hook. The hook is forged

of round bar flattened, as shown in the section on the left, to deepen the section where the bending is greatest. Taking δ as the opening of the hook, or diameter of the rope sling, the unit for the proportional numbers is 1·3 δ.

The diameter at the bottom of the screw thread (of external diameter d^1) which supports the hook is $\frac{3}{8}\,\delta$ very nearly. Taking the limiting stress at 3 tons per sq. in., the hook will support a load

$$P=\frac{\pi}{4}\left(\tfrac{3}{8}\,\delta\right)^2\times 3=0{\cdot}33\delta^2 \text{ tons.}$$

If, then, P is given instead of δ, we may design the hook from the value

$$\delta=1{\cdot}74\sqrt{P},$$

where δ is in inches and P in tons.

It is often more convenient to take as the unit of proportions of a crane hook the diameter of the iron out of which the hook is forged. For the minimum diameter of the hook at any section subjected simply to the tension P, we may take

$$d_1=0{\cdot}75\sqrt{P},$$

which corresponds to a stress of about $2\frac{1}{4}$ tons per sq. in. But the iron out of which the crane hook is made may be

$$d=\sqrt{P}, \text{ and hence}$$
$$P=d^2 \text{ tons},$$

and d will be taken as the unit in the figures 296, 297. Fig. 297 accords closely with proportions given by Mr. Towne for ordinary sizes of

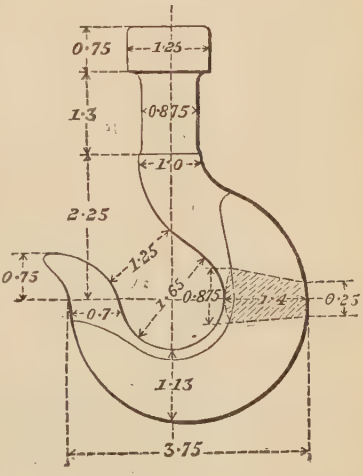


Fig. 296

crane hooks. But the mode of stating the proportions is different and less likely to lead to error in extreme cases. These proportions of Mr. Towne are based on large experience, and are no doubt convenient and practical.

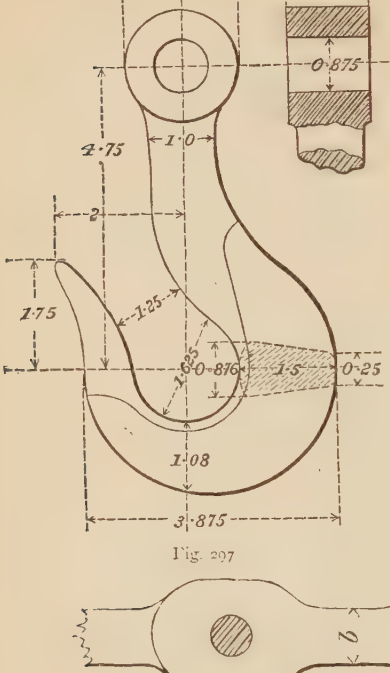


Fig. 297

299. *Plate-link chains.* — Chains of this kind are used when a great load is to be supported, as in the case of suspension bridges, patent slips, &c., or as gearing chains when the work transmitted is very heavy, as in the turning gear of cranes, dredger chains, &c.

Fig. 298

They are constructed with short or long links, according to the amount of flexibility required.

Let d be the diameter of the pin in the eyes of the links. Then the shortest convenient length of link is about 2·9 d. On the other hand, links are sometimes made 24 feet in length. Fig. 298 shows a simple flat-link chain with one and two links alternately, the double links being half the thickness of the single link.

Let b be the width of the link, δ the thickness of the single link, d the diameter of the pin, P the load on the chain. Then the stress on the link is given by the equation :

$$f_t b \delta = P \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The shearing stress on the pin is given by the equation :

$$\frac{\pi}{4} d^2 f_s = \tfrac{1}{2} P \quad . \quad . \quad . \quad . \quad . \quad (2)$$

And the bending stress on the pin is given by the equation:

$$\tfrac{3}{8} P \delta = f \frac{\pi}{32} d^3 \quad . \quad . \quad . \quad . \quad (3)$$

Suppose $f_t = 5$ tons per sq. in.; $f_s = 4$ tons per sq. in.; and $f = 5$ tons per sq. in. Then the equations become :

$$b\delta = 0{\cdot}2P \quad . \quad . \quad . \quad . \quad . \quad (1a)$$
$$d^2 = 0{\cdot}159P \quad . \quad . \quad . \quad . \quad (2a)$$
$$d^3 = 0{\cdot}764P\delta \quad . \quad . \quad . \quad . \quad (3a)$$

Equation (3a) will in general give a greater value of d than (2a). Equations 1a and 3a give

$$d^3 = 3{\cdot}82\, b\, \delta^2.$$

Experiment shows that if d is less than $\frac{2}{3} b$, the link crushes in the eye and is weakened. Hence if

$$\delta < 0{\cdot}2785\, b,$$

d must be taken not less than $\frac{2}{3} b$, and its bending resistance will be in excess.

Fig. 299 shows the forms of link eyes found by experiment to be strongest. A is the form arrived at by Mr. G.

Berkley, and B that arrived at by Sir C. Fox. If links are short, they will not generally be made of the most economical form. They should then have a form which includes the shape here indicated.

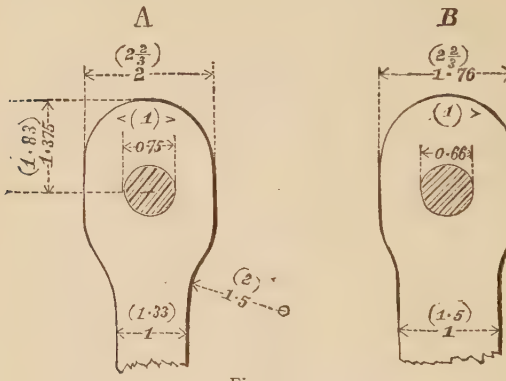


Fig. 299

Three modes of fastening the pin in flat-link chains are shown in fig. 300. At *a*, the pin is simply riveted over the outside link. At *b*, a washer-plate is interposed, which secures greater freedom of motion in the links. At *c*, a

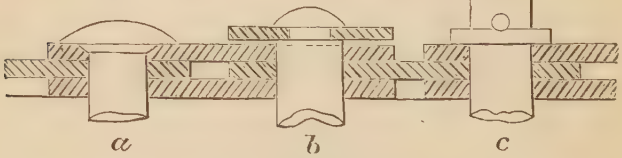


Fig. 300

washer and split pin are used ; and this probably would be the best plan but for the possibility of the split pin falling out when the chain is in use.

Gearing Chains

300. In cases where a considerable amount of work has to be transmitted between two shafts at a slow speed, the

tension in a flexible transmitter may easily be much greater than ordinary belts can sustain. In such cases metal chains may be used, so formed that the links fit into the projections of toothed wheels on the shafts. There can then be no slipping of the belt on the toothed wheels; and as the chains may have almost any strength, an extremely great force can be exerted through the chain. Such chains are used on a small scale in some forms of reaping machinery, and on a large scale in cranes and lifting gear. The chains carrying the buckets of large dredgers act in the same way. Such chains forming a class of transmitting organs intermediate between belting and gearing, are termed gearing chains or pitch chains. The chief objection to their use is, that however well they fit the toothed wheels at first, they are liable from stretching and wear to become of slightly greater pitch than the toothed wheel, and they then work very badly. To obviate this as far as possible, the links should be short.

The commonest form of such a gearing chain is a flat-linked chain, having two systems of links spaced some distance apart. The toothed wheel acts in the space between the two systems of links, and the teeth gear with the pins passing through the link ends. A mode of construction in some respects better than this is to divide the toothed wheel into two parts, between which the chain is placed. The teeth of the wheel then gear with the alternate link ends on each side. With a chain of this kind, however, a larger toothed pulley is required than for the ordinary form. Long-linked chains are sometimes used on a polygonal pulley without teeth. The polygonal pitch line of such a pulley has sides equal to the lengths of the links between the centres of the pins; and it has usually five or six sides. With four sides the twisting moment is too variable and the motion too irregular.

Fig. 301 shows the ordinary form of a flat-link gearing chain and its toothed pulley. The side views also show

chains with one set of links in each system and with two sets.

301. *Strength of gearing chains.*—Let 2 T be the total tension on the loaded span of the chain, and $2i$ the whole number of links in the width of the chain. Then the tension in each half of the chain, having i links, is T. Let δ be the thickness, b the breadth of a link, and d the diameter

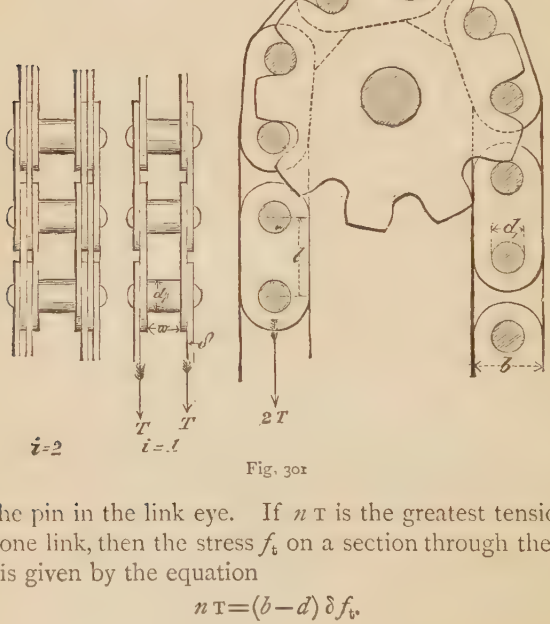


Fig. 301

of the pin in the link eye. If n T is the greatest tension in any one link, then the stress f_t on a section through the link eye is given by the equation

$$n\,\mathrm{T}=(b-d)\,\delta f_t.$$

Usually $b=2{\cdot}5\,d$, and then

$$n\,\mathrm{T}=1{\cdot}5\,d\delta f_t \quad . \quad . \quad . \quad (1)$$

The average bearing pressure of the pin in the link eye, f_c is given by the equation,

$$n\,\mathrm{T}=d\delta f_c \quad . \quad . \quad . \quad . \quad (2)$$

The pins are subjected to bending action which will increase with the total stress T and with the thickness δ of the links. Hence, whatever the distribution of the tension T amongst the links, the greatest bending moment on the pin will be

$$M = m\,T\,\delta,$$

and the stress due to bending will be given by the equation

$$M = m\,T\,\delta = \frac{\pi}{32} d^3 f \quad . \quad . \quad . \quad (3)$$

where f is the greatest intensity of bending stress, and m a coefficient depending on the distribution of the tension in the links.

Consider a part of the chain hanging freely between the toothed pulleys. For such a portion of chain the links may be assumed to carry equal portions of the load, and $n = \frac{1}{i}$. Supposing the tension in each link to act at the centre of the link, we get for the greatest bending moment on the pin

$$M = \frac{T\,\delta}{i}\left(\tfrac{1}{2} - \tfrac{3}{2} + \tfrac{5}{2} - \ldots - 4\frac{i-1}{2}\right)$$
$$= T\,\delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence, equations 1, 3 and 4 give

$$1{\cdot}5\,d\,\delta\,i f_t = \frac{\pi}{32}\frac{d^3}{\delta} f;$$

and supposing the stress in the link and pin equal, and taken at $f = 10{,}000$ lbs. per sq. in., we get

$$\frac{\delta}{d} = \frac{{\cdot}256}{\sqrt{i}}$$

$$d = 0{\cdot}01614\sqrt{\frac{T}{\sqrt{i}}} \quad . \quad . \quad (5)$$

and if $f = 14{,}000$

$$d = 0{\cdot}01365\sqrt{\frac{T}{\sqrt{i}}} \quad . \quad . \quad (5a)$$

Prof. K. Keller[1] has pointed out that the links in gear with the toothed pulley are subjected to a very different straining action. Consider the link just leaving the driven or coming into gear with the driving pulley on the loaded side of the chain, and suppose for the present that the whole tension 2 T in the chain acts through the pin A on the first tooth of the wheel. Then if t_1 t_2 t_3 are the tensions in the links,

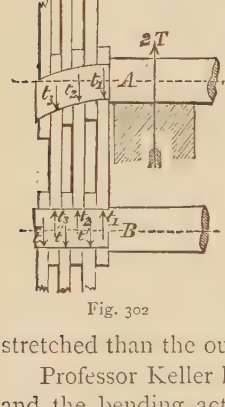


Fig. 302

$$t_1+t_2+t_3 \quad . \quad . \quad . \quad . \quad =T;$$

under the action of the tensions the pin A will bend, and its flexure will be much greater than that of the second pin B in the free part of the chain, which is subjected to two sets of tensions in opposite directions. Consequently the inside links will be more stretched than the outside links, and $t_1 > t_2 > t_3$ . . . .

Professor Keller has worked out the stresses in the links and the bending action on the pin on the assumptions: (1) That the whole tension comes on the first tooth of the wheel; (2) that the bending of the second pin B may be neglected. If we put Δ_1 Δ_2 Δ_3 for the deflections of the pin at the centres of the links, l for the length of the link when unstrained, ω for the section of a link, λ_1 λ_2 λ_3 for the elongations of the links; then

$$\lambda_1 = t_1 \frac{l}{\omega E}; \quad \lambda_2 = t_2 \frac{l}{\omega E} \quad . \quad . \quad . \quad .$$

$$\lambda_1+\Delta_1=\lambda_2+\Delta_2=\lambda_3+\Delta_3 \quad . \quad . \quad . \quad .$$

equations which, with the known equation for the neutral axis of a bent bar, suffice to determine the tensions t_1 t_2 t_3, and the bending moment on the pin. The calculation is too long to be given here, but it leads to the result that if

[1] Zeitschrift des Oester. Ing. Vereins, 1878.

the links are numerous, the outermost links may happen to be in compression instead of in tension, and thus do not add anything to the strength of the chain, but on the contrary weaken it. In designing the chain the condition may be introduced that the tension in the outermost link should not fall below $\frac{1}{10}$ T. This condition fixes a relation between d and δ. Using this condition Prof. Keller gets:

for $i=1$	$n\,T=T$		
$=2$	$=\cdot9\,T$	instead of $\cdot5\,T$	for equal distribution of the tension;
$=3$	$=\cdot55\,T$	$\cdot33\,T$	
$=4$	$=\cdot38\,T$	$\cdot25\,T$	

and for the greatest bending moment on the pin:

for $i=1$	$M=1\cdot5\,T\,\delta$		
$=2$	$1\cdot7\,T\,\delta$	instead of $2\cdot5\,T\,\delta$	for equal distribution of the tension.
$=3$	$2\cdot6\,T\,\delta$	$3\cdot5\,T\,\delta$	
$=4$	$3\cdot6\,T\,\delta$	$4\cdot5\,T\,\delta$	

Taking $\omega=2\cdot5\,d\,\delta$

$$l=2\cdot9\,d$$

The value required for δ to satisfy the condition above is

$i=$	2	3	4
$\frac{d}{\delta}=$	2·85	4·90	7·1

and the values of m and n in the equations above are:

$i=$	1	2	3	4
$m=$	1·5	1·7	2·6	3·6
$n=$	1·0	0·9	0·55	0·38

Comparing the results of the application of these rules with experience in the use of large gearing chains, Prof. Keller is led to the conclusion that the whole tension 2 T is not carried by a single tooth of the pulley, but must be distributed to two teeth. He assumes that the one tooth takes $\frac{2}{3}$ and the other tooth takes $\frac{1}{3}$ of the whole tension. If this correction is made $\frac{T}{3}$ must be substituted for T in cal-

culating the bending moment on the pin. The correction, however, only requires to be made if $i=4$ or more. For less values of i the bending moment on the pin is greater in the free part of the chain than on that in gear with the pulley. The following table gives the proportions of gearing chains calculated by Prof. Keller's rules. The general proportions are :

Breadth of link	$b=2{\cdot}5\,d$
Thickness of middle part of pin . . .	$d_1=1{\cdot}2\,d$
Width of thickened part of pin . . .	$w=1{\cdot}7\,d+0{\cdot}2$
Width of eye of link between hole and end of link	$=0{\cdot}85\,d$
Length of link, centre to centre of pins .	$=2{\cdot}9\,d$

Table of Proportions of Flat-Link Gearing Chains (Keller)
Dimensions in inches

Load on chain 2 T lbs.	No. of links $2i$	Pin diameter at ends d	Pin diameter at centre d_1	Length of middle part of pin w	Link thickness δ	Link breadth b	Link length l	Tooth thickness $a=l-d_1$ $=1{\cdot}7\,d$
220	2	0·16	0·20	0·43	0·060	·39	·55	0·35
550	2	0·26	0·32	0·67	0·098	·65	·83	0·51
1100	2	0·37	0·47	0·83	0·118	·93	1·10	0·65
1650	2	0·45	0·55	0·94	0·158	1·12	1·42	0·75
2200	4	0·49	0·59	1·02	0·158	1 24	1·42	0·83
3300	4	0·61	0·75	1·22	0·177	1·52	1·77	1·04
4400	4	0·69	0·83	1·38	0·216	1·79	2·01	1·18
5500	4	0·85	1·02	1·61	0·256	2·11	2·44	1·42
8820	6	0·85	1·02	1·61	0·177	2·11	2·44	1·42
12100	6	0·94	1·14	1·81	0·197	2·36	2 76	1·61
16500	6	1·08	1·30	2·05	0·216	2·70	3·15	1·85
22050	6	1·26	1·54	2·32	0·256	3·15	3·66	2·14
33070	8	1·44	1·73	2·60	0·197	3·60	4·17	2·44
44100	8	1·65	1·97	3·00	0·236	4·13	4·80	2·72
55100	8	1·85	2·24	3·31	0·256	4·61	5·35	3·13
66100	8	2·03	2·44	3·62	0·295	5·05	5·87	3·43

302. *Recent use of pitch chains for transmitting power.*— Some years since flat link pitch chains came into extensive use in the driving gear of tricycles, and electrical engineers have also adopted it to gear down from the motor to the first motion shaft in electric tramcars. A high velocity ratio is obtained in this way more simply than by any other method except worm gear. When run constantly at high speed, the ordinary pitch chain proves to be defective from the small bearing surface of the pins on the links. This difficulty has been overcome very ingeniously by Mr. Hans Renold, who invented the form of pitch chain shown in

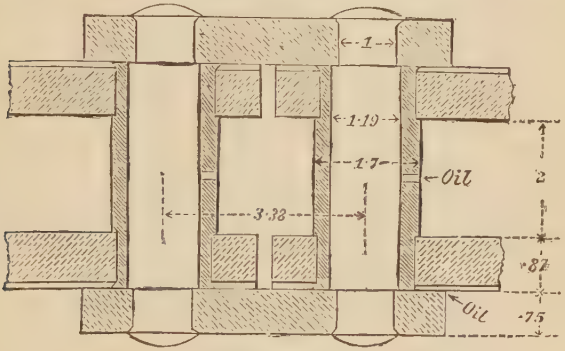


Fig. 303

fig. 303. A tube is riveted between the pair of inside links and this is traversed by a pin riveted to the outside links. The pin gets a bearing surface on the whole length of the tube. In the case of a tramcar on the Bessbrook and Newry Tramway [1] the motor is 20 H.P. and the power developed sometimes exceeds this. To transmit this when the car is running at 7 miles an hour the tension on the chain is 1,430 lbs., and the speed 460 feet per min. At starting the tensions may reach 3,400 lbs., and the maximum speed may reach 1,300 feet per min.

[1] Hopkinson, 'Proc. Inst. Civil Engineers,' vol. xci. p. 206.

A steel chain is used of the pattern shown in fig. 303, with the tubes keyed and riveted to the inner links and the pins keyed and riveted to the outer links. The tenacity of the steel is 43½ tons per sq. in. Pitch of chain 2⅛ ins. Weight 8·5 lbs. per foot. The breaking stresses are 13·8 tons (shear) on the pins; 10·2 tons on outside links, and 14·8 tons on inside links. The wearing surface is 16 sq. ins. per foot of chain.

303. *Toothed pulley.*—A gearing chain runs on a pulley the pitch line of which is strictly a polygon. Usually it is desirable to make the pulley as small as possible, and the minimum number of teeth adopted in practice is about as given in the following table:—

2 T not greater than		1,000	7 teeth
,,	,,	4,000	8 ,,
,,	,,	18,000	9 ,,
,,	,,	70,000	10 ,,

If l is the length of the links from centre to centre of pins, z the number of teeth in the wheel, R the radius of the wheel,

$$l = 2 \text{ R} \sin \frac{\pi}{z}$$

$$\text{R} = \frac{l}{2 \sin \frac{\pi}{z}}.$$

The diameter of the part of the pin which acts on the toothed wheel may be $d_1 = 1{\cdot}2\, d$. (§ 301.)

304. *Form of teeth of wheel.*—As the chain unwinds from the wheel, the centre of the pin in the chain describes the involute of the pitch circle. Let $a\,b$, fig. 304, be the pitch line of the wheel, $a\,c$ that of the chain. Draw the circle $d\,d$ at ⅔ the intended height of the wheel tooth, cutting $a\,c$ in e. Take $a\,g = \frac{1}{4}\, a\,e$. Then a circle $h\,e\,k$ struck from g with radius $g\,e$ will sensibly coincide with an involute of the pitch line, and a circle $l\,m$ struck from g touching the circumference of the

pin will be a suitable form for the wheel tooth. The part of the tooth below the pitch line may be a circle struck from *h* with radius equal to the radius of the pin. As, however, gearing chains are not quite so exact as toothed wheels, it will be better to take a circle for the tooth form such as *l m'* lying a little within the curve *l m*, and a circle *l n'* for the bottom of the tooth lying a little outside of *l n*. If the wheel teeth are tenoned into the wheel, the parts below the pitch line may be tangents to *l m*.

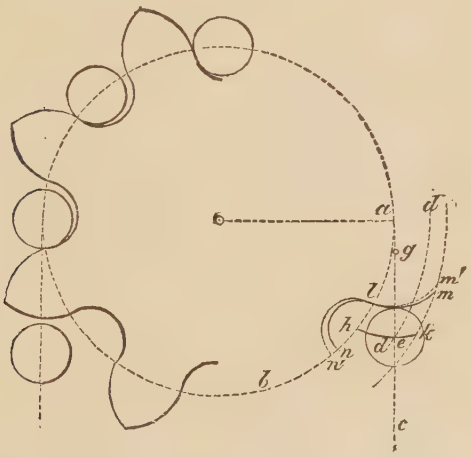


Fig. 304

Karl von Ott, comparing the weight, cost, and strength of the three materials, hemp rope, wire rope, and chains, arrives at the conclusion that for equivalent strength the cost is proportional to the numbers 2 : 1 : 3. That is, hemp rope is twice as costly and chains three times as costly as wire rope of the same tensional strength.

The following rules are added for comparison with the rules for chains. They give the weight and strength of hemp ropes and wire ropes used for supporting a simple

tension Let G = weight of rope in lbs. per fathom; γ = girth of rope in inches; Δ = diameter of rope in inches; P = breaking weight of rope in tons,

$$G = \alpha \gamma^2 = \beta \Delta^2$$

	$\alpha =$	$\beta =$
Tarred hemp ropes . . .	0·25	2·47
White ,, ,, . . .	,,	,,
Iron wire ropes	0·785	8·6
Steel ,, ,, ...	,,	,,

$$P = \kappa \gamma^2 = n \Delta^2$$

	$\kappa =$	$n =$
Tarred hemp ropes . . .	·36 to ·42	3·5 to 4·2
White ,, ,, . . .	·49 to ·69	4·8 to 6·7
Iron wire ropes	1·75	17·28
Steel ,, ,, ...	3·00	29·61

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